

Breaking Down Problem Solving for All Students

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Introduction

When I first began teaching, I worked with the lowest level seventh and eighth graders at a Title I school. Their problem solving skills were low, but I expected this. I attributed their low problem solving skills to a lack of computation skills, a general dislike for math, and a lack of motivation to do well in school. I spent that year creating graphic organizers, scaffolding every concept, and doing whatever I could to help them through the math curriculum. I realized that children who are weak in math computation, and school in general, may have weak mathematical problem solving skills.

I spent my second year of teaching working with sixth grade English Language Learners (ELLs). They too had weak problem solving skills, but I attributed this to their language deficits and overall struggles of being ELLs adjusting to a new culture and school system. I again spent the entire year implementing strategies that would help them to get through the curriculum. I ended that year still believing that children who are weak at math computation have weak mathematical problem solving skills, but I also added ELLs to the list of students who struggle with problem solving.

In my third year of teaching, I moved from teaching ELLs to teaching two mainstream classes and providing special education push-in support for two classes. I expected my special education students to have some problem solving deficits due to their classified learning disabilities in the areas of reading comprehension and mathematics, but I expected my mainstream students to be just fine. As I had expected, my special education students had some difficulties with problem solving, but what surprised me was that my mainstream students had the same difficulties. Some of them even had more difficulties than the special education students. At the end of this year I had a new belief. I started to believe that the majority of middle school students struggle with mathematical problem solving. The first two months of this school year have strengthened this belief.

This year we have moved to the Inclusion model, and I have a mix of ELLs, special education, and mainstream students in my three math classes. It has been the perfect opportunity for me to see if my belief that the majority of middle school students struggle with problem solving is true. Throughout these past two months I have observed and listened to my students as they adjusted to middle school and tried to make their way through a new math curriculum. I have paid very close attention to their conversations while they work in pairs or small groups, and I have noted several comments that seem to

be made over and over. These comments include: “I don’t understand how to start this problem.” “What is this problem asking me for?” “I am confused.” “I don’t even know where to begin. “Why does this problem have so many words and not a lot of numbers?” From these conversations and my previous teaching experience, I have decided that my belief is most certainly true. I have also decided to take my opportunity with DTI to design a unit that focuses on mathematical problem solving strategies so that I can begin to help my students become better problem solvers.

The objectives of this unit are for students to learn an acronym (MATCH) for a problem solving process that consists of five steps. These steps were adapted from the four-step model listed below, and they include:

1. Make Sense of the Problem
2. Adopt a Strategy/Make a Plan
3. Try Your Strategy and Plan
4. Check your Work
5. Have a Back-Up Plan

Additionally, students will learn specific problem solving strategies that can help them to navigate through the sixth grade Common Core mathematics curriculum. These strategies include making a drawing or diagram, looking for a pattern, constructing a table, and working backward.

Rationale

I decided on these objectives after I began to read a book over the summer titled *Helping Children Learn Mathematics*¹. This book dives into the importance of teaching students problem solving skills, and it breaks it down for teachers. I was drawn to many points in the book, but I was particularly drawn to the presentation of the four-step model of problem solving:

1. First, understand the problem
2. Second, devise a plan for solving it
3. Third, carry out your plan
4. Fourth, look back to examine the solution obtained²

I was immediately drawn to this simple four-step process because I think that it is something that can easily be turned into an acronym that my students can relate to and remember. I have seen similar strategies, such as RACE (restate the question, answer the question, cite evidence, and explain your evidence) work so well for students in ELA that I am confident that a similar strategy will help my students to become better problem solvers.

My idea to create a unit on the process of problem solving was also strengthened when we read *How to Solve It*³ by Polya throughout our seminar. This book also focuses on using steps to solve problems and helping students to identify these steps and use them when working with difficult problems. Additionally, our seminars led to thoughtful discussions with teachers from many different grade levels who all agree that a model for problem solving is helpful for students of all ages. We also agree that teaching specific problem solving strategies is beneficial for students, particularly special education students and ELL students who need extra scaffolding and modeling to be successful.

In addition to the four-step model listed above, *Helping Children Learn Mathematics* also focuses on particular problem solving strategies to teach to your students. In reading about these strategies and thinking about the sixth grade curriculum, I have decided to focus my unit on teaching four of these strategies:

1. Make a drawing or diagram
2. Look for a pattern
3. Construct a table
4. Work backward⁴

Since the sixth grade Common Core State Standards for Mathematics (CCSSM) have a large focus on ratio relationships, fraction and decimal relationships, and linear relationships, I feel that these four strategies, in addition to the MATCH acronym that focuses on a five-step problem solving process, will help my students to “attack” challenging problems. The CCSSM also focuses on students “persevering” through difficult problems, and I believe that these strategies will help my students to be much more successful in this strand.⁵

Background Information of the School and Students

Conrad Schools of Science is a mathematics and science magnet school in the Red Clay Consolidated School District that houses students in grades 6-12. It is considered to be an urban school located in the city of Wilmington. Conrad has recently reached its full student capacity of approximately twelve hundred students after adding a grade and many more students and teachers each year. In addition to being a choice school, Conrad is also a Red Clay feeder school for English Language Learners (ELLs) of various Hispanic ethnicities.

In previous years, all ELLs at Conrad took the majority of their classes together with a small group of ELL teachers who taught the same subject to the three middle school grade levels. The only exceptions to this schedule were the students’ exploratory classes in which they took these with mainstream students and teachers. This year, however, Conrad began the inclusion process of ELLs into the mainstream classroom. This process is being piloted with the sixth grade class. Since I am a sixth grade teacher, I now teach

three classes which serve half of the sixth grade population. I am both special education and ELL certified so I have the majority of the special education and ELL students split amongst my three classes. This design makes for very diverse groups of learners with varying ability and language levels.

Overview

When I first began the DTI seminar, I thought that I was going to create a unit that focused on content and embedded problem solving strategies. I decided to change my focus because the Red Clay school district uses the *Connected Math* curriculum. The *Connected Math* curriculum is a very rich curriculum, which like recommended by NCTM in the book *Teaching Mathematics through Problem Solving*⁶, aims to teach mathematics through the presentation of real-world problems that embed content. While I think that the *Connected Math* curriculum is great, my students will tell you otherwise.

My students struggle greatly with the *Connected Math* curriculum. They often become so frustrated in trying to solve the given problems that they miss the content that is actually being taught through the problem. In turn, I frequently need to supplement the curriculum with specific notes and other materials that gives them a step-by-step guide to the content that was supposed to be presented through real-world problems. Every time I have to do this, I feel like I am taking a giant step backwards because instead of learning the content through problem solving, my students are learning the content through direct instruction. I want to give my students a process and a toolbox for attacking these problems so that they are able to learn the content in a more meaningful way. That is where this unit comes in to play.

This unit focuses on a five-step process for problem solving that I adapted from the four-step processes that I found in the books *Helping Children Learn Mathematics* and *How to Solve It: a new aspect of mathematical method*. The five-step process will use the acronym MATCH which stands for:

- M-Make Sense of the Problem
- A-Adopt a Strategy/Make a Plan
- T-Try Your Strategy and Plan
- C-Check Your Work
- H-Have a Back-Up Plan

My unit also focuses on teaching four problem solving strategies that are applicable to the sixth grade curriculum. These strategies are: make a drawing or diagram, look for a pattern, construct a table, and work backward. My plan is to teach this unit at the start of the school year so that my students can take what they learn in this unit and apply it to the *Connected Mathematics* curriculum. I want to teach them a series of mini-lessons that focus on each of the steps in the MATCH process and the four specific problem-solving

strategies. My goal is to combine the mini-lessons with activities, such as discussions, solving problems in pairs or groups, sorts, etc. that help my students to implement the MATCH process and to develop an understanding of what problem solving strategies are best to use with different types of problems. I think that by giving them a process for problem solving and a tool box of problem solving strategies, I will be able to help close the gap in their lack of problem-solving skills so that they can learn more mathematics through problem solving instead of learning mathematics through direct instruction.

In order to understand the purpose of the unit, it is important to gain a deeper understanding of the steps of problem solving and the problem solving strategies themselves. I also feel that it is important to understand the teaching strategies that I plan to implement throughout the unit.

Steps of Problem Solving

Understanding the Problem (Make Sense of the Problem)

Polya lists the first step in problem solving as “Understanding the Problem.” It sounds simple enough, right? But what does understanding the problem really entail? It entails comprehending the written or verbal language of the problem and identifying the key parts of the problem which include the unknown, the data, and the condition. It is important for students to pick out these key parts and to attend to them and refer back to them while working through the problem. Without a true understanding of the problem, students will also lack interest in solving. If students do not desire to solve the problem, positive learning is difficult to attain.⁷

Devising a Plan (Adopt a Strategy/Make a Plan)

Polya states “Devising a Plan” as the second step in the problem solving process. Devising a plan requires much thought, and coming up with a plan can be a very challenging process. A plan requires an outline that includes the computations or calculations that are required to get to the unknown that was found through the first step of understanding the problem. Students will have great difficulty devising a plan if they have not correctly identified the key parts of the problem which are listed above. Even students who fully understand the given problem may have to adjust their plan multiple times to develop an outline that focuses on the correct calculations and computations needed to solve a given problem. It is critical for students to understand that plans can change and that they need to be flexible in order to develop a plan that works for a given problem.⁸

Carrying Out the Plan (Try Your Strategy and Plan)

The third step listed by Polya is “Carrying out the Plan.” To an outsider, this step may seem simple. Some may think that you can just take your outline and complete the computations or calculations listed. Although this is part of the process, it is important to note that precision to detail is extremely important in this step. It is particularly important because if part of the plan is flawed, students need to be able to go back and target which piece or pieces of the plan need to be fixed. In addition, it is important for students to understand that carrying out their plan is like completing a proof. They have to be convinced that the solutions obtained by carrying out their plan are correct and that they truly answer the unknown obtained in step one, Understanding the Problem.⁹

Looking Back (Check Your Work)

The final step in Polya’s process is “Looking Back.” Polya states that even higher-level students often skip this step as many students are satisfied after they have carried out their plans and obtained a solution. It is important for teachers to push students to go back and examine their work. Looking back accomplishes many things, among them being finding errors in methods and calculations, further developing the ability to solve similar problems, and looking at the problem through another angle to obtain the same solution using different calculations and strategies. More importantly, looking back allows students to make connections with other problems that they have worked on which helps them to further understand content and techniques for solving problems related to that content.¹⁰

Have a Back-Up Plan

Although this step is not described in Polya’s process, I felt that it would be a great addition to the problem solving process that I am going to teach my students because I think that it forces them to try another strategy when their original strategy does not work. Often times, my students will check their work after using a strategy, and if their work is wrong, they go back to the same strategy but try doing different calculations. Having a back-up plan will force students to think about solving the problem using a different strategy as opposed to trying the same strategy over and over.

Problem Solving Strategies

Make a Drawing or Diagram

At some point in our lives, we have most likely used a diagram or a picture to solve a real-world problem. I do it at the start of every year to try and figure out a way to arrange the desks and tables in my classroom to maximize space. According to Reys, Suydam, and Lindquist, teaching this strategy to your students is important because it “provides a way of depicting the information in a problem to make the relationships apparent.” By

drawing a diagram or picture, students are able to visualize the information presented to them. It is important to stress to students that the details of the drawing are not important when teaching this strategy. For example, if students are to draw a ferris wheel, you should stress that the details, such as the spindles, cars, and other details of the ferris wheel, are not essential in helping them to solve the problem. Additionally, you can reverse this strategy at times and provide students with a drawing or diagram and ask them to create a problem that relates to what is presented so that they are able to apply this strategy in a different, yet still meaningful, way.¹¹

Making a drawing or diagram will help my students tremendously when they work in the *Prime Time* unit. *Prime Time* focuses on common factors and multiples, and students are asked to solve real-world problems that involve calculating factors and multiples. For example, students are asked to calculate when two ferris wheels traveling at different speeds will meet at the bottom and to determine the greatest number of children they can invite to their birthday party if they have a certain number of party favors and they want each party bag to be fair. Constructing drawings or diagrams is a very useful strategy in helping students to visualize these problems and calculate solutions.

Look for a Pattern

Children work with patterns often in the early elementary grades and throughout elementary school. They are usually asked to visually identify a pattern that is presented through images or numbers. As students enter middle school, it is important to extend the teaching of patterns to include finding patterns presented in word problems and higher-level thinking problems. In *Helping Children Learn Mathematics*, the authors emphasize that it is important for teachers to model for students that looking for patterns in more difficult problems is an “active search.” It is important to show them that drawing diagrams, pictures, and tables can help them to identify patterns in a problem that can be used to solve for the unknown.¹²

This strategy will be helpful for my students when they work in the *Bits and Pieces I, II, and III* units. As part of these units, students are asked to discover the algorithms for adding, subtracting, multiplying, and dividing fractions. By looking for a pattern when solving problems involving these operations, students can identify the steps of solving operations with fractions problems in order to develop their own algorithms for solving.

Construct a Table

According to Reys, Suydam, and Lindquist, “organizing data into a table helps children to discover a pattern and to identify information that is missing.” By teaching students to use a table to problem solve, you provide them with a tool that allows them to organize and analyze information and data. Data organization and analysis is a crucial skill to teach your students because it helps them to recognize patterns and operations necessary

to solve difficult problems. It also gives them a concrete model to reference as they work their way through the problem solving process. Constructing tables is a problem solving strategy that can be used through high school and college level math courses as data organization and analysis is an integral part of higher –level mathematics. To this day, I make tables often when solving challenging math problems.¹³

In thinking about the sixth grade curriculum, I think that this strategy will be particularly meaningful to my students when they work in the *Models You Can Count On* unit which focuses on ratios and ratio relationships. Ratio tables are extremely helpful in solving ratio problems and helping students to see the relationships among equivalent ratios. Additionally, constructing tables will be a useful strategy for my students when they work in the *Variables and Patterns* unit that focuses on linear relationships. The tables will be particularly useful when the students are asked to graph linear relationships and to determine linear equations.

Work Backward

Often, students are given the solution to a problem and are asked to find the original conditions. In order to solve problems like these, students need to be taught how to work backward. By teaching this process, students will be able to take an endpoint and work their way back to determine the unknown. This is a great real-life tool as well since we often know what we want our end result to be in many situations, but we need to determine where to start in order to get there. Working backward can also allow students to take the original conditions of a problem and the solution of a problem and work from the beginning and the end until they meet in the middle.¹⁴

Working backward will be a perfect strategy for my students to use when they work in the *Covering and Surrounding* unit. During this unit, students are asked to formulate equations for calculating the area and perimeter of rectangles, triangles, parallelograms, and circles in which they can utilize the look for a pattern strategy. After formulating the equations, students are asked to apply their skills to find dimensions of different shapes with given areas or perimeters. In these problems, students will be able to utilize the working backward strategy because they are starting with the end result and trying to find the beginning dimensions.

Teaching Strategies

Acronyms

Throughout my special education and ELL methods courses, we discussed the use of acronyms as a useful strategy to help struggling students to remember important processes. An acronym is a combination of letters that are used to help promote the memory of steps or information. Familiar acronyms include PEMDAS (please excuse my

dear Aunt Sally) for memorizing the order of operations and Roy G Biv (red, orange, yellow, green, blue, indigo, and violet) for memorizing the colors of a rainbow. By implementing an acronym for problem solving, students can more easily remember and refer back to the steps of problem solving so that they are able to apply these steps to become more successful problem solvers.¹⁵

Cooperative Learning

Cooperative learning is an integral part of any classroom, but I feel that it particularly important in a math classroom. Cooperative learning is the grouping of students into either heterogeneous or homogeneous pairs or small groups. These pairings and groupings allow them to work together to strengthen thinking and learning. In math, cooperative learning is particularly useful because it allows students to both analyze each other's thinking and to work together to find solutions to difficult problems.

Cornell Notes

I attended an AVID training during the summer of 2012. During this training I was introduced to Cornell Notes. Since then, I have been using Cornell notes as part of my classroom. I adapted the note taking system to include the following sections: key points/examples, key words, summary, and questions. My students have had great success with this method of note taking, particularly my special education and ELL students. Using this type of notes also allows me to make connections to other areas of my classroom, such as the word wall. All of the words that are listed as key words in our notes become part of the classroom word wall. Additionally, the summary section forces students to synthesize the information presented, and the question section forces students to reflect on the information and discover any gaps that they may have in their learning. They can then further reflect on their learning in their math journals and make connections to their notes in their journal reflections.

Think, Pair, Share

Think, pair, share is a strategy in which students complete three simple steps. First, they think about an answer to a given question or prompt. Second, they pair up with another student. Third, they share their answers with the student that they are paired with. I find this strategy to be particularly useful with my special education and ELL students because it gives them an opportunity to share their thoughts with one student instead of having to share in front of the entire class. I find that my special education and ELL students are often intimidated to share their answers with the entire class because they have a fear of being incorrect or they are not confident enough in their language abilities to speak in front of a classroom filled with native speakers. Think, pair, share takes the stress of sharing away for these students and provides them with an opportunity to discuss their thoughts and analyze the thoughts of others.

Graphic Organizers

Graphic organizers are an integral tool in any classroom. Many middle school students are familiar with graphic organizers in writing, but they are less familiar with graphic organizers in math. According to an article by Alan Zollman, “a graphic organizer is an instructional tool students can use to organize and structure information and concepts and to promote thinking about relationships between concepts.” He also states the importance of using graphic organizers in mathematics and explains that they help students to make connection between concepts, organize information, and classify given details of a problem as either essential or non-essential in obtaining the solution. Additionally, graphic organizers are extremely helpful for special education and ELL students. All throughout my undergraduate and graduate degree programs, we discussed the importance of using graphic organizers with special education and ELL students because they provide them with a place to organize information and see connections to previously learned material.¹⁶

Journals

I have been making a conscious effort to promote the use of math journals in my classroom. I find the journals to be extremely beneficial for students as they provide a place for them to express their successes and frustrations. They also provide a place for them to explain what they need more help with without feeling embarrassed to let me know that they are struggling in front of the entire class. By implementing the journals at the start of the year during this unit, I feel that it will force them to be part of our classroom routine which is what I have been trying to establish for the past couple of years.

Differentiated Instruction

With the varying ability and language levels of the students in my classroom, it is crucial for me to implement differentiated instruction. Differentiated instruction takes into account the variety of ways that students learn. It also takes into account the level of understanding that students have of a particular concept. In order to create groupings to provide differentiated instruction, I will implement a variety of formative assessment strategies such as thumbs up-thumbs down, exit tickets, teacher observation, and pre-assessments. By providing students with differentiated instruction within this unit, I will be able to ensure that all students are able to learn the content so that they will be able to apply the problem solving steps and strategies learned to the problems and content presented throughout the year.

Classroom Activities

Objectives

The classroom activities for this unit are based on the following objectives:

1. Students will be able to persevere through difficult problems by implementing a five-step process for problem solving
2. Students will be able to choose and apply appropriate problem solving strategies to solve problems
3. Students will be able to compare and contrast problem solving strategies

Essential Questions

Students will focus on the following essential questions when working through this problem solving unit:

1. How does a five-step system of problem solving help me to make sense of and solve problems?
2. Which problem solving strategies are best for solving particular types of problems?
3. How can I push through difficult problems when I have hit a “road block?”

Activities

Activity One

Materials Needed: Student journals, graphic organizers (Appendix B)

During this activity, students will be asked to complete a journal entry using the following prompt: Think of at least two times in your life when you needed to implement problem solving. Describe the problem solving process that you used, and keep in mind the following questions as you answer: How did you solve your problem? How did you decide the steps to take to solve your problem? Did you need to revise your plan? Following the journal entry, students should pair up with another student and discuss their entries. After discussing, students should complete the graphic organizer in pairs and share out ideas with the class after completion.

Activities Two-Ten

Materials Needed: Student notebooks, Cornell notes template (Appendix C), MATCH graphic organizer (Appendix D), math problems (Appendix E)

During these mini-lessons, students will be presented with each of the five steps of the MATCH acronym and each of the four problem solving strategies described above.

These steps and strategies will be presented one at a time on separate days. Students will take notes on these steps and strategies using the Cornell notes template. After each set of notes on the MATCH acronym, students will be asked to complete the section of their MATCH graphic organizer that corresponds to the step that they learned. Additionally, students will be asked to apply the step or problem solving strategy by completing part of a math problem. These problems may be completed individually, in pairs, or in small groups.

Activity Eleven

Materials Needed: Math problems (Appendix E), sort graphic organizer (Appendix F)

After students have completed the mini-lessons listed above, they will complete a sort activity in which they are presented with math problems and asked to sort them into categories based on which problem solving strategy they feel would be best to use to solve the problem. The sort will be completed in pairs or small groups, and there will be a class discussion following completion in which students will be asked to explain where they put each problem and why.

Activity Twelve

Materials Needed: Math problems (Appendix E), computers, MATCH Graphic organizer (Appendix D), project rubric

Following the sort, students will be asked to choose a problem from the sort to complete a presentation on how to solve the problem using the MATCH problem solving process. Students should work in pairs or small groups to create a presentation in the form of a power point, video, podcast, or any other creative form which shows how to solve the problem using each step of the MATCH process. Students will present these to the class and will be assessed using a rubric. The rubric should be created as a class so that students are involved in the process and understand the expectations.

Conclusion

As a result of the research and readings done inside and outside of my seminar, I truly believe that teaching this unit to my students at the start of the year will help them to become better problem-solvers. It is my hope that through the MATCH process my students will gain the skills that they need to be able to learn through problem-solving as opposed to learning through direct instruction. I am confident in this unit, and I believe that it will be able to help my students, as well as the students of my fellow teacher colleagues, to reach new heights in mathematics!

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Appendix A

CCSS.Math.Content.6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

CCSS.Math.Content.6.RP.A.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

CCSS.Math.Content.6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express $36 + 8$ as $4(9 + 2)$.*

CCSS.Math.Content.6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

CCSS.Math.Content.6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

The common core standards listed above are covered in this unit by giving students strategies that can be used to solve the types of problems covered in each standard. These problem types include ratios and proportions, common factors and multiples, writing and solving equations, and finding the area and perimeter of complex shapes. The standard of mathematical problem solving is addressed in this unit through the use of the MATCH strategy which will give students a method of problem-solving that will help them to persevere through the increasingly more difficult sixth grade curriculum.

Appendix B

| Problem/Scenario | How did you figure out what was needed to be able to solve your problem? | What strategies did you use to solve? How did you choose these? | What happened when you implemented your strategies? | Looking back on your process, how could you improve your strategies? | Did your original plan work or did you need to implement another plan? |
|------------------|--|---|---|--|--|
| | | | | | |
| | | | | | |

Appendix C

| Date | Title |
|------------|----------|
| Key Words: | Notes: |
| | Summary: |

Appendix D

| Letter | What does this letter stand for? | How can I apply this step when solving a problem? |
|--------|----------------------------------|---|
| M | | |
| A | | |
| T | | |
| C | | |
| H | | |

Appendix E

1. A large ferris wheel makes one revolution every 60 seconds. A small ferris wheel makes one revolution every 30 seconds. After how many seconds will the two ferris wheels meet back at the bottom? How many times has each ferris wheel gone around?
2. How many hours would it take to drive 90,000 miles if 30 miles were driven every 15 minutes?
3. How many different ways can you make change for a quarter?
4. Jack paid \$10 to join the Game Stop club. If he pays \$5 to rent a video, find the total cost of renting 0-10 videos.
5. The perimeter of a square is 28 cm. How long is each side? What is the area of the square?
6. You enter an elevator on the ground floor. You go up 5 floors, down 4 floors, up 9 floors, down 3 floors, down 2 more floors, then up 6 floors. On what floor do you get off the elevator?
7. Joey is making party bags to give to his guests at his birthday party. He has 30 party horns, 15 stickers, and 75 pieces of candy. He wants all of the bags to be the same, and he does not want anything to be leftover. What is the greatest number of bags he can make? How many of each item will be in the bag?
8. If two whole numbers have a sum of 14 and a product of 48, what are the numbers?
9. Fairy Tale Town has a population of 3,000 people. The population doubles every 20 years. What will the population be in 20 years? 40 years? 60 years? When will the population be over a million people?
10. If it takes 3 hours to write 6 pages, how long will it take to write 52 pages?
11. The area of a rectangle is 60 in. The perimeter is 38 inches. What are the dimensions of the rectangle?
12. A plane has 12 rows of seats with 3 seats in each row and 12 rows of seats with 2 seats in each row. If there are 4 seats with broken seatbelts, how many tickets can be sold for a flight?

13. The ratio of blue to yellow marbles in a jar is 5:3. There are 15 blue marbles in the jar. How many yellow marbles will need to be added to the jar to make the ratio of blue to yellow 1:1?

14. A baker makes cookies in packages of 3 and packages of 4. If he baked 31 cookies, how many of each type of package did he make?

Appendix F

| Make a Table | Look for a Pattern | Draw a Picture | Work Backwards |
|---------------------|---------------------------|-----------------------|-----------------------|
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- ¹ Reys, Suydam, and Lindquist, *Helping Children Learn Mathematics*.
 - ² Reys, Suydam, and Lindquist, *Helping Children Learn Mathematics*, 32.
 - ³ Polya, *How to Solve it; a New Aspect of Mathematical Method*.
 - ⁴ Reys, Suydam, and Lindquist, *Helping Children Learn Mathematics*, 35-37.
 - ⁵ "Common Core State Standards for Mathematics."
 - ⁶ Schoen, *Teaching Mathematics through Problem Solving: Grades 6-12*, 42.
 - ⁷ Polya, *How to Solve it; a New Aspect of Mathematical Method*, 6-7.
 - ⁸ Polya, *How to Solve it; a New Aspect of Mathematical Method*, 8-12.
 - ⁹ Polya, *How to Solve it; a New Aspect of Mathematical Method*, 12-14.
 - ¹⁰ Polya, *How to Solve it; a New Aspect of Mathematical Method*, 14-19.
 - ¹¹ Reys, Suydam, and Lindquist, *Helping Children Learn Mathematics*, 35.
 - ¹² Reys, Suydam, and Lindquist, *Helping Children Learn Mathematics*, 35.
 - ¹³ Reys, Suydam, and Lindquist, *Helping Children Learn Mathematics*, 35-36.
 - ¹⁴ Reys, Suydam, and Lindquist, *Helping Children Learn Mathematics*, 37.
 - ¹⁵ "Study Guides and Strategies."
 - ¹⁶ Zollman, "Students Use Graphic Organizers to Improve Mathematical Problem-Solving Communications."

Curriculum Unit
Title

Breaking Down Problem Solving for All Students

Author
Mandy Reis

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Understand a five-step process for problem-solving, adopt and use specific problem-solving strategies, apply the problem-solving process and strategies to the *Connected Math* curriculum used to teach the sixth grade CCSSM

ESSENTIAL QUESTION(S) for the UNIT

1. How does a five-step system of problem solving help me to make sense of and solve problems?
2. Which problem solving strategies are best for solving particular types of problems?
3. How can I push through difficult problems when I have hit a “road block?”

CONCEPT A

CONCEPT B

CONCEPT C

MATCH Problem-Solving Process

Problem-Solving Strategies

Persevering through Difficult Problems

ESSENTIAL QUESTIONS A

ESSENTIAL QUESTIONS B

ESSENTIAL QUESTIONS C

How does a five-step system of problem solving help me to make sense of and solve problems?

Which problem solving strategies are best for solving particular types of problems?

How can I push through difficult problems when I have hit a “road block?”

VOCABULARY A

VOCABULARY A

VOCABULARY A

MATCH

Table, pattern, problem-solving strategies

Perseverance

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Materials needed: problem sets, graphic organizers, Cornell notes templates, journals