Teaching Geometry through Investigation and Modeling

Kathleen G. Gormley

Introduction

What is the best way to teach geometry to third grade students? What a great question! Some basic premises should be addressed. Students who are engaged and who can make personal connections to their learning develop a better understanding of the presented concepts. Instruction that is presented in a variety of ways and that addresses the wide variety of learning styles is effective. Manipulating physical models is essential. Each of these premises will be utilized in my unit. Students will create models of shapes, manipulate these models to investigate a variety of concepts, including symmetry, congruence, perimeter, area, and more.

When I began this venture, my perception of geometry focused mainly on lines, angles, and shapes. As I have moved forward, my view has broadened to realize that geometry instruction should include so much more. Geometry can incorporate number sense and number operations, data gathering, as well as reasoning. All of these skills and concepts will be introduced and expanded upon as we move toward a real-life application of geometry.

Vocabulary acquisition is essential to ensure success and using accurate and specific vocabulary will prevent misconceptions at the onset. Furthermore, research suggests that vocabulary acquisitions improves not only reading achievement, but is also central to the development of new conceptual frameworks and is crucial to the understanding of more sophisticated ideas. Knowledge and achievement is directly related to vocabulary development. Therefore it is imperative to provide instruction on geometric vocabulary and provide multiple opportunities of exposure to students in order to solidify learning. This unit will begin with introducing students to famous mathematicians throughout history with an emphasis on the accomplishments in the field of geometry. In one of our weekly seminars, my seminar leader, Cristina, started telling us about the life and discoveries of Evariste Galois and how he was killed in a duel for questionable reasons. This sparked my curiosity and I went home and looked up more information about him, now I do not truly understand what “to solve an equation in radicals” means but this story prompted me to want to learn more as I hope it will for my students and it is an excellent way to create cross-curricular connections! Then we will move toward investigating basic definitions, explanations, and models of geometric concepts. Students will investigate shapes and their attributes, they will sort polygons based on these attributes. According to the Common Core State Standards, students should be able to “reason with shapes and their attributes. Recognizing rhombuses, rectangles, and squares as examples of

quadrilaterals and draw examples of quadrilaterals that do not belong to any of the sub categories." Students will use geo-boards to construct polygons, classify the polygons, and then justify the classifications. These investigations will lead us into discussions involving symmetry and congruence. The models created on the geo-boards will provide visual affirmation of these concepts.

As students investigate the shapes, we will begin to experiment with the concepts of perimeter and area. Again referring to the Common Core Standards, students will "solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different area or with the same area and different perimeter." As students begin to understand the concept of area, we will relate area to addition and multiplication.

Another facet of this unit of study will deal with the partitioning of shapes into parts with equal areas. This will develop the concepts of fractions to my students. Students will determine the area of each part as a unit fraction of the whole. Again, students will use geo-boards to build models and then partition the models and prove that the area of the parts will equal the area of the whole.

This scaffolding of concepts will help to insure students are building their understanding on a solid base. The use of predicting and building of models will enable students to see and manipulate the concepts before moving on. Students will work in a variety of formats, including independently, pairs, and groups. Each step of the way, students will participate in discussions explaining the many different strategies used to solve the problems.

Demographics

The Red Clay Consolidated School District is located in Northern New Castle County, Delaware with a combination of urban and suburban settings. Some of its elementary schools are located in the heart of the largest city in the state. The district is comprised of 28 schools with approximately 1000 teachers. It services over 16,000 students. Of those students, 27% are African American, 4% are Asian, 20% are Hispanic, and 49% are White. Students' needs vary, with almost 15% receiving Special Education Services and 10% receiving English Language support. In addition, 41% of the students come from families with low incomes.

Highlands Elementary is an urban school in the city of Wilmington, Delaware. We are a small K-5 school with an enrollment of an average of 320 students. Our minority population represents 86% of our student body with 81% of the students falling into the low socio-economic status. I am a third grade teacher with a class size varying between 24-28 students which is representational of the make-up of the school.

Geometers

In an attempt to develop a cross-curricular unit, reading passages detailing the life of several famous geometers will be included and students will be able to try to unlock the mysteries of their findings themselves. It is easy to see how students will enjoy unlocking the patterns of Pascal's triangle or proving Gauss' explanation of adding numbers in a series. Don't just read the information, have students play with the concepts, reason with them, prove them!

Thales (THAY-leez) c636-c546 BC

Thales is recognized as one of the first Greek philosophers and scientists. He used deductive methods in his mathematical discoveries. One of his more widely known formulas was developed on a trip that he took to Egypt. He wanted to know the height of the pyramids and no one at the time was able to figure this out. As Thales stood in the sun-filled location, he used deductive reasoning to find the solution to his problem. He saw his shadow and realized he knew several facts that would aid him. He knew his height, he knew the height of his shadow, and finally he knew the height of the pyramid's (Height of Thales)/(Height of Thales^' shadow)=(height of pyramid (unknown))/(height of pyramid^' s shadow). Thales constantly was asking "How" and "Why" when he came upon a problem that needed solving. Every time we observe a pattern and predict an outcome, we follow his example.

Euclid (YOO-klid) c333-275 BC

Considered one of the greatest mathematicians of antiquity. Euclid realized how difficult it was to teach mathematics with the textbooks available at the time. They were a collection of muddled, disorganized concepts. Euclid set out to organize all existing mathematical knowledge. He systematically arranged the discoveries and accomplishments of other mathematicians. His textbook, "The Elements", a thirteen volume tome, has been the standard textbook around the world for over 2000 years.

Leonard of Pisa or Fibonacci (fee-boh-NAH-chee) ca1180-1250

Fibonacci introduced the use of Hindu-Arabic numerals (0,1,2,3,4,5,6,7,8,9) throughout Europe. He also gained famed for exploring a sequence of numbers, the Fibonacci sequence, 1,1,2,3,5,8,13,21,34,…. In this sequence each number is the sum of the two previous numbers. As mathematicians investigated this sequence, surprising applications in nature began to take form. A few examples are, patterns of leaf buds on many stems, spirals of seeds in the heads of sunflowers, petals on an artichoke, or scales on a pineapple.

Blaise Pascal (pahs-KAHL) 1623-1662

Pascal is widely recognized as the inventor of the first calculator. He wanted to create a calculating machine to help is father to balance figures. Due to this “arithmetic machine” he is regarded today as the “father of the computer age” and a valuable computer language is named after him, Pascal. Pascal enjoyed finding patterns in an array of numbers shaped in the form of a triangle. They later named the array Pascal’s Triangle

					1							
				1		1						
			1		2		1					
		1		3		3		1				
	1		4		6		4		1			
1		5		10		10		5		1		
	6		15		20		15		6			

Have students use Pascal’s triangle to find a variety of patterns.

Leonard Euler (OY-ler) 1707-1783

Euler was a Swiss mathematician who became blinded as an adult and had assistants transcribe his thoughts and formulas for him. Many formulas are attributed to this mathematician and are named for him. One specific formula that fits nicely into this unit is named Euler’s Formula. He realized that for certain solids you could add the number of faces to the number of vertices and this would equal the number of edges plus 2. (F + V= E + 2)

Carl Friedrich Gauss (GOWS) 1777-1855

Gauss proved at an early age that he was particularly adroit with numbers. During an exercise given to him by an angry teacher, Gauss found, by using logical reasoning and knowledge of patterns, that he could quickly add all of the numbers from 1 to 100. His teacher was amazed and later bought him an expensive textbook on arithmetic and encouraged him to seek out a private tutor. His work leads us to realize the ease in adding numbers in a series. The details of the story on the method Gauss used to compute this sum are uncertain. Some history books say that Gauss realized that by pairing and adding the terms from opposite ends of the list we get identical intermediate sums: 1 + 100 = 101, 2 + 99 = 101, 3 + 98 = 101, and so on. Then we add these pairs we get twice the sum of all the numbers from one to 100 to be 100 x 101=10100. So the sum will be 5050. Others texts present a different approach and suggest a way to apply this idea as follows. If there are an odd number of terms in the series, find the median, or middle number and multiply it by the total amount of numbers (terms) in the series; i.e. 1+2+3+4+5= 3x5=15; (3 is the median number and there are 5 numbers in the series). If there are an even amount of numbers in the series, add the two middle numbers together and then

multiply by half of the numbers in the series; i.e. 6+7+8+9+10+11+12+13= (9+10)x4=76; (9 and 10 are the middle numbers in the series then multiply by 4 since there are eight total numbers in the series).

Investigating and Reasoning with Shapes

Teachers need to decide before beginning any exploration dealing with shapes and attributes whether they will begin with 2-D shapes or 3-D shapes. Misconceptions can occur if the two are interchanged without distinction. A 2-D shape occurs in only one plane and has two dimensions, length and width. A 3-D shape occurs in more than one plane and has three dimensions, length, width, and height. For my unit, I choose to begin with 2-D shapes. The use of picture books, folding and cutting paper, drawing shapes on grid paper, and geoboards will help students to manipulate shapes, explore the attributes, and create rules that will group figures. As students become proficient with the 2-D figures, 3-D figures can be introduced and explicitly named. Connections can be made based on the faces of the 3-D shapes. Students can trace the faces of the 3-D shapes to solidify the connection.

I believe that oftentimes we give students cute names for mathematical terms that are designed to help them and end up lending to misconceptions as the student proceeds through the years of math instruction. In order to prevent misconceptions, some terms need to be explicitly taught. As students begin their examinations, they will be therefore given explicit instruction on basic geometric terms. Using the specific terms will help to avoid confusion of concepts. The following terms also appear in the appendix and can be used as word wall entries and flashcards for memory or tic-tac-toe type games to help solidify learning. The following terms and ideas will be explained: A **point** is that which has no parts. A **line** is a two-dimensional object that has no endpoints and continues on forever in a plane. **Angle**s can be explained as two rays that come together at the same endpoint. There are three types of angles that we will highlight; a **right angle** is an angle whose measure is 90 degrees, an **acute angle** is an angle whose measure is greater than 0 degrees and less than 90 degrees, and an **obtuse angle** is an angle whose measure is more than 90 degrees and less than 180 degrees. **Parallel lines** are two or more lines that have no points in common. **Perpendicular lines** are two lines that intersect forming a right angle. A **polygon** is a plane figure made up of three or more line segments, called the sides of the polygon. A **quadrilateral** figure is a four sided polygon. A **rectangle** is a quadrilateral whose angles are all right angles. A **square** is a quadrilateral whose sides are all equal in length and all angles are all right angles. A **rhombus** is a quadrilateral with both pairs of opposite sides parallel and all four sides equal in length. A **trapezoid** is quadrilateral that has at least one pair of parallel sides.

Congruent versus Similar

Two figures are congruent if they match exactly with each other by having corresponding sides of the same length and corresponding angles of the same measure. Students should be given manipulatives or allowed to cut figures out of paper in order to prove congruency. Side lengths and angles should match. Two figures are similar if one is a bigger or smaller version of the other. The corresponding angles must have the same measure. Again, students should be allowed to prove and justify whether two figures are congruent or similar. The use of geoboards is beneficial during the teaching of this concept. It allows students to compose and decompose figures.

Symmetry

A figure is said to have symmetry if it were folded in half over a line and the two halves were identical. A square has four different ways to fold it and have the edges match, it has vertical, horizontal, and two diagonal symmetry. A circle has an unlimited number of symmetries. Students can use the upper case alphabet to explore symmetry, for example fold the letter "T" a vertical axis, it appears the same. Which other letters have symmetry?

Transformations

Flips, slides, and turns are common ways to describe transformations in an elementary classroom yet students would be more successful in future years if we gave them the correct, and in my opinion, descriptive terms of reflection (flip), translation (slide), and rotation (turn). A reflection is, simply put, a flip over a line, yet it is more than that. It means that every point one the pre-image is equidistant with the corresponding point on the image from a central line, or mirror line. The reflection of a figure is a figure that needs to be the same size and the original figure, it just faces the opposite way. Think of what your reflection in a mirror shows. You can use handheld mirrors up to a figure or design and show students reflection symmetry. Challenge students to then create reflection symmetry of designs. A translation is the moving of a figure or design, not resizing, not rotating, just moving. Every point of the figure must move the same distance and in the same direction. A rotation happens when we move a figure or design around a center point by a certain angle.

Perimeter and Area

Making real world connections to perimeter and area help to build conceptual understanding for students. Pose the question; why would we need to measure around a shape? Students will come up with many imaginative reasons, if they get stuck be sure to plug in examples of your own, putting up a fence creating a border on a bulletin board. **Perimeter** is defined as the distance around a figure. Start at a point, go around and stop when you get back at the point where you started. **Area** of a figure measures the size of the region enclosed by the figure.

Teaching Strategies

Vocabulary

Research tells us that direct instruction of vocabulary improves comprehension. I believe that the changing of mathematical terms to make them more “kid friendly” can lead to confusion and requires re-teaching as the student progresses through the grades. If a student cannot create a schema for a math term, their understanding of the concept becomes questionable. Students should be introduced directly to the mathematical terms repeatedly and should participate in open discussions of the concept. They should participate in hands on exploration of the concepts and terms applied to them. Students should also develop and understanding of similarities and differences between related vocabulary.

One instructional strategy to adopt and adapt is the Frayer Model. This strategy helps students learn new concepts through the use of attributes and non-attributes. Students learn a concept by seeing examples and non-examples of the concept.
Here are the steps:

- Define the concept by showing the properties of the concept.
- Show students how this concept differs from other similar concepts. (Highlight non-critical properties.)
- Give examples and explain why these are examples.
- Give non-examples and explain why these are non-examples.
- Give students examples and non-examples and ask them to decide whether they are examples or non-examples.”

Differentiated Instruction

In order to meet the needs of all the learners in my classroom, I will use Differentiated Instruction. Differentiated Instruction is an approach to teaching content in ways that address a variety of learning styles and needs of students while maximizing the potential of all learners. This will help me to accommodate the diversity of academic needs present in my classroom. My instruction as well as the students' research can be differentiated in a variety of ways. I will be able to differentiate according to content, process, or product. Through differentiated content students will have access to a varied level of texts and/or websites and could possibility be "buddied" with a partner at a different level to assist with the learning. Differentiated process refers to activities that help the students make sense of the skills being taught. These activities can be modified to enable students of varied readiness to work on their level. When differentiating products, students are given choices to create different products based on their individualized learning style.

Cooperative Learning

Cooperative Learning is a successful teaching strategy in which teams of learners of varied abilities and interest come together to help each other to improve their understanding of the subject matter. Cooperative Learning is used to promote student learning and retention of material, foster positive social relationships within the classroom community, and enhance student satisfaction with the learning experience. The following are examples of a variety of cooperative learning activities that I will make use of throughout this unit.

Think-Pair-Share

This approach involves three steps. During step one, students "think" silently about the topic or a question about the topic. After a pre-determined time frame, generally one to two minutes, students "pair" up and discuss their thoughts or responses on the topic. After another minute or two, students "share" their thoughts or response with other "pairs" or the entire group. Informal assessments are key to checking for understanding and a good way to accomplish this is to pose higher order thinking questions at the end of a daily lesson. A question will be posed by me and the students will answer the question using this method. Participating in this activity helps students analyze and evaluate their learning as well as clear up confusion or misconceptions in a non-threatening way as they answer a daily question in smaller less threatening groups.

Team-Pair-Solo

This approach is similar to but opposite of Think-Pair-Share. During this approach, students first complete a problem as a "team". Next they complete a similar problem as a "pair". Finally students complete problems "solo". This approach is designed to help students engage and succeed with problems which can initially be beyond the ability but can be solved within the context of a group, this is based on the concept of mediated learning. By allowing students to work on problems they could not do alone first as a team and then with a partner, they move to a position that they can do alone what at first they perhaps could not complete or could only do with help. This approach will be a helpful way to differentiate and assist students who may feel overwhelmed with the task individually. Only the students who need support will take part in this approach.

Three Minute Review

In this approach, the teacher stops at any time during instruction or discussion, a timer or other signal may be used to begin, and gives students three minutes to review what has been said or ask or answer questions about the lecture. This activity will be useful when new vocabulary is being introduced. Students can then discuss, act out, illustrate, or choose a way that will help them to remember the new words or concepts they are

learning. Since this activity is timed, the focus is not on the product it is on the varied way students choose to represent the vocabulary.

Groupings

The class gets divided into teams of three. One member of each team is given an assignment to master to be able to teach to the other members, for example students will explore the concept of transformations. Team members work to learn the material, they may consult with other members in the group groups working on the same material. Teams go back together with each member teaching the other members the concept. Team members can quiz and tutor teammates. Three stations will be set up to allow students to investigate the concepts of transformations; reflections, translations, and rotations. The students will participate in a small variety of activities to introduce the concepts. They will then return and share their knowledge. Each student becomes the master teacher yet each concept is covered within the group.

Journals

Journal often gets overlooked in the math classroom. However, the use of journals can be valuable to students as they work to develop their understanding, mathematical thinking, and communication skills. Journals provide teachers with an opportunity to informally assess students' learning and provide students with an opportunity to express what they have learned and detail any residual confusion. When a student makes an entry into their math reflection journal, they have to think about they have done and then communicate mathematically their experience. This helps to solidify the experience and creates multiple pathways in their brains. In their journals, students can represent their learning using words, numbers, and pictures. As they become comfortable, they will begin to develop arguments that enable them to justify their findings. Teachers should set aside 5 to 7 minutes at the end of class to allow students to reflect. Journals should not be used daily, however used when a new topics have been covered and students are expected to synthesize this new learning. If you have never used journals before, be patient, it will take time for students to develop proficiency in using this tool. There is no right or wrong way to complete a journal as it is an entry into the mathematical thinking processes. See appendix for a variety of journal prompts for students.

Classroom Activities

Lesson 1

Lesson Overview

During these lessons, students will be introduced to two-dimensional polygons and will work to sort them based on a variety of attributes. Then will move into identifying three-dimensional shapes.

Enduring Understanding: Understand that shapes in different categories may share attributes.

Essential Question: Can you justify why you placed a polygon into a specific group?

Materials

Large variety of polygons, Venn diagram, journals, camera

Procedure-Activity 1

This lesson is designed to be an introductory lesson enabling students to investigate, manipulate, and categorize two dimensional shapes. I will model the exercise a few times to demonstrate the procedure. Using paper cut outs of a variety of polygons in a variety of sizes, I will place some polygons into one side of a Venn diagram and others in the other side. I will then ask students to develop a "rule" for my diagram. As students justify the rule they have stated, I will encourage them to correctly use the vocabulary to explain the shapes. After a few turns, students will be paired up and given bags of shapes and a Venn diagram. They will take turns placing shapes into the Venn diagram while the other partner will justify the placement. I will circulate the room listening to students as they participate in the activity.

Assessment

As an assessment for this activity, students will glue the shapes to the Venn diagram inside their journals. Students will then justify the placement of their shapes.

Procedure-Activity 2

After students have investigated two-dimensional shapes, they will work to identify three-dimensional shapes in their environment. In pairs, students will take a digital camera around the school in search of examples of solid shapes. Upon their return, students will present their slide show. Other students will try to identify which solid they have captured in the picture. The presenting pair will have to justify their choices.

Assessment

The assessment for this activity is embedded in the activity and discussion. Students participating in the discussion and justifying their decisions will provide me with information about their understanding.

Lesson 2

Lesson Overview

This lesson will help students to understand the concept of transformations. Students will manipulate hand cut out to represent a series of transformations.

Enduring Understanding

Predict and describe the results of translations, reflections, and rotations of two dimensional figures.

Essential Question

Can you identify the motion or series of motions of the two-dimensional shapes.

Materials

Hand cutouts, sentence strips, lists of a series of transformations, journals

Procedure

Students will be given a card with a series of transformations listed on it. Students will need to manipulate the hands to create the pattern. Next, the student will create a pattern of their own and their partner will label the transformations. Finally, the students will be given the beginning figure and the end figure with the number of steps in between. Students will need to get to the end figure in the required number of steps.

Assessment

As a culminating activity for this lesson, students will glue the hands to a sentence strip and label the transformations. After they will create a journal entry detailing what they have learned during this activity.

Lesson 3

Lesson Overview

This activity is designed to be a culminating activity for the unit. Students will be given one of several plans for a classroom. Using the requirements of the plan, students will use their geometric understanding to create the classroom design.

Enduring Understanding

Understand concepts of area and perimeter and attributes of plane figures.

Essential Question

How can you design a classroom using a variety of geometric concepts?

Materials

Centimeter grid paper, several different plans, journals

Procedures

Students will be given one of several classroom design sheets. Using grid paper, students will design their classroom following the requirements set out of the design sheet. The design sheets will contain information such as; the area the classroom must be and figures it must include. Students will be required to determine the perimeter of the classroom. See Appendix D.

Appendix A

Standards

Common Core State Standards

Standard of mathematical Practice

1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
3 Construct viable arguments, and critique the reasoning of others.
4 Model with mathematics.
5 Use appropriate tools strategically.
6 Attend to precision.
7 Look for and make use of structure.
8 Look for and express regularity in repeated reasoning.

Content Standards

Measurement and Data 3.MD

Geometric measurement: understand concepts of area and relate area to multiplication and addition.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry 3.G

Reason with shapes and their attributes.

Appendix B

Vocabulary

point	has no parts
line	one-dimensional object that has no endpoints and continues on forever in a plane
right angle	an angle whose measure is 90 degrees
obtuse angle	an angle whose measure is more than 90 degrees and less than 180 degrees
Acute angle	an angle whose measure is greater than 0 degrees and less than 90 degrees
Parallel lines	two or more lines that have no points in common
Perpendicular lines	two lines that intersect forming a right angle
polygon	a figure made up of three or more line segments called sides
quadrilateral	a four sided polygon
rectangle	a quadrilateral whose angles are all right angles
square	a quadrilateral whose sides are all equal in length and all angles are right angles
rhombus	a quadrilateral with both pairs of opposite sides parallel and all four sides equal in length
trapezoid	a quadrilateral that has at least one pair of parallel sides

A**ppendix C**

Journal prompts

I knew I was right when…
The easiest part for me is…
The hardest part for me is…
The thing you need to remember for this type of problem is…
Use a different strategy to solve this problem.
If someone did this problem wrong, I would…
What help would you give a friend to help them solve this problem?
What strategy did you use to solve this problem? Why did you do it this way?
Is math getting easier or harder for you? Why do you think this?
Is geometry easier or harder than other kind of mathematics for you?
Write a "how to" paragraph to explain to someone how to solve this type of problem.
How do you feel about your work?
What would happen if you missed a step?

Appendix D

Plan set 1

Your classroom must include all of the following requirements:

- Area: 100 square centimeters
- 14 squares; six of which are congruent
- 2 similar quadrilaterals
- 1 polygon with and area of 24 square centimeters
- 3 non-congruent triangles

What is the perimeter of your classroom? _____

Plan set 2

Your classroom must include all of the following requirements:

- Area: 120 square centimeters
- 15 quadrilaterals
- 3 non-quadrilateral polygons
- 3 figures that are congruent and have parallel sides
- 3 figures that are similar and with perpendicular corresponding sides

What is the perimeter of your classroom? _____

Plan set 3

Your classroom must include all of the following requirements:

- Area: 150 square centimeters
- 3 right triangles
- 10 quadrilaterals, four congruent and four similar figures
- 4 non-quadrilateral polygons containing parallel sides
- 1 polygon with an area of 25 square centimeters

Teacher Resources

Math is Fun
http://www.mathsisfun.com/geometry/transformations.html
This website gives a great explanation of transformations to use in the elementary classroom

Bright Hub
http://www.brighthubeducation.com/teaching-elementary-school/16636-help-teaching-geometry/
A website with articles and lesson ideas for the classroom

Institute for Mathematics and Education
http://ime.math.arizona.edu/
A website connected with The University of Arizona with access to a variety of articles and resources for teaching mathematics

Instructor Web
http://www.instructorweb.com/lesson/geometryshapes.asp
Another website with lesson plan ideas for the geometry classroom

Math Forum
http://mathforum.org/geometry/geom.units.html
Geometry lesson plans

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KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Understand that shapes in different categories may share attributes., Predict and describe the results of translations, reflections, and rotations of two dimensional figures., Understand the concepts of area and perimeter and attributes of plane figures.

ESSENTIAL QUESTION(S) for the UNIT

Can you justify why you placed a polygon into a specific group?, Can you identify the motion or series of motions of the two-dimensional shapes?, How can you design a classroom using a variety of geometric concepts?

CONCEPT A	CONCEPT B	CONCEPT C
Classifying polygons	Identifying and understanding transformations	Determining perimeter and area of plane figures
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
Can you justify why you placed a polygon into a specific group?	Can you identify the motion or series of motions of the two-dimensional shapes?	How can you design a classroom using a variety of geometric concepts?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
Polygon, quadrilateral, rectangle, square, rhombus, trapezoid, angle, right angle, obtuse angle, acute angle, congruent, symmetrical	Translation, rotation, reflection, symmetry, similarity, congruent	Perimeter, area, line, parallel lines, perpendicular lines

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Materials: large variety of polygons, journals, venn diagram, centimeter grid paper, classroom blueprints with a variety of areas