Beyond the Mirror: Symmetry All Around Us

Nancy Ventresca

Introduction

I teach at Thurgood Marshall Elementary School. It is a large suburban school in Newark, Delaware. Our demographics are diverse, with approximately 42% of students reporting their race as African American, 41% as White, 12% as Asian American, and 6% as Hispanic (DE Department of Education, 2010). Other characteristics include 7% of students that are English Language Learners, 40% from low income families, and 8% identified as Special Education students. Ours is the largest district in the state of Delaware and it encompasses both urban and suburban populations.

I am responsible for challenging our top academic students as Thurgood Marshall's Enrichment Teacher. This means that I work with the students who perform in the highest 10% in the areas of Math and Reading. My groups are representative of the school's diverse population in terms of race and socioeconomic status. Although being in the program one year does not guarantee that a student will be eligible the next, many of my students qualify for my program year after year. Data used in the eligibility criteria include Delaware Comprehensive Assessment System (Delaware's State Test) and Terra Nova (norm referenced) scores in each subject area, along with teacher recommendation. The program begins in the first few weeks of school and students may enter the program at any time after a review by myself, the classroom teacher and the principal.

This unit has been created for my fifth- grade Mathematics Enrichment students. They are seen each day for a period of 30 minutes during an intervention time. Therefore, my students continue to have access to the regular classroom curriculum, in addition to the Enrichment program.

Rationale

This unit was developed as an extension of the Geometry strand of my curriculum. We use a series from Kendall Hunt called "Mentoring Mathematical Minds", fondly referred to as "M Cubed."[1] The final unit for the year, "Funkytown Fun House" focuses on proportional reasoning and similarity. Topics covered include ratio, proportion, similarity, congruence, scale factor, perimeter, area and volume of similar figures. Between this unit and the regular classroom using Math Connects, the standards seem to be covered well for the grade level. However, my students thirst for more and I have written this unit to provide them with some experiences that they might not otherwise have until middle school.

The Journey to Symmetry

Any discussion of basic geometry no doubt sends us back to fond thoughts of kindergarten. The identification of circles, triangles and squares is as basic as the identification of letters and numbers. As students move along the academic pathway, their knowledge builds upon this basis, becoming more refined and specific in its proofs. This is true for all the mathematical strands, and in particular, geometry.

The creation of this unit began with a look at the mathematics standards. I began my journey with the Principles and Standards of Teaching Math, 2000, published by the National Council of Teachers of Mathematics. This document was written and discussed by teachers and clearly defines standards across all grade levels. There are five process strands:

1. Problem Solving
2. Reasoning and Proof
3. Communication
4. Connections
5. Representation

According to the NCTM, the goal of all programs is to give students the ability to build new mathematical knowledge through problems solving, solve problems that arise in Math and beyond, apply and modify a "toolkit" of problem solving strategies, and finally, to monitor and reflect on their process of mathematical problem solving.[2] This applies to programs beginning in prekindergarten and continues through the twelfth grade. Taking a close look at the Geometry standard, I was reminded of the expectations for my target group: fifth grade. I needed to know just how much my students would know when they began our symmetry work together. Given that they are all very capable students, I can expect that they will possess a solid knowledge that began in Kindergarten and continued through second grade (recognizing, building, drawing comparing and sorting two and three dimensional shapes; describing attributes and parts of those shapes and them extending their thinking by investigating and predicting results of combining and deconstructing these shapes). This analysis, classification and further investigation continues from the third through fifth grades. But I was especially interested in the expectations that pointed to symmetry: along with the previously mentioned tasks, students should also be presented with the concepts of congruence and similarity. Indeed, by the end of the fifth grade, students should be able to apply transformations, to use symmetry to analyze mathematical situations, to predict and to describe the results of sliding, flipping and turning two dimensional shapes. In addition, they should be able to identify and describe line and rotational symmetry in two and three dimensional shapes and designs. My unit began to take shape.

Since my students have demonstrated their strong mathematical abilities, I wanted to push them beyond their grade level expectations. Looking ahead at the grade 6-8 expectations, I found that their futures held expectations that they would be able to describe sizes, positions and orientations of shapes under those rather informal transformations (flips, turns, slides and scaling) and to examine the congruence, similarity, and line or rotational symmetry of objects using transformations. I also want them to begin using the proper mathematical terms for the isometries.

Next I went to my Common Core Standards for Mathematics, and Geometry in particular. Searching the Common Core for "symmetry", I found that it only stands on its own in the 4th grade: Standard 4.G.3. *Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry*[3]. Middle school standards mention similarity and transformations, but symmetry is not specifically addressed. In fact, it seems that time spent on geometry in general is rather minimal until high school. When I discussed this with my District curriculum specialist, she told me that in the fifth grade, the chapter on geometry is actually optional, as those standards had been moved to eighth grade in the Common Core. This is rather ironic, because one could argue that symmetry is one of the most important concepts in mathematics! It is evident in calculation and equations, the writing of proofs and in geometric design. Given this, it seems to me that this unit is all the more important for my students!

However, there is more to this unit than just the Common Core standards for mathematics. With the use of Geometer's Sketchpad and computer applications, it also addresses the importance of allowing students to become experienced with a variety of mathematical tools. In fact, the Common Core specifically requires that students should use technological tools to explore and better understand mathematical concepts and to be able to use appropriate tools strategically.

But first, I needed to take a look at the history of this realm of mathematics in order to continue the creation of my unit.

The Origins of Geometry

"If there can be such a thing as a household name in mathematics, "Euclid" is it."[4]

Despite its antiquity, "The Elements" by Euclid is unquestionably the most revered and universally famous work that is still in use today. In fact, it holds the distinction of being the world's oldest continuously used mathematical textbook. With the advent of the printing press, "the Elements" was among the first books to be created in printed form and is now available in over a thousand different languages. It lags only behind the Bible. Many historic figures have been devoted to its teachings, including Abraham Lincoln, who is said to have studied the Euclid’s Elements to assist him with logic, arguments and the construction of proofs.

So what are “Euclid’s Elements”? Written by the Greek mathematician Euclid somewhere around 300 BCE in Alexandria, it is a voluminous text whose main subjects are geometry, proportion and number theory. The theorems are not all truly Euclid’s discoveries. In fact, many of the theorems were the work of earlier greats, such as Pythagoras, Hippocrates, Theeaetetus and Eudoxus, that were collected, refined and recorded by Euclid. Regardless, he is given due credit for his arrangement of these theorems in a logical way and for extending explicit proofs of previously discovered theorems.

Everything explained in the Elements are geometrical constructions that can be achieved using only a straight edge (ruler) and a compass. It is comprised of thirteen books, each taking on a particular topic and associated theorems. Book one outlines the fundamentals of plane geometry, including congruent triangles, parallel lines, the sum of angles in a triangle and the Pythagorean theory. Book two deals with geometric algebra, Book three investigates circles, Book four focuses on regular polygons, Book five is about the arithmetic theory of proportion, Book six applies proportion to plane geometry and theorems about similarity, Book seven deals with elementary number theory (such as prime numbers, greatest/least common denominators, Book eight discusses geometric series, Book nine includes theorems on the infinitude of prime numbers, Book ten provides an prelude to integration with a “method of exhaustion”, Book 11 deals with the fundamental propositions of three dimensional geometry, Book twelve uses the method of exhaustion in reference to the relative volumes of cones, pyramids, cylinders and spheres and finally, Book thirteen investigates Platonic solids.

However, symmetry is not mentioned a great deal in the Elements. This may seem somewhat confusing, because, after all, it is taught along with Geometry and is contained in our Mathematics and Geometry standards. So where did it come from, if not from Euclid?

Much of the theory about symmetry can be traced back to a young political revolutionary and a mathematical obsessive named Evariste Galois. Unfortunately he died at the tender age of twenty-one, after a duel at dawn over a young woman. Fortunately for the rest of the world (and future mathematicians!), he left a sixty page manuscript that revolutionized mathematics. He invented a language that described symmetry in mathematical structures, which is now known as "group theory", and it has its roots in algebra, not geometry. Essentially, Galois took the ancient tradition of algebra and redesigned it to be used as a tool in the study of symmetry.[5]

Symmetry in the World Around Us

Symmetry is found all around us. We may not regularly notice it, but it is in our very bodies: two eyes, two ears, two arms and two legs. The human body, by design, is symmetrical. Think of our internal organs: two lungs, two kidneys, two hemispheres of the brain. Even aspects of our evolution seem to be hardwired to appreciate symmetry. Have you ever really looked at the faces of celebrities who are considered the "most beautiful"? Most of their faces demonstrate bilateral mirror symmetry. Studies have been undertaken where researchers showed test subject photos of people, and the most common choice was the face that looks the most evenly balanced. They are deemed the most attractive and perhaps that is a reflection of a genetic language signaling good health and favorable genetic makeup.

Consider other examples found in the animal kingdom. Many motor functions and abilities rely on a perfect symmetry. The animals with the best symmetry will likely run the fastest and thus catch the most prey. Creatures lacking a limb tend to become victims to the stronger, more balanced predators. The honeybee offers us another perfect example. It is attracted to the pentagonal symmetry of the honeysuckle and the hexagonal shape of the clematis flower. These plants themselves are communicating with the bees they attract. The eyesight of bees is quite poor, and yet they can pick out these specific flowers (food) by their shape. Again, the most symmetrical flowers sing out their message, while those that are damaged will fail to be pollinated and wither.

Even animal activists have used symmetry in their argument against animal cruelty. It seems that free range chicken eggs are more symmetrical than those of their sister hens who are forced to live in tightly packed cages, suggesting that it requires more energy and comfort to produce perfection.

This admiration for perfect balance is also evident throughout history in our art and architecture. The palace of the Alhambra in Spain is a remarkable structure and a fantastic illustration of this. Because the Moorish artisans were forbidden to draw things that had souls, they turned to geometric shapes. From the reflective symmetry of the pool in the front, to the walls that are tiled in dizzying examples, it is a magnificent combination of art, mathematics and architecture. I want my students to see and appreciate this wonder.

Perhaps the most common example of symmetry in art is found in the works of M.C. Escher (1898-1972). Escher was a Dutch graphic artist who, despite having no formal training in math or science, created beautiful repeating geometric patterns,(known as tessellations) that continue to fascinate their viewers today. Lithographs and woodcuttings, Escher's pieces are respected for their intricate repeating patterns and mathematical structures. My particular favorite is Escher's *Night and Day*, a complex print that illustrates birds flying over a country side, with a fabulous symmetry that viewed from one perspective, looks like daytime and from another, looks like night.[6]

Similarly, the use of tangrams can be used in the unit as well. Tangrams come from China and are a form of shape puzzle that is made up of 5 triangles, a square and a parallelogram. Legend has it that the tangram puzzle is attributed to a Chinese nobleman named Tan who created a beautiful square tile to be gifted to his emperor. However, he dropped the tile and it broke into seven pieces. Despite his desperate attempts to repair the tile, Tan could not put it back together. He did discover, however, that the seven pieces could be assembled to make other shapes, such as buildings, animals and ships. The seven pieces are referred to as "tans" and the tangram has been a favorite puzzle of many geometry enthusiasts and children since around the nineteenth century.

Learning the Language of Symmetry

"The universe cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometric figures, without which means it is humanly impossible to comprehend a single word."

-Galileo, Opere Il Saggiatore p. 171.[7]

As Galileo indicates, the language of geometry must be precise. In fact, there are many misunderstandings and misuses of theories that can be attributed to incorrect use of vocabulary. Young students may insist that an orange is a "circle", when in fact, it is actually a sphere. It is very important that the proper terms are presented and used early on. In order to teach this unit, the following concepts and vocabulary must be understood and clearly demonstrated to the students. As a teaching strategy, I have my students create vocabulary foldables for review and quick reference (these foldables will be further discussed in the lesson plan section of the unit).

Plane symmetry involves moving all points around the plane so that their positions relative to each other remain the same, although their absolute positions may change. Symmetries preserve distances, angles, sizes, and shapes.

Rotation

To rotate an object means to turn it around. Every rotation has a center and an angle.

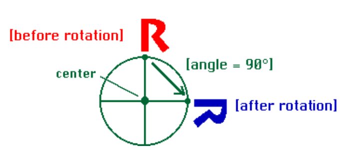


Figure 1[8]

Reflection

To reflect an object means to produce its mirror image. Every reflection has a mirror line. A reflection of an "R" is a backwards "R".

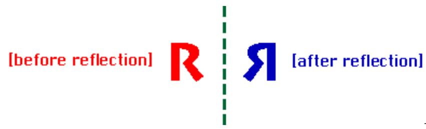


Figure 3[9]

Glide Reflection

A glide reflection combines a reflection with a translation along the direction of the mirror line. Glide reflections are the only type of symmetry that involves more than one step.

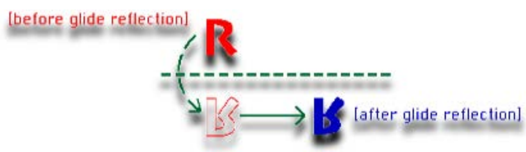


Figure 4[10]

Translation

To translate an object means to move it without rotating or reflecting it. Every translation has a direction and a distance.

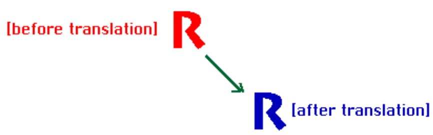


Figure 2[11]

These are the four kinds of isometries that will be the focus of this unit: reflection, rotation, translation and glide reflection. Each will be taught and students will have the opportunity to explore them using a variety of mediums.

Tessellations

A Tessellation is also known as a “tiling”. It is created when a shape is repeated continuously, covering a plane without leaving any spaces or overlaps. A regular tessellation means that the tessellation is composed of congruent, regular polygons. In other words, all sides and angles of the polygon are equivalent and all the polygons used are the same size and shape. In the Euclidean plane, only triangles, squares and hexagons tessellate:

a tessellation of triangles

a tessellation of squares

a tessellation of hexagons

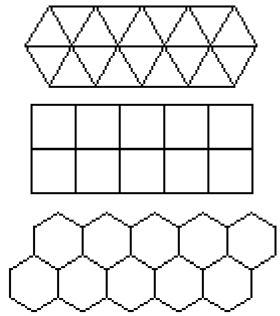


Figure 5[12]

Using Geometer's Sketchpad in the Elementary Classroom

Although more commonly used in middle and high school math classes, I was able to access this dynamic geometry software in my own room by requesting it from my District's technology department. At first, it seemed a bit intimidating, but after some practice using it on my Smart Board, I found it to be an outstanding educational tool. It allows the construction of very precise figures that can then be moved and changed interactively. Through its use, my students will become adept at using an important technological tool to deepen their understanding of the new concepts. In addition, by introducing them to Sketchpad in elementary school, they will be better prepared when they encounter it later on in middle school.

The program has a Sketchpad Learning Center that includes videos, tutorials, tips and online links. Another valuable resource that I used in the development of this unit is a text from Key Curriculum Press called "Exploring Geometry and Measurement with The Geometer's Sketchpad (2012). Not only does it address the Common Core Standards, it offers very clear and concise activities that facilitate the teaching of this content. In particular, Chapter 5 focuses on transformations and includes magnifying and shrinking, mosaic tile designs and tessellations. I highly recommend this resource!

Strategies

This unit will begin my unit with a pretest to assess my students' general level of knowledge of the necessary concepts. If I am able to have access to our school's interactive response system, this will be utilized. Pre-assessments are powerful tools for all teachers, and especially for me, as a resource teacher. My students are drawn from several different classrooms and it is not unusual to have students transfer from other districts and states. By using a pre-assessment, it allows me to determine which objectives my students have mastered prior to the beginning of the unit. Beyond this, the results assist me in differentiating instruction, planning my activities that address varying readiness levels, and forming flexible groups. In addition, it avoids wasted time teaching what students may already know, guides instruction to challenge students appropriately, and can motivate students to be more involved and attentive. It can even identify students who have extensive background knowledge to form expert groups.

Activities in the unit will include individual and partner work. The act of communicating content knowledge with a peer is crucial to deepening students' understanding of a given topic. This component of the reasoning and proof process is extremely important, as is the ability to effectively communicate mathematical thought in writing. By laying the groundwork in elementary school, the transition to more difficult proofs and explanations will be more comfortable for students.

The unit also includes cross curricular activities to encourage students to see symmetry as not just something that consists in mathematics. Social studies and art can be addressed by examining the art of M.C. Escher, the flags of many countries, First Nations Native American artwork, and architecture. Given internet access, students can virtually travel the globe to experience symmetry in the world around them.

The final activity in the unit will be differentiated by choice. Students will be given a menu from which they can choose 3 activities to showcase their knowledge of symmetry. These activities will be representative of Bloom's levels of higher order thinking. They will also use the Learning Focused framework. In addition, there are opportunities for the use of technology and media (computers, video, interactive whiteboard activities and cameras. By being given a choice, the learning is also differentiated by learning style.

Activities

Introduction: Assessing Prior Knowledge

Since my students come from different classrooms and prior knowledge may vary, I will begin the unit with a quick and fun pre-assessment. They will be given a Geometry Word Puzzle that addresses plane geometry terms. This will allow me to reteach if necessary to fill in any "holes" in their prior knowledge.

This type of activity could be created at www.puzzlemaker.com, which offers a variety of puzzle types, or students could be given the vocabulary terms and instructed to create their own for another student to solve. Crosswords, word scrambles and fill in the blanks are possible options.

Vocabulary to include:

Triangle, square, rhombus, parallelogram, polygon, circle, congruent, similar, image, segment, midpoint, intersection, vertex, perpendicular, diagonal, parallel, symmetry

Activity Two

Goals: Students will further develop their understandings of plane figures

Using appropriate technology, students will solve problems that arise in mathematics and in other contexts

Students will create and use pictures, manipulative, models and symbols to organize mathematical ideas

Essential Questions:

Where can we find symmetry in our environment?

How is symmetry used in areas such as architecture and design?

Vocabulary: Line symmetry, rotational symmetry, plane

Materials: digital cameras, projector

Procedure:

1. Activate prior knowledge with a discussion of the term “symmetry”. Build connections to geometry and symmetry in the real world, guiding the discussion towards symmetry in nature, anatomy and architecture. Examples may be shown using PowerPoint or other media. Possibilities include animals (starfish, and spiders are neat!) snowflakes, leaves, etc. For architecture, the Alahambra in Spain and famous buildings such as the Parthenon provide nice examples.
2. Students will pair up for this project and each pair will be given a digital camera. If your school does not have access to school cameras, perhaps some students might bring in their own.
3. Students will be sent out in the school in search for examples of symmetry found in and around the school building. Encourage them to record the location of each example.
4. Upon returning to the classroom, pictures will be uploaded to the classroom computer for discussion and presentation purposes.
5. This activity could be extended to the creation of student presentations. Should you choose to do so, allow students to print their best photos and have them identify the symmetry evidenced in each. Another option would be to incorporate the photos into a Powerpoint presentation.

Activity 3: Fun with Tangrams

Essential Question: How do plane geometry shapes relate to each other?

Geometry is the only area of math where students can model and construct. This activity requires students to cut, draw and build a tangram from a set of instructions written in the language of geometry. Students should work individually, but if a student needs scaffolding, they may work with a partner to assist them.

Begin with some background about the history of tangrams. Tell them that a tangram comes from China and that it is a puzzle that is made up of 5 triangles, a square and a parallelogram. After reassembling them back into the large square, these shapes can be rearranged to create other shapes and figures.

Instructions for Constructing a Tangram

1. Draw an 8 inch square on drawing paper or cardstock.
2. Starting at the upper left hand corner, label each vertex clockwise and alphabetically where two line segments meet. Start with point A and proceed through point D.
3. Draw segment BD.
4. Find the midpoint of AB. Label it E.
5. Draw EF.
6. Find the midpoint of EF. Label if G.
7. Draw CG.
8. Find the intersection of GC and BD. Label it H.
9. Find the midpoint of HD and label it J.
10. 10. Draw FJ.
11. Find the midpoint of HB and label it K.
12. Draw GK.
13. Cut along the lines. You should have 7 pieces, or "tans"

There are many resources available for constructing tangrams. Folding instructions can be found on the web at sites such as Math Forum: http://mathforum.org/trscavo/tangrams/construct.html. Another interesting site can be found at http://tangrams.ca. This site provides a history of tangrams, instructions and puzzle shape challenges. Students may also enjoy the opportunity to play tangram games online, at sites such as http://www.kidscom.com/games/tangram/tangram.html.

Activity 3: Learning the Language of Symmetry
Essential Question: What is the vocabulary that I need to know to better understand symmetry in my world?
This activity will introduce students to the language of symmetry and allow them to explore its evidence all around them.
Vocabulary:
Line of reflection, reflectional symmetry
Vertical symmetry, horizontal symmetry, rotational symmetry

Materials: mirrors, rulers, drawings/pictures of items that are clear demonstrations of symmetry (butterflies, faces, hearts, etc).
Demonstrate using an overhead or document camera. Then allow them to work in pairs to identify the types of symmetry evidenced in the samples given.

Examples to provide may include letters from the alphabet:
Vertical reflectional symmetry: A, H, I, M, O, T, U, V, W, X, Y
Horizontal reflectional symmetry: B, C, D, E, H, I, K,O,X
When the page is rotated 180 degrees, rotational symmetry: H, I, N, O, S, X, Z
Both vertical and horizontal symmetry: H, I, O, X
**All letters that have both horizontal and vertical symmetry also have rotational symmetry.
Extension/Homework: Given the name "ANNA", students will identify the symmetry found in the name. Other words that can be used include BOB, TOM, MOM, DAD, HOME.

Activity 4: Symmetry in the Real World
Essential Questions: Where can I find examples of symmetry in the every-day world? How is symmetry used by advertisers, artists and communities?
Symmetry is evident in the world around us, including advertising, architecture and art. Begin with displaying several examples of signs, such as the symbols for men's and women's restrooms, the Recycling triangle, and Volkswagen or Chevrolet symbol. International flags, quilt squares, Native American Art also provide some nice options for discussion.
Challenge students to identify the types of symmetry they see in the examples. This could be a group activity, with small groups each being presented with a variety of examples to analyze. After group work, students could present to the class for a whole group discussion.
Extension: Students will search for symmetry in symbols in the everyday world. They will produce a poster of one (or more) symbols and identify the symmetry within it.

Activity 5:
This activity is differentiated by choice and offers several opportunities for assessing student learning.   Students will choose from a *learning menu*, which will include a variety of activities that encourage higher level thinking, while allowing students to demonstrate their knowledge and understanding of the unit.
See Appendix 2.

Appendix 1

Figure 1

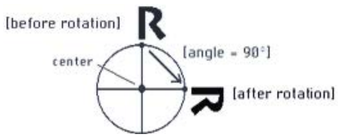


Figure 2

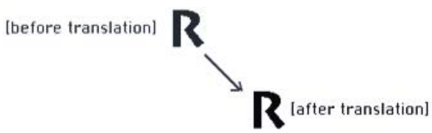


Figure 3

Figure 4

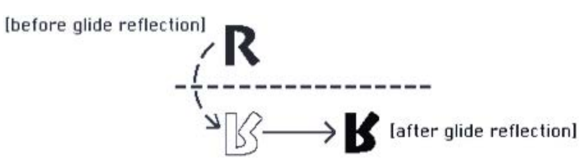
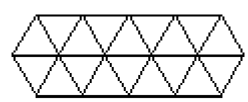
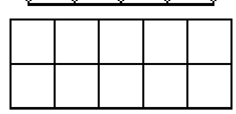


Figure 5

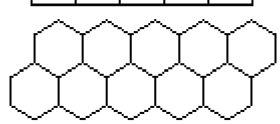
a tessellation of triangles



a tessellation of squares



a tessellation of hexagons



Appendix 2

Menu Planner

Menu: Symmetry All Around Us

Due: Main, side and dessert dishes must be completed by:________ You may choose to complete more than one selection from each.

Main Dish *(Choose one)*

1. Create a brochure featuring a transformation of your choosing, incorporating information about its structure, what you find interesting about it, and where it can be found.

2. Help build our Symmetry Trivia Game! Write 10 trivia questions and their correct responses and prepare them on index cards.

3. Create a presentation to demonstrate the most important concepts about symmetry for younger students. You may write and act out a skit, use PowerPoint, or posters.

Side Dish *(select one)*

1. Diagram at least 4 examples of symmetry found in nature. Categorize and distinguish between the types of symmetry in your choices.

2. Illustrate examples of symmetry found in modern daily life. Be sure to include at least 4 different situations/items.

3. Research the Alhambra. How would you design a similar structure today? Where would you find your inspiration? Write a letter to a company who is considering building such as structure in New York City, applying for a job as a designer.

4. Use the vocabulary from this unit to create a crossword puzzle, including the design and matching clues.

Dessert *(select one)*

1. What is the Fibonacci Sequence? What does it have to do with patterns and symmetry in nature? Cite examples and explain.

2. Research the work of M.C. Escher. Which is your favorite and why? Create your own tessellation and write clear instructions so that you might teach a classmate to do the same.

3. Write a story or comic using the concepts of symmetry. For example, you might have a character searching for its reflection.

4. Name some careers that use symmetry and geometry. Give evidence of how these topics are important in these jobs.

GLOSSARY:

Terms and Concepts That May Be Helpful

Congruent: Identical in shape and size. In other words, equal measures of angles and equal lengths of sides.

Corresponding Angles: When a transversal intersects two parallel lines, non-adjacent angles on the same side of the transversal are called corresponding angles. These corresponding angles are congruent.

Diagonal of a polygon: A line segment whose end points are nonconsecutive vertices of the polygon.

Icosohedron: A three dimensional object built from equilateral triangles. Having 20 triangular faces, this is part of the family of 3 dimensional objects called *polyhedra*.

Image: the output, or result of a transformation.

Isometry: A transformation of a figure that produces an image congruent to the original figure.

Line of symmetry: a line that divides a figure into halves that are mirror images.

Parallel: parallel lines are lines that never intersect.

Perpendicular: two lines that intersect and form right (90 degree) angles.

Reflectional Symmetry: a characteristic of a figure or design that has at least one line of reflection.

Vertical reflectional symmetry: A characteristic of a figure or design that has a *vertical* line of reflection

Horizontal reflectional symmetry: A characteristic of a figure or design that has a *horizontal* line of reflection.

Rotational symmetry: A characteristic of a figure or design that will look the same after being turned less than 360 degrees.

Translation: A motion that slides a figure or design in a fixed direction by a given distance.

Glide reflection: A motion that reflects a figure or design in a straight line then slides it in a direction parallel to that line.

Translational symmetry: A characteristic of a design that is able to look just like itself after sliding in a fixed direction.

Glide-reflection symmetry: A characteristic of a design that is able to look just like itself after reflection in a straight line, followed by sliding in a direction parallel to that line.

Strip pattern or frieze: A design that demonstrates translational and/or glide-reflectional symmetry in just one fixed direction.

Similar: Having congruent corresponding angles and having corresponding sides that are proportional. Similar figures may be congruent, but they're usually different sizes.

Transformation: A repositioning of a pre-image object to its image.

Vertex: The common endpoint of the two rays that form an angle.

Standards Addressed in This Unit

This unit integrates standards in mathematics, technology, and communication. Further explorations can easily incorporate standards in Art, should you decide to included work with tessellations, architecture, flags, masks, etc.

Grade 5: Geometry

- Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category.
- Classify two-dimensional figures in a hierarchy based on properties.
- Use appropriate tools strategically

Grade 6: Geometry

- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
- Use geometric models to represent and explain numerical and algebraic relationships.

Grade 7: Geometry

- Draw, construct and describe geometrical figures and describe the relationships between them.

Grade 8: Geometry

- Understand congruence and similarity using physical models, transparencies or geometry software.

Listening and Speaking:

Comprehension and Collaboration

- Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.

Presentation of Knowledge and Ideas

- Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
- Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.

Resources for Teachers:

There are many resources available on the internet. If you are fortunate enough to have an interactive whiteboard, there are also great activities available for free. Although some links may expire, be sure to check out favorites such as :

Smart Exchange: http://exchange.smarttech.com/#tab=0 Smart has a treasure trove of interactive lessons and games on many, many topics. Easily searchable and free.

Illuminations: http://illuminations.nctm.org/ This site, sponsored by NCTM, is easy to use and has many great ideas and activities. Specifically, take a look at http://illuminations.nctm.org/LessonDetail.aspx?id=U104, for a look at "Paper Quilts" and http://illuminations.nctm.org/ActivityDetail.aspx?ID=168 for "Frieze Patterns.

Math Forum: www.mathforum.com. A wide variety of great interactive lessons and materials.

Manga High: www.mangahigh.com this site allows teachers to register students and assign challenges that they complete online. Medals are earned and competitions between schools can occur. My students really enjoyed these games and I liked the administrative tracking abilities that it provides. In addition, there are brief instructional videos that allowed my students to explore new mathematical topics. Best of all, it's free!

Great Maths Games: www.greatmathsgames.com This site has many good resources, including very nice representations of symmetry and including the Tajmahal.

www.learner.org/courses/learningmath/geometry/session7/part_a/index.html This link includes video, notes and printable PDFs on line symmetry and rotations.

Books

Bennett, Dan. Exploring Geometry and Measurement with the Geometer's Sketchpad Version 5. Emeryville CA: Key Curriculum Press, 2012. This is an awesome resource if you have access to Geometer's Sketchpad.

Britton, Jill. Symmetry and Tessellations. New York: Dale Seymour Publications, 2000. This is a valuable resource with a variety of activities.

du Sautoy, Marcus. Symmetry: A Journey into the Patterns of Nature. 1 ed. New York: Harper Perennial, 2008. This was a wonderful read to get you hooked on symmetry in the world around us!

Escher, M. C. M.C. Escher, 29 master prints. New York: Abrams, 1983. There are lots of places to find images of tessellations, but this is really beautiful.

Muschla, Judith, and Gary Muschla. Geometry teacher's Activities Kit: Ready to Use Lessons and Worksheets for grades 6-12. New York: Center for Applied Research in Education, 2000. This large volume offers many wonderful activities and reproducible pages for students.

Stewart, Ian. Why beauty is truth: the history of symmetry. New York: Basic Books ; 2008.

Notes

[1] Gavin, M. Katherine. *Project M3: mentoring mathematical minds series*. Dubuque, Iowa: Kendall/Hunt Pub. Co., 2006
[2] Geometry Standard." National Council of Teachers of Mathematics. http://www.nctm.org/standards/content.aspx?id=314#analyze (accessed November 11, 2011).
[3] "Common Core State Standards Initiative | Home." Common Core State Standards Initiative | Home. http://www.corestandards.org/ (accessed November 10, 2011).
[4] Stewart, Ian. *Why beauty is truth: a history of symmetry*. New York: Basic Books, a member of the Perseus Books Group, 2007. P. 19
[5] Stewart, Ian. *Why beauty is truth: a history of symmetry*. New York: Basic Books, a member of the Perseus Books Group, 2007
[6] Escher, M. C.. *M.C. Escher, 29 master prints*. New York: Abrams, 19831981.
[7] Mathematical Quotations -- G." Department of Mathematics, Furman University. http://math.furman.edu/~mwoodard/ascquotg.html (accessed November 13, 2011)
[8] Addington, Susan. Math Forum, "The Four Types of Symmetry in the Plane." Accessed December 15, 2011. http://mathforum.org/sum95/suzanne/symsusan.html.
[9] Addington, Susan. Math Forum, "The Four Types of Symmetry in the Plane." Accessed December 15, 2011. http://mathforum.org/sum95/suzanne/symsusan.html
[10] Addington, Susan. Math Forum, "The Four Types of Symmetry in the Plane." Accessed December 15, 2011. http://mathforum.org/sum95/suzanne/symsusan.html
[11] Addington, Susan. Math Forum, "The Four Types of Symmetry in the Plane." Accessed December 15, 2011. http://mathforum.org/sum95/suzanne/symsusan.html
[12] Addington, Susan. Math Forum, "The Four Types of Symmetry in the Plane." Accessed December 15, 2011. http://mathforum.org/sum95/suzanne/symsusan.html

Curriculum Unit Title: Beyond the Mirror: Symmetry All Around Us

Author: Nancy Ventresca

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

A shape may be asymmetrical, have one line of symmetry or many lines of symmetry.

Symmetry exists in nature and in man-made objects.

Symmetry is an important concept in art, architecture and in many other fields.

ESSENTIAL QUESTION(S) for the UNIT

How do you determine lines of symmetry?

How do you determine rotational symmetry?

How is symmetry used in areas such as architecture and art? In what other areas is symmetry important?

CONCEPT A	CONCEPT B	CONCEPT C
Identifying symmetry in plane geometry	Identifying symmetry in nature and art	Identifying transformations

ESSENTIAL QUESTIONS A	ESSENTIAL QUESTIONS B	ESSENTIAL QUESTIONS C
How do I determine lines of symmetry in regular and irregular polygons?	Where can I find examples of symmetry in nature? Where can symmetry be found in man-made symbols and architecture? Why do symmetric designs and logos appeal to so many people?	How can I identify different types of symmetry?

VOCABULARY A	VOCABULARY A	VOCABULARY A
Asymmetrical Reflectional symmetry Line of symmetry Rotational symmetry Axis of symmetry rotation	Architecture Alhambra logo Advertising tessellations M.C. Escher	Translation glide reflection translational symmetry rotational symmetry transformation

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Geometer's Sketchpad

Exploring Geometry and Measurement with the Geometer's Sketchpad, Key Curriculum Press