When Lines and Angles Intersect a Circle

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Overview

In this unit, students will come to discover and prove that lines and angles that intersect a circle result in special properties. Students will tap into their prior knowledge of trigonometric ratios, the Pythagorean Theorem, as well as the External Angle Theorem. Through exploration and hands-on construction, students will come to appreciate how the properties of circles are very much related to the properties of triangles.

Students will form conjectures about the many properties they will explore based on their observations of patterns, but they will be asked to prove their conjectures work for all cases by developing their ability to use geometric reasoning.

This unit is intended for students at the sophomore/junior level of high school. In the New Castle County VoTech School District, this unit would be part of Integrated Math IV curriculum, coinciding with Unit 6 of the Core Plus Book 3 series.

Rationale

In general, students struggle with the concepts of Reasoning and Proof. Most do not fully appreciate what is necessary to *prove* that an assertion is always true or that a strategy will always work, often because it seems obvious to them and unnecessary to prove. Even at the high school level, there is an immaturity of thought that can limit a student's ability to reason abstractly. They are more comfortable using concrete measurements and numerical patterns to help form their conclusions.

Geometry is a wonderful opportunity to enhance students' reasoning and proof skills. While shapes and figures are concrete enough for them to construct, measure, and manipulate, the relationships that exist in geometric figures are rich in opportunities to use abstractions.

In this unit, I hope to develop the students' confidence in using their innate abilities to recognize patterns into a curiosity about why those patterns exist. This curiosity will hopefully translate into a desire to prove that the patterns can be described abstractly using mathematical properties with which they are completely comfortable.

My lesson would address these Delaware State Standards:

Classification Grade 11 :Identify and apply the properties of circles as they relate to central angles, inscribed angles, and tangents.

Location and Transformation Grade 10: Use appropriate technologies to model geometric figures and to develop conjectures about them.

Measurement Grades 10 and 11:

- Apply trigonometric relationships to determine side lengths and angle measures of right triangle.
- Use trigonometric relationships to determine side lengths and angle measures of any triangle.
- Apply the Pythagorean Theorem and its converse.

Also deeply embedded in this unit are the Delaware and NCTM Mathematics Process Standards of Problem Solving, Reasoning and Proof, Connections, and Communication.

Mathematical Background

This unit will require that the teacher have a comprehensive understanding of the properties of triangles. Fundamental properties of triangles are the root of most, if not all, of the relationships that will be explored.

Properties of Triangles

It will be necessary for students to fully comprehend right triangle relationships such as the Pythagorean Theorem and Trigonometric Ratios. Additionally, students must have a complete comprehension of the External Angle Theorem. It is not enough for students to merely be able to use these skills, but they should spend time exploring why these relationships exist.

Especially in the cases of the Pythagorean Theorem and External Angle Theorem, students should invest time proving why these theorems work. This may be their first exposure to “formal proof” in a geometric context. The proofs do not need to be executed in a traditional two column format, but they need to be done in a way that clearly shows how the theorem can be applied universally and will work for all cases. It is especially relevant later in circle investigations to note that the external angle of the angle opposite the base in an isosceles triangle is double in measure one of the two base angles, because it is the sum of the angles not adjacent to it.

In investigations with chords, students will use the properties of isosceles triangles to solve for unknown angles, radii or chord lengths. It will be essential for students to understand that the perpendicular bisector of the chord represents the altitude and the perpendicular bisector corresponding to the base of an isosceles triangle that is formed by the chord with the radii at its endpoints.

Students should already have prior knowledge of how to prove triangles are congruent. At a minimum, time should be spent reviewing the topic, with special attention paid to the SAS, SSS, ASA, and HL congruence cases. These will prove useful when exploring properties of line segments in circles as well.

Circles

Once the knowledge of triangles is sufficiently established and the students begin to build a comfort level with very basic geometric proofs, it will be easier to introduce the circle into the unit. New vocabulary, such as radius, diameter, arc, chord, secant, tangent, inscribed, and circumscribed, will be used as they arise in the lessons.

Tangents to a Circle

The first property that will be explored is the relationship between the circle and a line tangent to the circle. Interactive software, such as Geometer's Sketchpad and/or GeoGebra allows students to use concrete measures of segment lengths to prove that tangent lines are perpendicular to the circle they intersect. This exploration is rooted in their prior experience with the Pythagorean Theorem.

It would also be beneficial for students to use compasses and straight edges to construct perpendicular lines by hand. This builds their understanding that the shortest distance from a line to a point not on the line is a perpendicular segment. Thus, the shortest distance from the center of the circle to the tangent line is the radius of the circle, and the radius of the circle is, therefore, perpendicular to the tangent line.

Conversely, students can construct circles that are tangent to a line by first constructing perpendicular lines, then constructing a circle whose center is on the perpendicular line and whose edge intersects where the two line segments intersect.

Another important relationship that students need to discover and prove is that there are exactly two congruent line segments that are tangent to a circle from any point external to the circle. This can and should be proven by drawing on students' previous knowledge of congruent right triangles. Once it has been established that the tangent is perpendicular to the radius, student can prove that two tangent segments drawn from the same point to the circle are congruent by constructing congruent right triangles. The radii will be one of the legs for each of the triangles, the tangent lines the other legs, and the segment connecting the external point to the center will be the hypotenuse. The hypotenuse will be shared by the two triangles. Using the HL congruence case, the tangent lines will be proved to be congruent.

Central Angles and Chords

The next property to be explored is the relationship between a central angle, its intercepted arc, and the chord that connects the points of interception. By the time students get to high school, they have acquired the knowledge that the measure of the angle around a point is 360 degrees. However, they may not be familiar with the concept of arcs. They may not know what an arc is or how it is measured. By definition, the central angle has the same measure as its intercepted arc.

A chord is a segment inside the circle determined by two points on the circle. Like a central angle, the length of a chord also has a special relationship with the arc it intercepts. Because of the complete symmetry of circles, a chord can be used as the base of an isosceles triangle whose remaining sides are radii of the circle. Using the symmetry of the isosceles triangle, the isosceles triangle can be bisected to form two congruent right triangles. From there, all properties of right triangles can be employed to solve for unknown side lengths, angles, etc. It can further be investigated and proved that congruent chords on the same circle intercept congruent arcs.

Perhaps the most compelling property of chords on a circle is that the perpendicular bisector of a chord passes through the center of the circle. This can be investigated and explored by having students construct a circle and then use paper folding to create chords and their perpendicular bisectors. GeoGebra/Sketchpad can also be used in a similar fashion to demonstrate this property. This property of chords can be used to determine the center of a circle that circumscribes a triangle. The three sides of the triangle would all be considered as chords on this circle. Students can use a straight edge to construct a triangle. By using paper folding or other tools available, students can find the perpendicular bisector of each side of the triangle. The three will meet at the center of the circle. Students can then use a compass to construct the circle by hand.

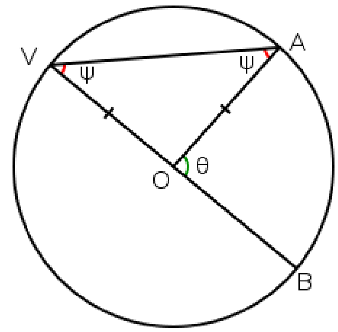
Inscribed Angles

An inscribed angle has its vertex on the circle and the sides of the angles are chords in the circle. Like central angles, an inscribed angle has a special connection to the arc it intercepts. It is half the measure of the arc. This property can easily be discovered by analyzing patterns in measurements by constructing inscribed angles using technology. Technology allows for easy and accurate measurements and it allows for the vertex and chords to move freely on the circle's edge to create an infinite number of angles for any circle. By exploring with technology, students should somewhat quickly be able to observe and conclude that an inscribed angle has half the measure of a central angle that intercepts the same arc.

This key idea can also be proved to be true using properties of triangles as shown in the diagrams below. Using constructions of this nature, students will prove that the relationship between inscribed angles and intercepted arcs is more than mere coincidence. It can be proved to be true based on properties of triangles that they are already familiar with.

This proof provides the opportunity to show how the use of drawing in supplemental lines to create triangles where they did not previously exist can aid in proving geometric relationships.

CASE 1: Inscribed Angle where one side is a diameter.



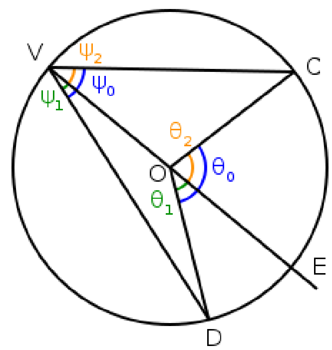
Prove that $m\angle BVA = \frac{1}{2}\, m\angle BOA$.

Since OA and OV are both radii, they are congruent, making ΔAOV isosceles. This also proves that $m\angle OVA = m\angle OAV$.

Angle θ is external to the triangle. Therefore, $m\angle BOA = m\angle OVA + m\angle OAV$, or $m\angle BOA = 2m\angle OAV$.

$\therefore m\angle BVA = \frac{1}{2}\, m\angle BOA$.

CASE 2: Inscribed Angle with the center of the circle in its interior



Prove that $m\angle DVC = \frac{1}{2}\, m\angle DOC$.

Segment EV is drawn in to create two isosceles triangles, ΔOCV and ΔODV, with segments OD, OV, and OC all being radii of the circle with center O. Using the reasoning provided in CASE 1 above, $m\angle DVE = \frac{1}{2}\, m\angle DOE$ and $m\angle CVE = \frac{1}{2}\, m\angle COE$.

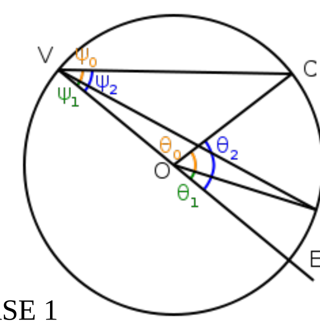
$m\angle DVC = m\angle DVE + m\angle EVC$

$m\angle DVC = \frac{1}{2}\, m\angle DOE + \frac{1}{2}\, m\angle COE = \frac{1}{2}\,(m\angle DOE + m\angle EOC)$

$m\angle DOC = m\angle DOE + m\angle EOC$

$\therefore m\angle DVC = \frac{1}{2}\, m\angle DOC$.

CASE 3: Inscribed Angle with the center of the circle in its exterior



Prove that $m\angle DVC = \frac{1}{2}\, m\angle DOC$.

Segment EV is drawn in to create two isosceles triangles, ΔOCV and ΔODV, with segments OD, OV, and OC all being radii of the circle with center O. Using the reasoning provided in CASE 1 above, $m\angle DVE = \frac{1}{2}\, m\angle DOE$ and $m\angle EVC = \frac{1}{2}\, m\angle EOC$.

$m\angle DVC = m\angle EVC - m\angle EVD$ and $m\angle DOC = m\angle EOC - m\angle EOD$

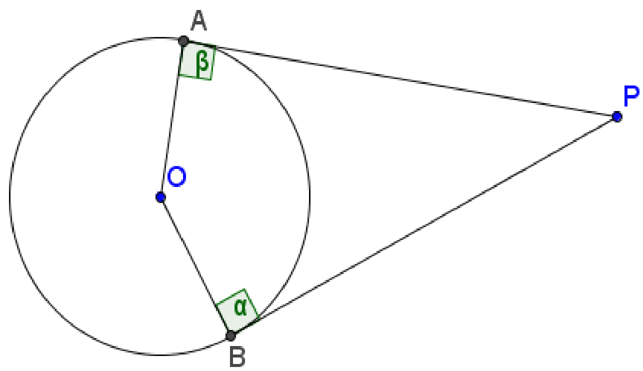
$m\angle EVC = \frac{1}{2}\, m\angle EOC$ and $m\angle EVD = \frac{1}{2}\, m\angle EOD$ as proven in CASE 1 above.

$m\angle DVC = \frac{1}{2}\,(m\angle EOC - m\angle EOD)$

$\therefore m\angle DVC = \frac{1}{2}\, m\angle DOC$

Exterior Angles Formed by Intersecting Tangents

Tangent lines that are not parallel will form an angle exterior to the circle. The angle will intercept the circle forming a minor arc (one less than 180 degrees) and a major arc (the remainder of the circle). Through exploration with constructions, either on paper or with technology, students can discover the unique relationship that the arc measures have with the measure of the angle formed by the intersecting tangent lines. The sum of the minor arc and the angle formed by the two tangents is always 180 degrees. Students should be challenged to attempt to prove why this is true.



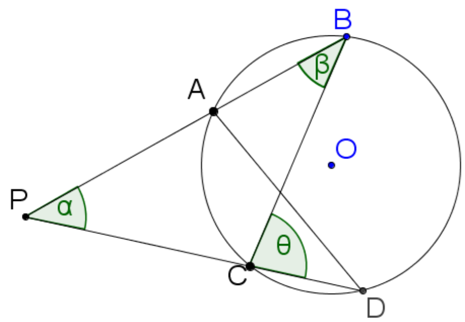
There are two commonly used approaches available to prove this statement.

1. Given that they have previously used supplemental lines to help demonstrate the existence of geometric relationships, they can construct a segment from the center of the circle to the point of intersection of the two tangent lines. From here, they should be able to use the HL congruence case to establish that the triangles are congruent. Given that the sum of the angles in any triangle is 180 degrees and that the tangent line is perpendicular to the radius (as previously discovered), the remaining angles in each of the right triangles must sum to 90 degrees. Thus, the sum of the central angle and the sum of the external angle must sum to 180 degrees, since each is double the angles of the congruent triangles.

2. Some students may have previous knowledge that the sum of the interior angles of any quadrilateral is 360 degrees. Given that the two known angles in the diagram above are always 90 degrees each, the other two unknown angles must sum to 180.

Exterior Angles Formed by Intersecting Secants

When two secant lines intersect outside of the circle, two arcs are intercepted by the angle they form. The measure of the angle formed by the two secants can be investigated and proved to be half the difference of these two arcs. Again, using technology for the flexibility in construction and ease of measurement, students will be able to experiment



with a variety of circles to analyze patterns and perhaps derive this relationship from inductive reasoning.

By drawing a supplemental line that connects a point on one of the intercepted arcs diagonally with a point on the other arc, students can use the previous relationship that they discovered regarding inscribed angles to prove that this is always true.

Prove that $m\angle\alpha = m\,\widehat{BD} - m\widehat{AC}$.

$\Theta = \frac{1}{2}\,\widehat{BD}$ and $\beta = \frac{1}{2}\,\widehat{AC}$ as previously proven using inscribed angle relationships.

$\Theta = \alpha + \beta$ and therefore $\alpha = \Theta - \beta$ from the external angle theorem.

$\therefore \alpha = \frac{1}{2}(m\,\widehat{BD} - m\widehat{AC})$

Interior Angles Formed by Intersecting Chords/Secants

When two chords intersect inside the circle, vertical angles are created and the chords intercept the circle at two arcs. Assuming that the point of intersection is somewhere other than the center, the two arcs will be different in measure, although the vertical angles intercepting those arcs are equal in measure. It can be explored and discovered that the sum of the intercepted arcs is equal to the sum of the vertical angles, or further, that half of the sum of the arcs is equal to one of the angles.

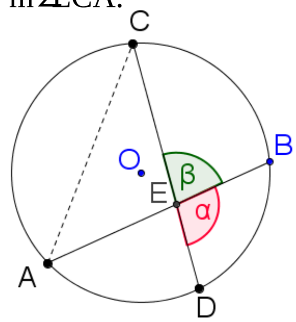
Using a geometric construction, this can be proved to be true for all cases by drawing on the previous proof that inscribed angles are half the measure of their intercepted arcs.

Prove that $\beta = \frac{1}{2}(\widehat{AD} + (\widehat{BC}))$.

Angle β is exterior to ΔAEC, therefore $\beta = m\measuredangle EAC + m\measuredangle ECA$.

$m\measuredangle EAC = \frac{1}{2} m\widehat{BC}$ and $m\measuredangle ECA = \frac{1}{2} m\widehat{AD}$.

$\therefore \beta = \frac{1}{2}(\widehat{AD} + \widehat{BC})$.



Strategies

Interactive software

When accessible, students should use computers with interactive geometry software. Students who are familiar with the CORE Plus curriculum may have previous exposure to the CPMP Tools, a free web-based software downloadable at http://www.wmich.edu/cpmp/CPMP-Tools/. GeoGebra is also free software that can be accessed and downloaded at http://www.GeoGebra.org. If available at your school, Geometers Sketchpad is also ideal for having students construct and explore the geometric properties of circles.

If a computer cart or lab is not available, it is still quite useful to use these interactive tools for whole class demonstrations of the properties. When combined with an interactive white board, students should still be afforded the opportunity to make and test conjectures about the properties.

Paper Folding

Paper folding allows for hands on manipulation and a more active engagement in the exploration of the properties of circles. Using a straight edge, a compass, and translucent patty paper, students can easily construct perpendicular bisectors of chords, prove congruence through overlapping segments, and bisect angles. All of these are key components used in problem solving with circles, lines and angles.

Graphic Organizers

It is important that students accurately document each relationship that they explore because the relationships are so inter-related. For example, the relationships between angles formed by secants or chords draw on the students' prior knowledge of the relationship between an inscribed angle and its intercepted arc. I intend to have a 3-column chart of *Vocabulary and BIG IDEAS* (Appendix). In column 1, the students title the big idea, in column two they describe it in their own words, and in column three they draw an illustration.

Classroom Activities

The following activities should come after a review of properties of triangles.

Activity 1: Tangent lines and circles

Three major understandings should result from this activity. First, students should know that a tangent line intersects the circle at exactly one point. Secondly, they should discover that the tangent line is perpendicular to the radius of the circle at the point of tangency. Finally, students should determine that there are exactly two congruent tangent linesegments from a point exterior to the circle.

Using Geometer's Sketchpad or some comparable software, the teacher should have the diagram of the circle and tangent line projected on the SMARTBoard. Point P should be moved around the exterior of the circle to a variety of positions to help students visually recognize that the tangent line is perpendicular to the radius. In a similar way, point O can be moved around to demonstrate the same. Refer to the Appendix for the student handout that accompanies this activity.

For question 1, students should be encouraged to come up with as many ideas as possible to prove the perpendicular relationship. For example, some may want to measure the angle. Others may want to measure distances and use the Pythagorean Theorem. Have students share with the class any suggestions that they come up with.

The teacher may lead the students in using paper folding to prove the perpendicular relationship if students did not use this as a method of proof. Students should trace the diagram onto patty paper. Have the students fold the tangent line onto itself making a crease at the point of tangency. The crease should coincide with the radius. When unfolded, students should clearly see that because the two angles formed are equal in measure as well as a linear pair, they must each be 90°.[i]

For question 2, when directed to draw the second tangent line, anticipate that many students will approximate where the point of tangency should be and then they'll draw in the line. Students who take this approach should be made to prove that their approximation works using methods similar to those used to answer question 1. Again, paper folding is a good option to help draw the line accurately. Students can draw a line connecting point P to the center of the circle O. Using $\overline{OP}$ as an axis of symmetry, the circle can be folded in half. The second tangent line should overlap the other tangent line and the two radii at the points of tangency should also overlap. This will show that the two tangent segments are congruent.

Students should be encouraged to prove that the two resultant triangles (ΔOAP and ΔOBP) are congruent using the HL congruency case. From there, the discussion can follow that segment OP bisects ∠AOB and ∠APB.

Students will then apply the knowledge gained to solve problems that involve tangent lines and circles. Additionally, students should document this BIG IDEA on their graphic organizer. It is key that they realize that tangent lines a) touch the circle at exactly one point and b) are perpendicular to the radius at the point of intersection with the circle.

Activity Two: Properties of Chords in Circles

This hands-on activity allows students to discover the important relationship between the perpendicular bisector of a chord and the center of the circle. Students can use this property to find the center of a circle that circumscribes a triangle.

Each student should be given several pieces of patty paper for experimentation. On one sheet, have students use a compass to construct a circle. Instruct students to draw at least three chords on the circle, one somewhat close to the center, one somewhat far from the center, and one about halfway from the center to the circle. These chords should not be drawn parallel to each other. Using paper folding, students find the perpendicular bisector of each chord, as they carefully overlapping the chord on itself and lining up the endpoints of each chord on top of each other. Using colored pencils, they should draw lines along their folds to discover that they all intersect at the center of the circle. (Note: Two non-parallel chords are sufficient to find the center of the circle.)

On another sheet of patty paper, students will use a straight edge to draw a triangle. If students are working in cooperative groups, encourage the group members to create a variety of different triangles (i.e., short and fat, tall and skinny, scalene, etc.). The objective for this portion of the activity is for students to construct a circle that will circumscribe the triangle they have drawn.

Again, using paper folding, students should find and draw the perpendicular bisectors for each side of their triangle. The point of intersection of the three lines they have drawn is the center of the circle and the distance from the center to each of the vertices of the triangle is the radius. Using a compass, students should be able to construct a circle around their triangles.

Students should reflect on the activity and write down their observations in the form of a conjecture. After a whole class discussion of the findings, the property should be documented on their BIG IDEAS graphic organizer.

Activity Three: Chords and Central Angles

There are several enduring understandings for this activity. First, central angles are, by definition, equal in measure to the arc they intercept. Additionally, a chord can serve as the base of an isosceles triangle whose vertex is the center of the circle and the perpendicular bisector will create two congruent right triangles. Additionally, the perpendicular bisector for the chord will also bisect the central angle opposite the chord. Finally, chords of the same length in a circle intercept congruent arcs.

Begin the lesson with an introduction to the concept of “arcs”. For most students, this may be a completely unexplored topic. Define the measure of an arc as being equal to the central angle that intercepts it. Discuss and demonstrate the proper form for documenting arcs using the endpoints of the arc and the arc symbol above it.

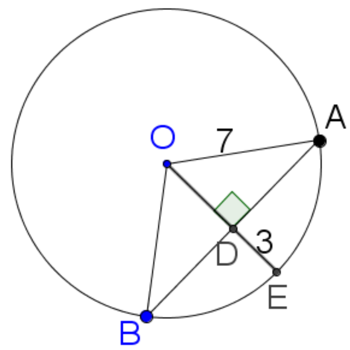
During this lesson, students should be able to use the relationships between the arc, the central angle, and the angle bisector to solve for the measure of the radius of the circle, the chord length, or the central angle.

In the accompanying student worksheet (Appendix), students are asked a series of guiding questions to help them solve for the various unknown measures for each example. In the first example, the radius is unknown. Students use their previous discovery of perpendicular bisectors to create congruent right triangles. Use trigonometric ratios, students can solve for the radius of the circle.

In the second example, students are given the intercepted arc measure and the radius to help them solve for the length of the intercepted chord. Again, students are prompted to create two congruent right triangles. Using trigonometric ratios, students can solve for the length of half of the chord which they will double to find the chord length.

In the final example, students are given the radius and the chord length in order to determine the measure of the intercepted arc. Creating congruent right triangles allows for students to use trigonometric ratios to solve for half of the measure of the central angle.Doubling that measure leads directly to determining the measure of the interceptedarc.

Other examples can be explored that would allow students to use the Pythagorean Theorem to solve for unknown measures as well. In this example below, the altitude of the triangle must be 4 since the radius is 7 and $\overline{DE}$ is 3. Half of the chord length can be found by $\sqrt{7^2 - 4^2}$. Doubling that result will give the chord length.



Students should demonstrate several examples on their BIG IDEAS graphic organizer. The recurring theme among almost all of the problems is the need to create congruent right triangles by adding supplemental lines. This allows for the properties of right triangles to be used to solve for unknown measurements.

Activity 4: Inscribed and Central Angles

The primary property to be mastered in this lesson is that the measure of an angle inscribed in a circle is equal to half of the measure of the central angle that intercepts the same arc. This property is easily discovered through observation using applets to demonstrate to the class. I have had success using CPMP tools: Interactive Geometry Explore Angles and Arcs. Points A, B and C lie on the same circle. Chords AB and BC create an inscribed angle with intercepted arc AC. Point B can move freely between A and C on the circle, but as long as A and C are not moved, the measure of angle B remains constant. From this, students can conclude that there are infinite angles of the same measure that can intercept the same arc.

This same applet also has the central angle which intercepts the same arc depicted. By manipulating the position of all three points, students can quickly recognize that the central angle is double the measure of the inscribed angle. The teacher should make certain to show all cases, where the center of the circle is on the angle, the center of the circle lies in the interior of the angle, and the center of the circle lies on the exterior of the angle. Once this concept is recognized, it should be documented on the BIG IDEAS graphic organizer.

Time in class should be devoted to exploring the proofs of these 3 cases. I would anticipate that the students will experience varying levels of success with the proof process, but it is worthwhile to expose all students to it. Some segments of the population may require more scaffolding than others.

The concept needs to be reinforced with several examples where it can be practiced. A student worksheet is attached in the Appendix.

Activity 5: Angles Formed by Tangent Lines

The objective of this lesson is for students to discover that the sum of the measures of the angle formed by the tangent lines and the minor arc it intercepts is 180°. Students will draw on their prior knowledge of tangent lines from earlier in the unit to realize that the segment connecting the center of the circle with the point of intersection of the tangent lines will form two congruent right triangles. This same segment bisects the minor arc, the central angle, and the exterior angle. Knowing that the sum of the angles in all triangles is 180°, students can easily calculate the unknown measure in a series of problems. A worksheet is attached in the Appendix.

Activity 6: Two Chords or Secant Lines Intersect in a Circle

The objective of this activity is for students to solve for measures of unknown arcs and angles when two chords intersect in a circle. A discussion and demonstration should take place to show how this is different than having two diameters intersect at the center. Begin with having a diagram with two diameters intersecting and two sets of vertical central angles result. Students should clearly recall that the measures of the intercepted arcs are the same as the measures of the central angles. Using interactive software, have students observe what happens to the arc measures as the vertex is moved from the center of the circle, keeping the angle measures constant. As the vertex moves closer to a given side of the circle, that intercepted arc becomes smaller while the one opposite it becomes larger. The vertical angles are congruent by definition, but quite obviously their intercepted arcs no longer are.

Using GeoGebra applets to explore, students can attempt to use inductive reasoning to arrive at the relationship between the angle measures and arc measures. To make this a bit more obvious to see, the teacher may demonstrate by making the vertical angles equal to such measures as 50°, 100°, and other such measures that may be able to make the pattern emerge more quickly. Once the pattern has been discovered, the teacher can facilitate a discussion of the proof using the concept of the external angle theorem.

This pattern should be documented on the BIG IDEAS graphic organizer. A practice worksheet for students is attached in the Appendix.

Resources

CPMP Tools found online at http://www.wmich.edu/cpmp/CPMP-Tools/.

GeoGebra found online at http://www.geogebra.org.

GeoGebra wiki file illustrating when two chords intersect inside a circle:

http://www.geogebra.org/en/upload/files/english/Guy/Circles_and_angles/Two_chords_angles.html

GeoGebra wiki allowing for an infinite number of examples to practice solving for unknown arc measures or vertical angle measures:

http://www.GeoGebra.org/en/upload/files/english/Guy/Circles_and_angles/Two_chords_angles_practice.html

GeoGebra wiki file illustrating when two secants intersect outside of a circle

http://www.GeoGebra.org/en/upload/files/english/Guy/Circles_and_angles/Two_secants_angles.html

GeoGebra wiki file allowing for infinite examples and practice problems involving secants and their intercepted arcs:

http://www.GeoGebra.org/en/upload/files/english/Guy/Circles_and_angles/Two_secants_angles_practice.html

Worksheet in PDF format for solving problems involving angles formed by secants, chords, and tangent lines:

http://www.kutasoftware.com/FreeWorksheets/GeoWorksheets/11-Secant%20Angles.pdf

Worksheet in PDF format for solving problems involving secants, tangents and arc lengths:

http://www.kutasoftware.com/FreeWorksheets/GeoWorksheets/11-Secant-Tangent%20and%20Tangent-Tangent%20Angles.pdf

Appendix A

VOCABULARY AND BIG IDEAS

BIG IDEA	IN WORDS	IN A PICTURE
Circle		
Tangent Line		
Central Angle		
Arc of a Circle		
Chord		
Inscribed Angle		
Secant Line		
When 2 chords intersect in the interior of a circle		
When 2 secants or tangents intersect in the exterior of a circle		

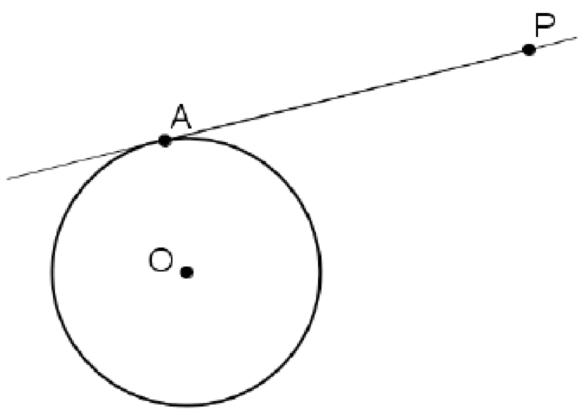
Appendix B

When Lines Intercept a Circle

Activity 1: Tangent Lines

TANGENT LINE: A line that touches a circle at exactly one point

In the diagram below, $\overleftrightarrow{AP}$ is tangent to $\odot$O. The shortest distance from the center of the circle to the tangent line is radius $\overline{OA}$.



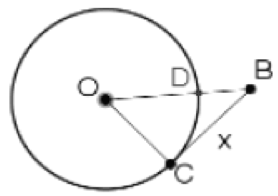
1. What Important relationship exists between the radius and the tangent line? How can you prove this?

2. Use the answer from #1 to help you accurately draw another line of tangency from point P. Label the point of intersection point B. What is the relationship between $\overline{AP}$ and $\overline{BP}$? How can you prove this?

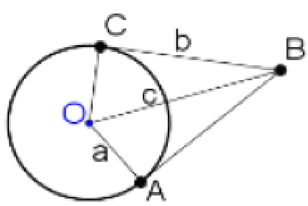
3. Draw a segment connecting point P to the center of the circle. Find two triangles that are congruent.

4. Refer to your finished diagram above. List all of the significant properties you have discovered about tangent segments AP and BP.

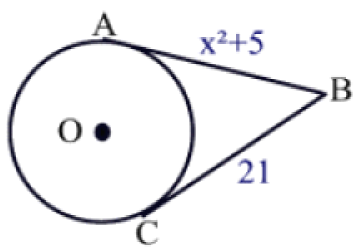
Use the properties that exist between circles and tangent lines to solve for each unknown quantity in the problems below.



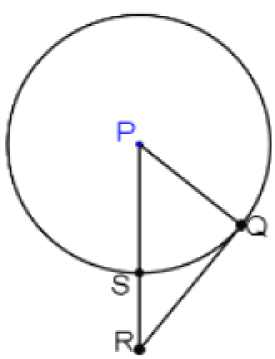
CB is tangent to circle O.
DB = 6
OC = 9
Find CB



BC and BA are tangent to cirlce O.
OC = 12
AB = 16
Find OA, BC, and OB.



AB and BC are tangents to circle O.
AB = x^2+5
BC = 21
Find x.

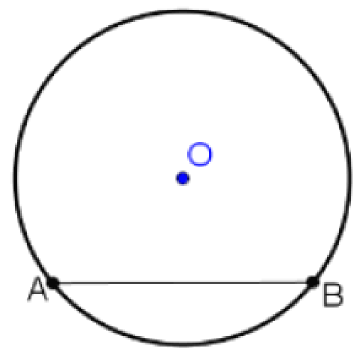


RQ is tangent to circle P.
RQ = 3
PQ= 3
Find RS

Name: ______________________________ Date: ____________________

The Relationship between Chords and Central Angles

Part 1: Solving for the radius of the circle



Given that m $\widehat{AB}$ = 98° and chord AB = 6 cm, find the radius of circle O.

Draw segments OB and OA to create a triangle.

1. What type of triangle is OAB? How can you prove this without measuring sides or angles?

Draw a segment from point O to chord AB that bisects angle BOA. Label the point of intersection C.

2. How long are segments BC and AC? How do you know for certain?

3. Prove that triangles OCB and OCA are congruent.

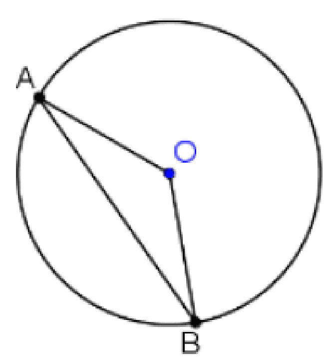
4. What is the measure of angles BOC and AOC? How do you know for certain?

5. Use the properties of triangles OCB and OCA to solve for the radius of the circle. Show work that supports your strategy.

Part 2: Solving for the chord length in the circle

Given that the m$\widehat{AB} = 130\,°$ and the radius of the circle is 8 inches, find the length of chord AB.

Draw in the angle bisector for AOB to create two triangles. Mark the point of intersection C.



1. What is the measure of angle OCB? OCA? How do you know for certain?

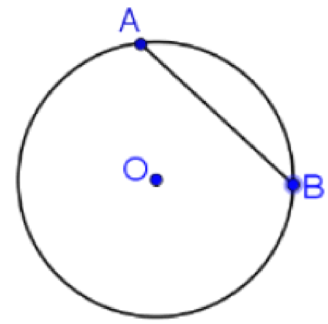
2. What is the measure of angle BOC? AOC?

3. Use this information to solve for segment BC. Show work that supports your strategy.

4. What is the relationship between segments BC and BA? Solve for the length of chord AB.

Part 3: Solving for the measure of the intercepted arc

Given that the chord length AB is 15 cm and the radius of circle O is 12 cm, find the measure of arc $\widehat{AB}$.



First, draw segments OA and OB to create an isosceles triangle.

Next draw the perpendicular bisector of chord AB. Label the point of intersection C.

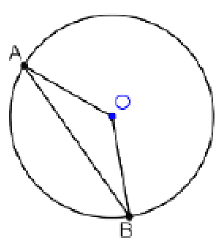
1. What is the measure of segment AC? BC? Include appropriate units in your response.

2. Use segments OA and AC to determine the measure of angle AOC. Include appropriate units with your response. Show work that supports your strategy.

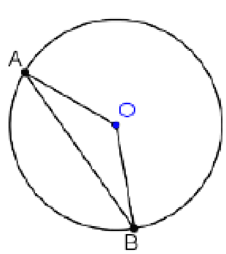
3. What is the relationship between the measures of angles AOC and AOB? Determine the measure of angle AOB.

4. What is the relationship between angle AOB and $\widehat{AB}$? Find the measure of $\widehat{AB}$.

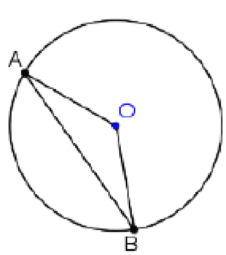
MIXED PRACTICE PROBLEMS (figure not drawn to scale) All responses should include appropriate units.



1. If the radius of the circle is 24 inches and $m\widehat{AB}$ is 140°, find the length of chord AB.



2. If $m\widehat{AB}$ is 110° and chord AB has a length of 16, what is the radius of the circle?



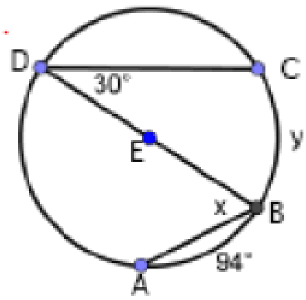
3. If chord AB is 48 units in length and the radius of the circle is 30, what is the measure of arc AB?

Name: ____ Date: _____________________

Inscribed Angles, Central Angles, and Intercepted Arcs

Use properties of central angles, inscribed angles, triangles, and semicircles to solve for the unknown measurement in the following problems.

1.



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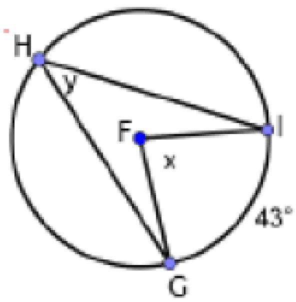

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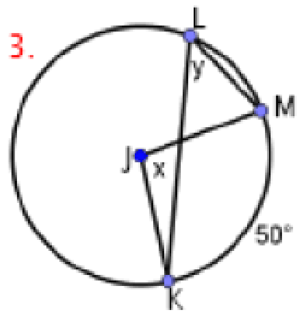

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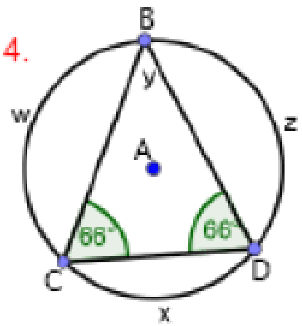
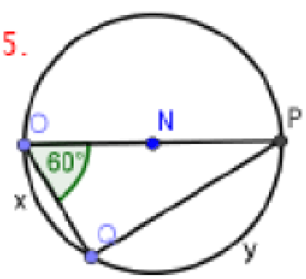
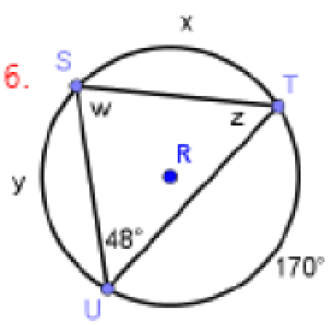


5.



6.



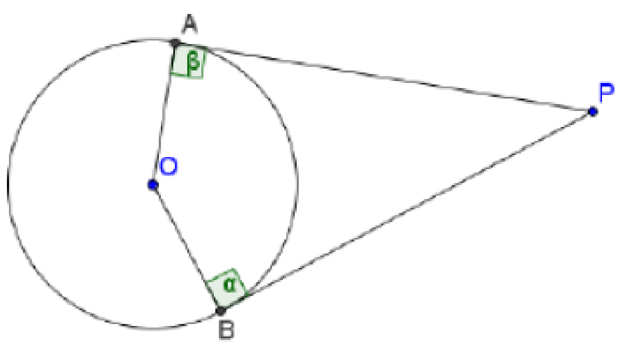
Name: ___________________________________ Date: ____________________

Relationship between Tangent Lines and the Arc They Intercept

OBJECTIVE: Discover and prove the relationship that exists between the measures of the angle formed by two tangent lines at their point of intersection and the minor arc that they intercept

- As explored earlier, from any point exterior to a circle, two congruent lines of tangency can be constructed.
- These lines each intersect the circle at exactly one point and are perpendicular to the radius of the circle at the point of tangency.

In the diagram below, draw an auxiliary line from point P to the center of the circle O, creating two congruent right triangles.



1. Given that the measure of angle AOB is 100°, find each of the following:

   a) m∠AOP = _______

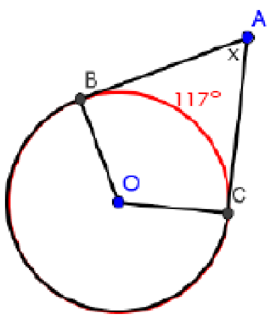
   b) m∠APO = _______

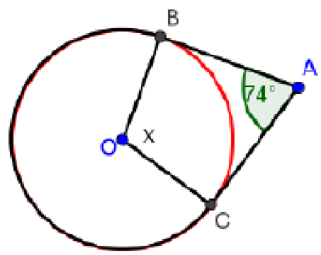
   c) m$\widehat{AB}$ = ________

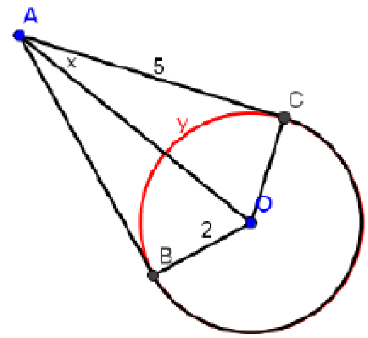
2. What is the sum of the measures of ∠AOP and ∠APO? Describe why this is ALWAYS the case.

3. What is the measure of angle APB? What is its relationship with angle AOB? Describe why this relationship will always be true.

4. In each diagram below, solve for the unknown measurement.







Name: ______________________________ Date: _________________________

Two chords intersect in the interior of a circle

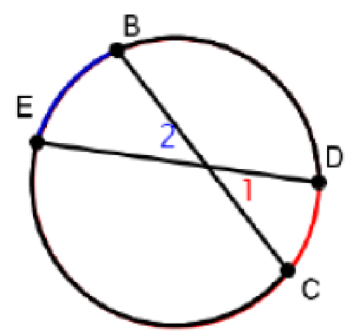
When two chords intersect in the interior of a circle, then the measure of each angle is equal to one half of the sum of the arcs that the angle and its vertical angle intercept. Another way to think of this property is that the sum of the two vertical angles is equal to the sum of the arc intercepted by the two vertical angles.

Example:

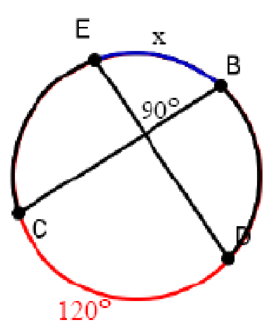
$m\angle 1 = \frac{1}{2}\,(m\widehat{DC} + m\widehat{BE})$

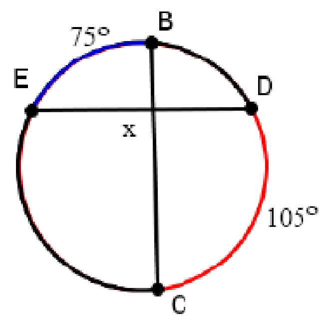
OR

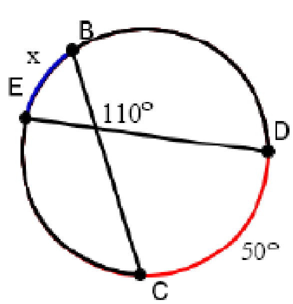
$m\angle 1 + m\angle 2 = m\widehat{DC} + m\widehat{BE}$

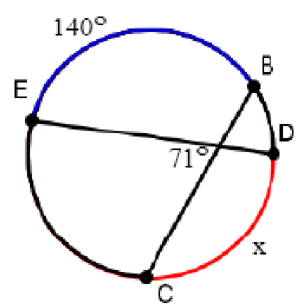


In each of the following circles, solve to find the unknown measurement.









Curriculum Unit Title: Investigating Circles

Author: Kathleen Wilson

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Properties of Circles can be described by theorems that integrate algebraic and geometric understanding, modeling, and proof.

ESSENTIAL QUESTION(S) for the UNIT

What properties exist when lines and angles intercept a circle and how can those properties be proved?

CONCEPT A	CONCEPT B	CONCEPT C
Tangents to a Circle	Chords, Arcs, Central and Inscribed Angles	Interior and Exterior Angles Formed by Tangents and Secants
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
What are the important properties of tangents to a circle and how can they be verified and applied to solve problems?	What are important properties of chords, arcs and central angles of a circle and how are they proven and applied? What is an inscribed angle in a circle and how is it related to the arc it intercepts?	How are the measures of other interior angles formed by secants related to the measures of the arcs they intercept? How are the measures of exterior angles formed by two tangents, two secants, or a tangent and a secant related to the measures of the arcs they intercept?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
Radius Diameter Tangent Line Point of Tangency Perpendicular	Chord Central Angle Major Arc Minor Arc Perpendicular Bisector Inscribed Angle Intercepted Arc	Secant Vertical Angles Interior Angle Exterior Angle

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES
