Exploring Allometry through Various Math Concepts

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Introduction

"Are you ready for the work place in the real world?" This is a question that most employers want to know about students when they graduate from high school. According to the article, *Are They Really Ready to Work*, "With significant numbers of workers retiring over the next 10 years, the United States is facing a serious challenge in preparing students to meet workplace demands in an increasing complex, knowledge and technology-based, global economy."[1] As teachers, we have to prepare our students academically in order to be successful. In particular, we must teach them the basic skills of reading, writing, science, social studies, and especially math. In the past, most students have not been performing adequately upon graduation due to lack of academic skills. The article cites certain areas that employers consider very important skills, but many students have not been 100% in the areas of writing in English, mathematics, reading comprehension, applied skills, and critical thinking/problem solving.[2]

Recently, the nation has been moving toward having common core standards for every student in every state. The Common Core State Standards in Mathematics (CCSS-M) is especially helpful for me as a 5th grade teacher at Elbert-Palmer Elementary School in the Christina School District because so many of my students move around. For example, they move from other states and then come back in the middle of the school year as well as change schools often. CCSS-M promises to provide consistency. Something has to change in order to improve the required skills mentioned above. My main goal in creating this unit is to engage my students academically in a real world application of mathematical by exploring allometry through various math concepts that focus on the CCSS-M and required job skills. This unit will be a step in the right direction due to the engaging aspects. Lessons that are engaging will allow self-exploration and led to deeper understanding of concepts. Also, the unit will address essential academic skills for any level of education such as mathematics, critical thinking, and problem solving.

Demographics

The Christina School District is a large school district located in Wilmington, Delaware. They serve the Wilmington and Newark area from pre-school through high school serving over 16,000 students. There are 3 inner city elementary schools. I teach 5th grade at Elbert-Palmer Elementary School. Enrollment data from the school's profile, according to the Delaware Department of Education, is shown in Figure 1.

Fall Enrollment

Kindergarten	48
1st	56
2nd	58
3rd	42
4th	42
5th	52
Total	298

Enrollment by Race/Ethnicity

African American	86.8%
Asian	0.3%
Hawaiian	0.3%
Hispanic/Latino	11.1%
White	1.5%

Figure 1[3]

According to the school's web site, my school contains 3.0% English language learners, and 94.6% low-income students. Special education students make up 10.7% of the school population, and 98% of the students receive free or reduced lunch.

Elbert-Palmer has held a superior school rating for the 2010-2011 and 2011-2012 school years. This means that academically our students have been performing well on the state testing. Test scores are shown in the Figure 2 below.

Student Achievement: % of Students meeting State Standards

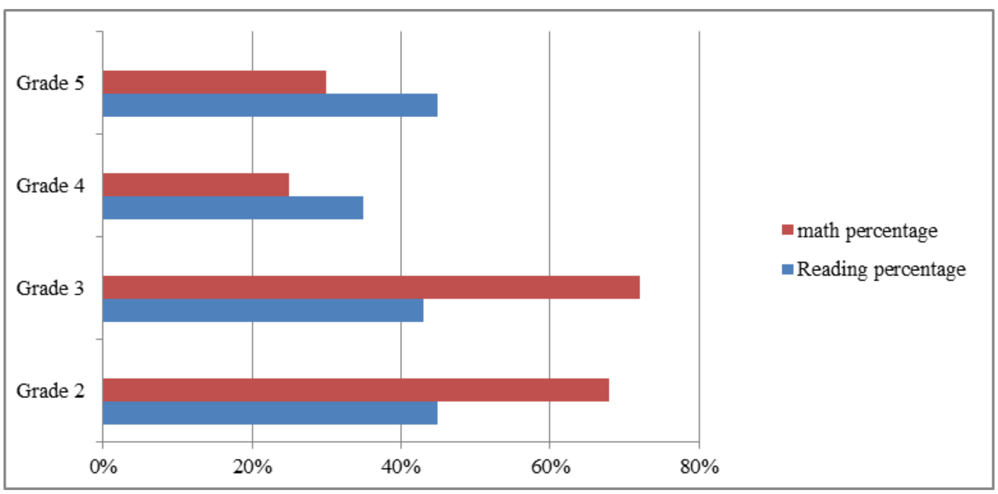


Figure 2

Objective

The objective I have for this unit is for students to increase their math skills through engaging activities utilizing the mathematical modeling cycling. Modeling is a mathematical practice standard for all grade levels in the Common Core State Standards for Mathematics (CCCS-M). Students will learn about the concept of allometry. According to the Wikipedia, allometry is a term used in biology that studies the relationship of body size to shape.[4] Allometry usually examines shape differences and the ratios between organisms or over time. The reason I decided to have my class study allometry to learn various math concepts is because I think it will be interesting to the students. They will actually like the concept of learning math through real life situations. Since the unit will be based on relevant hands on activities, students will be able to retain the knowledge longer. Also, the students are learning mathematics combined with biology, which will be useful for them later in life.

This unit will address the CCCS-M Grade 5 topics of analyzing patterns and relationships (CC.5.A.1), representing and interpreting data (CC.5.MD.1 and 2), and graphing points on the coordinate plan to solve real world and mathematical problems (CC.5.G.1 and 2). Currently, in my district's math curriculum, these standards are addressed, but this unit will put more emphasis on the topics.

Background

Mathematical Modeling Cycle

Prior to taking this course, I did not know much on modeling or the mathematical modeling cycle. When I would teach students, I used the mathematical model cycle without knowing. The students would have the problem/question as the initial starting point. Next, they would decide which model to use. Then they would analyze the model by solving it. If for some reason the process were unsuccessful, then they would start over again. I have learned through various activities what the mathematical modeling cycle consists of specifically. Some of the activities we were engaged in during the class were a M&M activity where the face up M's were counted and a model was determined, a Star Logo computer activity where an epidemic was observed using certain guidelines, and developing a model of the perimeter and area just to name a few of the predictable value exercises we performed. Each activity was analyzed utilizing the mathematical cycle. The 5 steps in the cycle are:

1. System, Reality
2. Problem/Questions
3. Formulate Mathematical Model
4. Analyze the model/Simplification
5. Prediction

The system, or reality is the experiment students will study. As in this unit, it can utilize real-world examples that allow them to observe relationships. The next step is to have students ask questions that they will be answering throughout the modeling process.

The modeling process also encourages students to ask more questions as they learn about the system.

During the "Formulate a Mathematical Model" stage, the students make tables and graphs from data and measurements they collect. In the "Analyze the model" stage, students observe the data and create equations that fit the data. They can use software such as Excel spreadsheets and to find the best regression equations for the data. The next step is to make predictions from their tables, graphs or equations for other situations.

Since it is a cycle, the final stage is to evaluate whether or not the model is good enough. If it is not a good predictor, students will have to collect more data to try to get a better model.

Besides being engaged in activities that involve the mathematical modeling cycle, another overall concept we learned throughout the course was the difference between descriptive/empirical models versus analytical/fundamental models. In class we learned that descriptive/empirical models are derived from data. Descriptive models are often a starting point for fundamental models. It uses a mathematical object such as a graph or equation to describe the data. We found the equation for best-fitting lines using Excel or graphing calculators. Analytical/fundamental models are derived from the physical world. They begin with a system and figuring out what physical laws are used. Analytical models require us to know more than just the mathematics; we need to understand the science, also.

Allometry

Students will study isometric allometry to help them reinforce graphing skills, problem-solving and critical thinking. Allometry involves the ratio of two measurements and studies it for a range of sizes, animals or ages. The study of allometry seeks to explain the reason for objects or organisms having a constant ratio over or across time/age or species. "Isometric scaling occurs when changes in size (during growth or over evolutionary time) do not lead to changes in proportion, as in maintaining a constant ratio. For example the length of a frog's legs is approximately the same length as its body. This one-to-one ratio generally holds true for small frogs and large frogs.[5] This term of isometric scaling is used throughout many of the sources that I have read. It reminds me of the human body and its relationship to arms – a person's height is the same as her arm span.[6]

In the article, "Allometry and Jumping in Frogs: Helping the Twain to Meet" the author was experimenting with whether or not size was able to determine how far a frog is able to jump. A. V. Hill's model predicted that jump distance would be the same no matter what the frog's size and body proportions were. Others thought the opposite - if the body size were larger then the frog would be able to jump further. Frogs of different species and different sizes were analyzed at different times of the day. The end result was that the larger the frog was in size, the longer it was able to jump. The conclusion was that either Hill's model was wrong or there were other allometric factors that affect the distance a frog can jump.[7]

Allometry can be analyzed in a variety of situations. In "*Physiologic Changes during Growth and Development*," O'Flaherty discussed allometry by analyzing human growth rate compared to a rat. The author was able to analyze how it takes many years for a human to multiply its weight 20 times, but only a few weeks for a rat. That makes me wonder if there's a relationship between how long it takes to multiply body weight is related to lifespan of different species. Allometry is a convenient descriptive tool. The allometric relationship has been found to be descriptive of the way in which many anatomic and physiologic functions depend on body size either between or within species.[8] O'Flaherty was able to calculate some math equations and plot graphs on arm length vs. body height, oxygen consumption rate vs. body mass, organ weight vs. body weight, glomerular filtration rate vs. body weight, and respiration rate vs. body weight. I was particularly interested in the first graph of infant arm length and height compared to adults. At later ages, arm length and height are proportional, with the slope of the isometric line being one.

This concept of body parts being proportional leads me to another online article titled, *Size Makes a Difference: Gymnastics and Swimming*. In this article Michael Joyner explained how much of elite athletic performance is due to genetic factors and how much is due to environmental factors. He highlights gymnastics and swimming. In this case, allometry applies to biology. "Gymnasts tend to be short and swimmers tend to be tall."[9] In the case of gymnastics, it is important to have a high strength to body weight ratio. Since strength to body ratio tends to decrease, as a person gets taller, elite gymnasts are usually short. Olympic swimming champion, Michael Phelps' body shape does not fit the general pattern of height and arm length being equal. In fact, there's something called an Ape Index. A high Ape Index (greater than 1) means you look more like a monkey; your body proportion has longer arms compared to height. Michael Phelps has a high Ape Index of 1.04.[10] I believe that the students will draw more interest to allometry knowing its relevance to the Olympics and sports.

Mathematics

In addition to allometry, some math concepts are necessary to review. This unit will discuss algebraic language, tables, equations, ordered pairs, data, graphs, measuring, and problem solving.

Algebraic language involves understanding of variables and recognizing that letters can be substituted to represent any number within an expression or equation, They will need to know that an equation includes an equal sign where as an expression does not. When they solve expressions or equations they are actually evaluating to get the solution.

Another math concept that should be reviewed is tables. It should be explained that the table is usually referred to as a T-table and has a pair of numbers. The pair of numbers includes an input number with one output number. Most of the time there is a pattern that can be recognized and it formulates to be a rule for the information. For example, when x=a number then y is 5 times the value. This helps them build upon recognizing patterns and helps to solve problems. When creating a table, information has to be collected to put inside of the table. As equations are being discussed with the students, it will be important to move from hands on manipulatives to actual abstract equations. This will assist in a better understanding.

When students are graphing the data collected, they will have to create ordered pairs. They will have to know that the x axis is horizontal and the y axis is vertical. The point where the axes meet in the middle is called the origin and looks like (0,0). All of the data should be positive numbers and be plotted in the first of the four quadrant (upper right quadrant). Data should be explained to students as collection from a survey to show information. Once tables and equations have been completed, students should interpret the data. They may have been introduced to mean, median, mode. Mean, also known as average, is adding the numbers then dividing by the amount of numbers used. Median is the number that lies in the middle after all the numbers are arranged in sequential order. Mode is the number that happens the most frequently. Outliers may need to be explained just in case some data is not within the range of the others.

Graphs are great tools because they provide a visual representation of data collected. A variety of graphs can be constructed such as a line graph, bar graph, or pie chart. It depends on the type of information that one is trying to show as to which graph to choose. By looking at the graphs one can tell if something is increasing or decreasing or the most or the least of something. For example, bar graphs and pie charts can be used to compare the amount of something, where a line graph shows changes over a period of time.

Knowing how to measure and read a ruler will also come in handy. The students will have to know that length is the distance of two points with a beginning and ending point. There are often 2 different measurements on a ruler. One side at the top measures in inches and the same side at the bottom measures in millimeters. Students need to know that 12 inches equal a foot, and a yard has 36 inches. They also need to know that 1 centimeter is equal to 10 millimeters, and one meter is equal to 100 centimeters, or 1000 millimeters.

Classroom Activities

Lesson Plan #1

Common Core Standards for Mathematics:

- CC.5.A.1 - Analyze patterns and relationships
- CC.5.MD.1 and 2 - Represent and interpret data
- CC.5.G.1 and 2 - Graph points on the coordinate plane to solve real-world and mathematical problems

Objectives

The students will determine if there is a relationship between height and width of a nail.
The students will measure nails in cm.
The students will create graphs using tables.

Lesson Opener/Motivation Warm up

Now that you have been introduced to the topic of allometry, we will explore the concept with nails. A variety of nail sizes will be placed on the table for the students to analyze. The students will have to make a prediction about the width (diameter) of the nail for a nail that is 5 cm long.

Procedures/Activities

I will explain to them that instead of using inches to measure the nails, we will be using cm due to the size of the nails. I will show them cm on a ruler and then ask them, “What types of items do you think would be measured in cm?

I will pull out items such as letters, gold fish crackers. Fingers could also be used. Then I would measure these items as an example and allow the students to do the same once I am finished. Since they have practice with measuring in cm, now the students can begin to measure the various sizes of nails.

The class would complete the table and start to graph independently or with a partner. The students should have some experience with this from previous lessons, but I will monitor for assistance.

Then each student would be responsible for creating a graph of the results. Once everyone is finished with their paper graphs, they will have to analyze the results and answer some questions based upon the graph. This activity will then be continued in the computer lab where the students will have the chance to create graphs.

Closure

The students will have to jot down in their journals what the relationship with width and height of nails is.

Assessment/Homework

The students will have to create a graph on paper with a table given to them of circumferences and height of a nail and answer some questions.

Modeling

The Mathematical Modeling Cycle explanation:
1. System, Reality = height and circumference of a nail activity
2. Problem/Questions = Can you determine how height and circumference relate?
3. Formulate Mathematical Model = Looked at the table and plotted data
4. Analyze the model/Simplification = observing and determining how to solve
5. Prediction = This is where you should be able to make future assumptions from the model. In this case, one should be able to use the equation above and solve how height and circumference of a nail have a relationship.

Lesson Plan #2

Common Core Standards for Mathematics:

- CC.5.A.1 - Analyze patterns and relationships
- CC.5.MD.1 and 2 - Represent and interpret data
- CC.5.G.1 and 2 - Graph points on the coordinate plane to solve real-world and mathematical problems

Objectives

1. The students will determine if there is a relationship between average height and arm span.
2. The students will measure height and arm span.
3. The students will create tables for data collected.

Lesson Opener/Motivation Warm-up

Have these questions written on the board. "What is your favorite summer Olympic event?" "Has anyone watched Olympic swimming?" "Do you remember who the winner was?" Then I will show a Michael Phelps race through You tube (http://www.youtube.com/watch?v=ax77_hHq9Dc). I will tell them about the Ape Index and that Michael Phelps has a high index and that is why he is a great swimmer.

Place a KWL chart on the board to have a class discussion to find out what the students know about allometry in relation to height and arm span, tables, and data, what they want to know, and what we will learn throughout the unit.

Know	Want to know	Learned
The students will have to make suggestions in this box.	The students will have to create questions in this box.	At the end of the unit, the students will have to discuss what they have learned.
They may know what a table and a ruler look like. They may have watched the Olympics. They may know how tall they are in inches.	Are your arms longer than your height? Why does arm length matter? How to read a ruler?	“Allometry is the study of the relationship of body size to shape, anatomy, physiology and finally behavior” Tables are necessary to input data.

Procedures/Activities

I would have a blank table on the Elmo to demonstrate to the class what a table looks like. Then I would label the table with x and y and explain that the x column is where we will record a student’s height and the Y column is where we would record a student’s arm span. “Does anyone know what the word is called for the information we are collecting?” Allow the students a chance to think. Hopefully, someone will say data and if not, then tell them.

Next, I will show the class a ruler and we will discuss the marks. I would point to the inches side and show exactly where one inch. I would explain that there are 8 cm inside of one inch and show and discuss ¼, ½, and ¾. Since a ruler is too small to measure I would show the students a tape measure, which has the same measurements, but is sturdy and long enough to use for height and arm span. The students would have time to use rulers to practice understanding how to use a ruler. They will have to measure an eraser, a pencil, and a finger to the nearest inch, half, quarter, and eighth with a partner. As a class we would review the findings to ensure that the students know are to measure with a ruler.

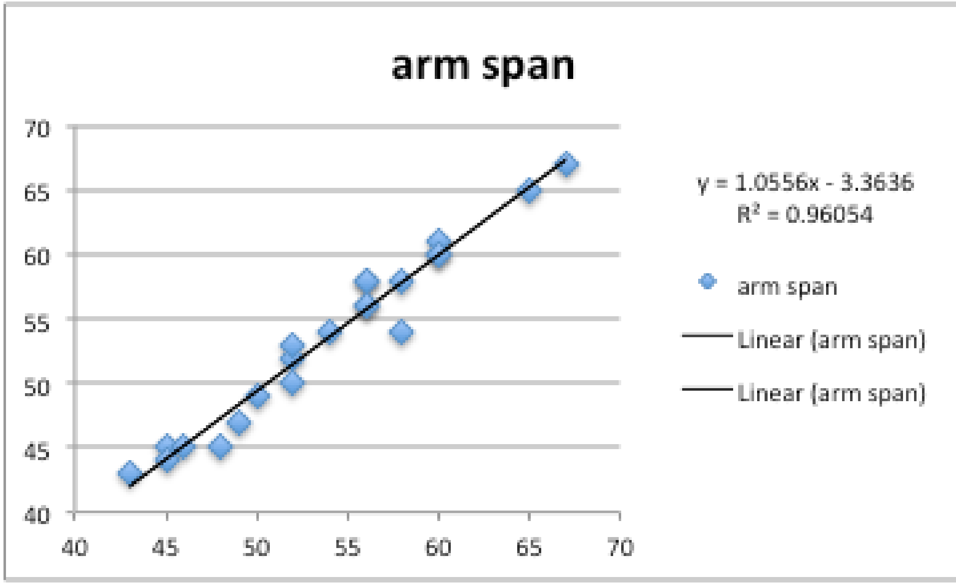
It would be good to demonstrate with a student how to use the tape measure to calculate height. I would have a student come to the front of the class to get measured and another to hold the tape measure at the bottom. I would then pull the tape measure and say and write the measurement on a piece of paper. The class would be split into groups of about 5 or 6 and they would take turns measuring each other's height and arm span. I would walk around monitoring to ensure accurate information is being collected. The entire class would then fill in the table together in order to get all the class data. Figure 3 is a sample data set from this year's students.

X-Height (inches)	Y=Arm span (inches)
43	43
67	67
56	58
45	45
52	52
65	65
56	56
60	60
48	45
60	61
54	54
46	45
56	58
50	49
58	58
45	44
52	50
60	60
58	54
49	47
52	53
56	56

Figure 3

Once everyone is finished filling in the table, they will be asked to analyze and make an assumption based on the information gathered.

The students should quickly realize how your height and arm length were about the same. During the activity, the students said some people were the same size, which was a good time to discuss mode, meaning most frequent number.



We will discuss the activity and they will write through their journals. The students will use measuring materials. Some questions will be asked of students such as, “What kind of pattern do you see?” “Do you think you are able to make predictions based off the data collected?

Closure

The students will complete the L portion from the KWL chart.

Assessment/Homework

I will informally walk around and monitor the students as they collect the data and complete the table. Students will calculate their own Ape Index.

Modeling

The Mathematical Modeling Cycle explanation:
1. System, Reality = height and arm span activity
2. Problem/Questions = Can you determine how height and arm span relate?
3. Formulate Mathematical Model = Looked at the table and plotted data
4. Analyze the model/Simplification = observing and determining how to solve

5. Prediction = This is where you should be able to make future assumptions from the model. In this case, one should be able to use the equation to predict the arm span for a person that is 66 inches tall.

Lesson Plan #3

Common Core Standards for Mathematics:

- CC.5.A.1 - Analyze patterns and relationships
- CC.5.MD.1 and 2 - Represent and interpret data
- CC.5.G.1 and 2 - Graph points on the coordinate plane to solve real-world and mathematical problems

Objectives

The students will discuss vocabulary terms.
The students will measure the relationship of head circumference and height.
The students will create graphs using tables.

Lesson Opener/Motivation Warm up

The students will review the term allometry (proportions, trends). Please write down in your journals what allometry is. “Do you think the ratio of the size of your head to your height changes as you get older? Why do you think so?”

Procedures/Activities

I will explain to them the vocabulary words that we will be learning are data, interpret, graph, axis, scale, and variables. They will have to brainstorm about the terms prior to us writing the definitions down on paper as to what they think each term is and how it can used.

I will show them a line graph and explain to them how to set it up on their paper. The X axis is horizontal and the Y axis is vertical. Today we will be measuring the size of our heads and comparing the relationship to your height.

I will ask them if we are measuring our head, “Will we be using a tape measure again?” “Yes, but not the hard type, but the soft type this time. I would demonstrate by measuring a student in front of the class and recording the data.

Then I would measure the students’ height with the help of another student. The class would have time to break into groups again and collect the data from the head circumference.

As a whole class, we would complete the table and start to graph. They will watch me label the axis and write the numbers to scale. I would then explain that the scale depends on the data that was collected because I want to cover the smallest and largest number in portion, so the graph looks presentable. Then I would begin to plot the points from my table and have them do the same at the same time as me.

Once everyone is finished with their paper graphs, they will have to analyze the results and answer some questions based upon the graph. This activity will then be continued in the computer lab where the students will have the chance to create graphs using the computer in order to record the equation.

Closure

The students will have to jot down in their journals how useful a graph can be and why.

Assessment/Homework

The students will measure and record the head circumference and height of everyone in their house and anyone else they can find. We will put all of the data together as a class and look for patterns.

Modeling

The Mathematical Modeling Cycle explanation:

6. System, Reality = height and head circumference activity
7. Problem/Questions = Can you determine how height and head circumference relate?
8. Formulate Mathematical Model = Looked at the table and plotted data
9. Analyze the model/Simplification = observing and determining how to solve
10. Prediction = This is where you should be able to make future assumptions from the model. In this case, one should be able to use the equation above and solve how height and arm span have a relationship.

Appendix A - District Standards

Analyze patterns and relationships:

Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

Measurement and Data

Convert like measurement units within a given measurement system.
Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5cm to 0.05), and use these conversions in solving multi-step, real world problems.

Represent and interpret Data

Make a line plot to display a data set of measurements in fractions of a unit (1/2, ¼, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Geometry

Graph points on the coordinate plane to solve real world and mathematical problems.

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate y-axis and y-coordinate.

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KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Algebraic language, tables, equations, ordered pairs, data, graphs, measuring, and problem solving

ESSENTIAL QUESTION(S) for the UNIT

How can you use Algebra in everyday life? How does data drive the world we live in? How do you use the metric system to measure in everyday life? How can probability help us to predict the events in life?

Where have you used equations in your life?

CONCEPT A	CONCEPT B	CONCEPT C
algebra	data	prediction
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How can you use Algebra in everyday life? Where have you used equations in your life?	How does data drive the world we live in? How do you use the metric system to measure in everyday life?	How can probability help us to predict the events in life?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
Variable, expression, evaluate, equation, graph	Graph, ordered pair, x and y coordinate, data, mean, median, mode, centimeter	probabilty

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

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