Discovering Physics with a Mathematical Model

Ellen Shackelford

Introduction

When I apply more energy to a vehicle I build, it travels a greater distance, but can I predict how much farther it will travel? I believe that the vehicle I build travels faster when I apply more energy to it, but can I predict how much faster it travels?

In my unit, students will use mathematical modeling to predict the effects of stored energy on a standard model vehicle they build from K'nex. They will formulate questions and determine if they can use a mathematical model to answer the questions..

I have taught a unit with the STC science kit, Motion and Design, that explored what happened when stored energy in a twisted rubber band was applied to a standard vehicle. The students merely compared how far their standard vehicles traveled with varying rubber band turns. The students could easily predict the results without conducting the experiment. But, what they could not predict was a relationship between additional energy applied and additional distance traveled or how the speed of the vehicle changed. In fact, students would probably predict that if you the rubber band turns were doubled, the distance traveled would double or the speed would double, and this is not the case. In this unit, the students will create a predictive, mathematical model to determine the relationship between the additional energy and the distance traveled or the relationship between the additional energy and the speed of the vehicle.

"Model with Mathematics" is one of the Mathematical Practices in the Math Common Core State Standards, I thought that I already included this in my mathematics instruction. I use manipulatives when I teach math and I model math strategies to the students, so I thought I must model with mathematics. I did not realize then how limited my thinking was and how I needed to incorporate this into my instruction. As Abrams noted, "Many challenging and exciting skills used in the development of models of applied settings have been ignored by traditional school mathematics."[i]

Many of my students struggle to find relevance or interest from textbook generated math textbook problems, yet they love science. Why not join the two? By adding mathematical modeling to the science investigation, the students will find relevance and add much more detail to the science concepts by adding predictive, quantitative data.

Demographics

I teach 5th grade in a diverse, Title I school, (62% low income), just outside of the city of Wilmington, Delaware. We have students from a range of socioeconomic backgrounds with 20% Hispanic, 16.4% black, 58.5% white, and a number of students with mixed backgrounds. Of particular concern is the lag of African American students in math assessment performance as measured by the state DCAS. Yet, when comparing the DCAS math and DCAS science scores from 2011-2012, 5th grade students at my school, the difference between African Americans and all 5th grade students in math was nearly twice the difference between African

Americans and all 5th grade students in science. Perhaps, infusion of math into science will help close the gap. While the 2011-2012 standardized test scores for 5th graders at my school as a whole defied the typical socioeconomic demographics by rivaling the more suburban elementary schools in our district, graduation rates and college matriculation rates still lag at most of the district high schools. Our new district vision includes the goal that all students will be college and career ready upon high school graduation. Better experiences in elementary school, particularly in STEM fields can only increase the likelihood of better high school experiences. Student motivation is a huge factor in future school success. Giving students relevant problems will help provide such motivation.

Fifth grade at my school is still predominantly a self-contained classroom, with all subjects taught by the same teacher with the same heterogeneous student groupings. My school and district support the inclusive student model. Most special education students remain with their regular education peers. Inclusion teachers support the special education students in reading and math, but not for science or social studies. Because I teach a self-contained classroom, I will be able to easily integrate the mathematical modeling with science since I teach all the subjects and have some flexibility in my schedule, if needed. My special education students have not had problems participating in the science classroom, since cooperative grouping provides support for them, particularly with reading tasks. I believe that they will be able to participate in this unit, even though it contains mathematics as long as they work within their groups.

Background Information

Energy is defined as, "the ability to do work."[ii] And "work is the transfer of energy."[iii] There exist multiple sources of energy, including many natural sources such as the sun (solar), wind, geothermal and water. In addition there are other sources of energy such as chemical, electrical, fossil fuels, and nuclear.

Energy can be stored or released. Stored energy is called potential energy while released energy is called kinetic energy. However energy "cannot be created or destroyed,"[iv] which is what is called the conservation of energy. Plants need solar energy to produce food through photosynthesis. Animals eat plants and the energy stored in the plant is then stored in the animal as potential energy. Later, the animal releases the energy through activity so the energy becomes kinetic energy. Electrical energy can be stored in a battery and converted into various forms of kinetic energy such as light or the movement of a motor. With the vehicle in this unit, the potential energy is stored in the rubber band which then transfers the energy to the vehicle as kinetic energy as the vehicle moves.

Energy is part of physical science and physicists have found ways to measure kinetic energy. Physicists developed measurements based on heat that is produced from energy. For example Btu (British thermal unit) is the amount of heat energy to raise the temperature of one pound of water by one degree Fahrenheit. The Btu unit is comparable to the standard measuring system used in the United States. The United States and Canada use this system but most other countries do not. The unit used internationally is the joule. 1,000 joules = 1 Btu. To measure the kinetic energy of a vehicle, the formula is $KE = \frac{1}{2} mv^2$, where KE is kinetic energy, and m is the mass of the vehicle, and v is the velocity or speed of the vehicle. Velocity = d/t, where d is distance and t is time. This can also be converted into joules: $1 \text{ joule} = 1 \text{ kg } m^2/s^2$, where m is miles the vehicle travels and s is the seconds for the vehicle to travel the distance. You may have observed that things that use energy heat up after using the energy. Basically, for elementary students, we simplify the process by concluding that, the greater the distance the vehicles travel, the greater the kinetic energy being used, as long as the mass of the vehicle remains the same. (This is another reason that the students adhere to a standard vehicle to keep the mass of the vehicle constant.)

One way that energy can be stored as potential energy is in a rubber band. When a rubber band is stretched or twisted, energy is stored in the rubber band. The more a rubber band is stretched or twisted, the more energy is stored as potential energy. The more potential energy in the rubber band, the more kinetic energy will result when released. When a rubber band is stretched and then released, it will travel farther than if it is not stretched. Let the students feel a wide rubber band that is stretched and it will feel warmer than one that is not stretched. The heat is a byproduct of energy. When the students twist the three connected rubber bands on their vehicles, the potential energy stored in the rubber bands will be transferred into energy of motion when they release their vehicles. The more the rubber bands are twisted, (until they break), the more energy is stored and the more energy is transferred into energy of motion. By measuring the distances traveled by the vehicles, the amount of energy can be compared- the greater the distance, the greater the energy transferred into energy of motion. (Note that the rubber bands can only be turned on the axle about 15 times. They will likely break if wound more than about 15 times.)

In order to perform an investigation and make conclusions from it, students should keep as many variables constant as possible. My students use K'nex building materials using a standard vehicle design, (see Figures 1,2,3 and 4), and a standard testing surface for their experimentation. If the K'nex building materials are not available other construction materials will work as long as certain design specifications are used. The front axle needs to be fixed with wheels that can move freely. The back axle needs to be able to turn with fixed wheels. That way the rubber bands can be fixed to the front axle and twisted onto the back axle. We use three rubber bands that are about 1 millimeter in width each and about two inches long. The rubber bands are connected by looping them into a slipknot to form a chain which is attached by a slipknot on the front axle of the vehicle.

Figure 1

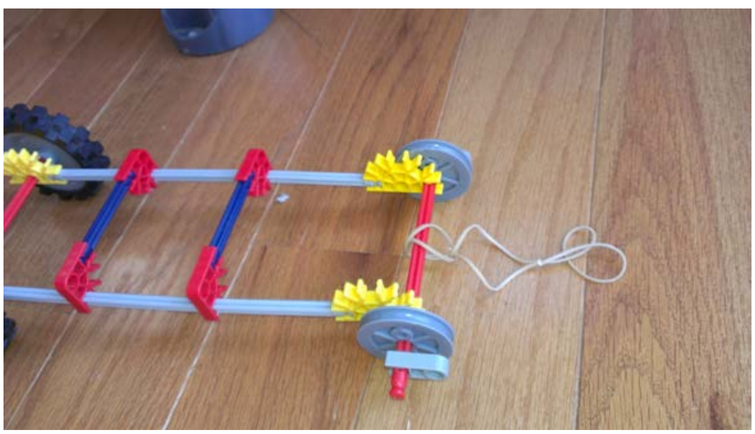
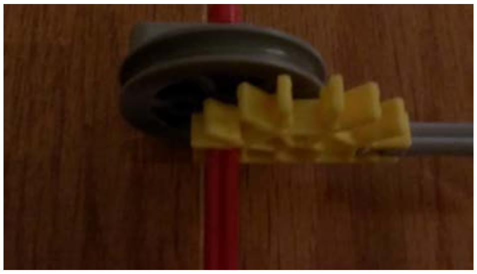
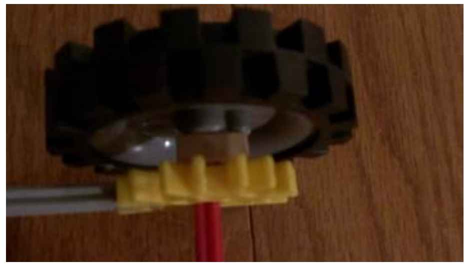


Figure 2

Figure 3 Figure 4





Students should demonstrate and agree on the procedure to make the rubber band turns on the axis. (I hold the end of the rubber bands on the axle and turn the axle until I see the end of the rubber band for each rotation. After a couple rotations, I do not have to hold the rubber band any more.) In addition, the class needs to discuss how to measure the vehicle distances, such as measuring from the front wheels of the vehicle. You should also make sure the students know how to measure distances greater than one meter, particularly if you are using meter sticks to measure. Students are not always sure how to measure distances greater than a meter stick. Students need to be challenged to attend to as much precision in their measuring as they can to provide accurate data. A difference in a centimeter or two can skew the data. However, 5th grade students are not always precise with measurements. Teacher guidance can be helpful here by observing the groups and guiding the groups that are not making precise measurements.

STC directs the students to take 3 measurements for each number of rubber band turns, (2,4, and 8), and to find the median of the data. I find that for the purposes of finding a mathematical model, it is better to use 5 measurements and to graph them all rather than find 1 average. This way, an inaccurate measurement does not affect the other measurements as it might with the averaging process.

It has been my experience with this investigation, that even though the vehicles are standard, their performances sometimes vary widely- some will travel a lot farther than others using the same exact procedure for no apparent reason. So, expect the data to vary between groups. Groups also need to be certain to use the same vehicle for all of their experimentation. While data may vary from group to group, it should be valid for each group because the students are comparing relative data between rubber band turns.

Mathematical Modeling Cycle

The Mathematical Common Core State Standards' document describes the practice as follows:

> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday live, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in

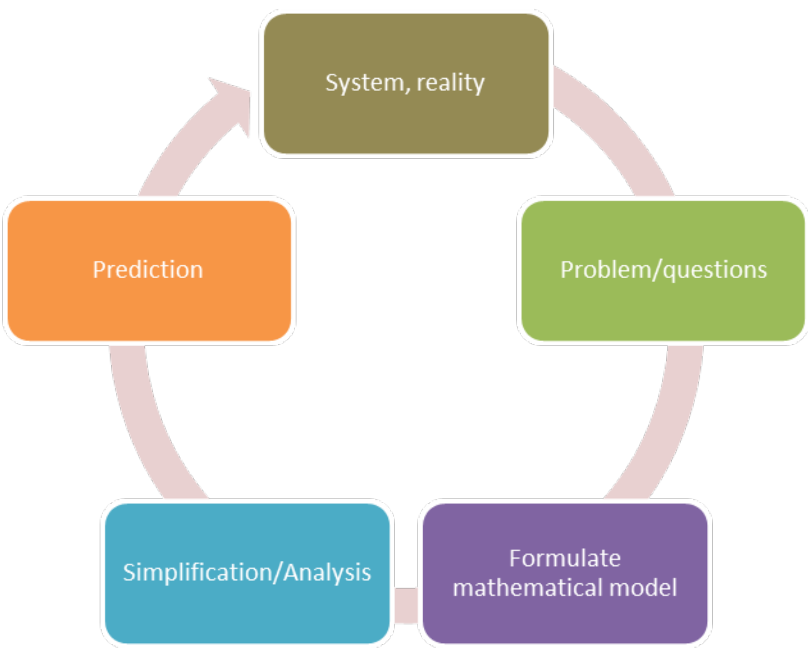
the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

The description involves much more than simply using manipulatives or demonstrating a mathematical strategy to my class or asking them to create a physical model. Mousouldes et al note, "Children need more exposure to situations where they explore informal notions of rate, ratio, and proportion…"[v] And English notes, "Traditionally, mathematical modeling …has been reserved for the secondary school years with the assumption that primary school children are incapable of developing their own models and sense-making systems for dealing with complex situations."[vi] Mousoulides et al note, "Modeling activities differ from traditional problem solving in at least two ways. First, in solving modeling problems students need to use and interconnect mathematical concepts and operations."[vii]

To incorporate effective mathematical modeling, students need real world problems that "require children to make sense of the situation so that they can mathematize it themselves in ways that are meaningful to them."[viii] Students are likely to find much more motivation to solve interesting real world problems than problems that exist only for computational practice, such as the problems they encounter in most of their math textbooks. In fact, De Corte found that students with real world problems tended to use knowledge that they had about the natural world. However students with contrived textbook problems tended to "forget" or go against what they already knew from the natural world when they attempted to answer the problems.[ix] Perhaps students will begin to see the relevance of mathematics clearly if we start teaching mathematical modeling.

In seminar, the mathematical cycle was presented in a diagram, (see figure 5). As shown, it is a repetitive cycle, repeating as many times as necessary, originating with the system, or reality, proceeding counterclockwise as many times as necessary.

Figure 5



System, Reality

The system or reality has to do with the physical system for which you want to apply mathematical modeling.

Problem, Questions

Once the system is defined, questions must be created that relate to the system. Students will brainstorm questions surrounding the system chosen. Questions should relate to questions the students have about the system. The questions chosen to examine in the modeling process should be those that can be answered with data from the system, or experiment. My students will brainstorm questions at this step and we will discuss them as a class. I can imagine the students might have the following questions:

Will the vehicles travel farther with more rubber band turns, (energy)?
How much farther will the vehicles travel with more rubber band turns, (energy)?
Will the vehicles travel faster with more rubber band turns, (energy)?
Why do the vehicles travel faster with more rubber band turns, (energy)?
Will the vehicles continue to travel faster with more rubber band turns, (energy)?
How much faster will the vehicles travel with more rubber band turns, (energy)?

The best questions for mathematical models are those for which a mathematical model can generate predictive values. So considering the possible questions above, the best questions for mathematical modeling are the second and the last questions. The second question can be answered with a mathematical model using the data from the vehicle trials and graphing them. The last question can also be answered with mathematical modeling, using data from timing the vehicles and measuring the distance traveled in that time, speed. The first and third questions above can be answered from the data without mathematical models and they are not asking for predictive values, which a mathematical model gives. The question that asks why the vehicles travel faster, is not asking for predictive values so a mathematical model is not appropriate. The problem with the question that asks if the vehicles will continue to travel faster is that the rubber band will break after about 15 turns. At that point it will not travel at all. The model cannot predict this data.

Formulate Mathematical Model

At this point in the modeling process, the students need to create a mathematical way to solve the questions created in the previous step. The correct model will be predictive.
This mathematical model for upper elementary students could be one of many graphs or formulas involving mathematical data. The variety of mathematical models is somewhat limited in the elementary years since the students have a limited range of mathematical tools they can use. Technology can help to supplement this deficit as long as the teacher exposes the students to it. In this unit, we will use the Excel program on the computer to graph the data and provide a trendline with its equation.

There are two different kinds of mathematical models: descriptive and empirical models which are those derived from data and analytical and fundamental models which are derived from various physical laws.

Simplification/Analysis

After creating the mathematical model and applying it to the data from the system, the model often needs to be simplified. However, this applies more to more advanced students with more advanced mathematical models. The data also needs to be analyzed. This is the step where the students will put their data into the Excel program and find a trendline to fit the data.

Prediction

After students successfully complete the analysis step, then they will evaluate the predictive value of the model. If they see that their model has successfully predicted the data of their system, then they have successfully completed the mathematical modeling cycle. If the students cannot prove that their model was predictive, they must redesign their model and repeat the steps in the cycle until they are satisfied with their models. You will need to lead the students through this step together since they have limited knowledge of the variety of mathematical models at this grade. They can experiment with their vehicles using 3,5, or 6 turns of the rubber band and see if they fall on the trendline that the computer created. If the new data falls on the trendline, they will have greater confidence that the model is predictive. This will be the point where the science and the mathematics really merge which will be an essential outcome of this unit.

> "The process of modeling constitutes the bridge between mathematics as a set of tools for describing aspects of the real world, on the one hand, and mathematics as the analysis of abstract structures, on the other; as such it is a pervasive aspect of mathematics. Given the increasing mathematisation of social as well as physical phenomena, an understanding of this aspect of mathematics is essential for informed citizenship."[x]

Strategies

Cooperative Learning

Mathematical modeling requires cooperative learning groups so that the students can collaborate with their peers as well as critique their ideas. We have used cooperative groups to solve math problems before and most of the students really value the support and the opportunities to work with another student. This is also a way to support some students who have learning disabilities or challenges with reading and writing by including them with others who are stronger in those disciplines.

Technology

Students will be given technological tools to use to help them with the mathematical models they want to use. Some of the actual math in the models they may use can be accessed by using technology. For instance, students will use an Excel program to create graphs and then use the program to create the equation for a line or curve that represents their graphs. The students may not understand what functions are yet, but they can see if the trendline the computer generates fits their data.

Student Oral Presentations

Student groups will be required to present their models to the rest of the class for feedback and constructive criticism. I think this will not only be instructive, but I think that many students who did not really care about math will find reasons to do a better job if they have to present it to the rest of the class. This may seem like an odd step, since the students are modeling the same activity. However, as stated before, the data between groups can vary greatly and the models may vary as well. Some groups may not attend to precision in their data collection and get unusual results. These results will become obvious as they evaluate their mathematical models.

Students will be guided in ways to give useful constructive criticism to their peers to aid in the reflective, evaluative phases of the modeling cycle. They should be able to make connections between mathematical constructive criticism and that they give to their peers during writing. Students need to be able to communicate and defend their mathematical thinking.

Classroom Activities

Each of the lessons require 45-60 minute class periods.

Lesson 1

Essential Questions: If I twist the rubber band more, how much farther will the vehicle travel? If I twist the rubber band more, how much faster will the vehicle travel?

Students can find mathematical models for one or both questions, or you can split the class and have one half find a model for each question.

Students take their standard vehicles, (see Figure 1), and fasten the rubber bands, 3 small rubber bands, connected together by looping one over the other, to the front, fixed axle of the vehicle. Students stretch the rubber band to the back axle that is not fixed. Then the students will hold the end of the rubber bands with one hand and twist the axle, so that the rubber band is rolled onto the axle. Each time the student sees the end of the rubber band will count as one complete rotation.

For the distance question, students will complete 5 trials with each different amount of rubber band rotations. They will stretch out a roll of narrow paper, such as adding machine paper. Next they will mark a line at the beginning of the paper. They will line up the front wheels of the vehicle on the starting line, still holding the back axle so the rubber band does not unwind, and then release the vehicle. They will mark the paper where the front wheels of the vehicle top. Students will measure the distance of each mark, but this can be done later, after all the experimentation is completed. Groups will complete 5 trials for 2 rubber band winds, 4 rubber band winds, and 8 rubber band winds.

Students will measure the 5 distances for each of the rubber band winds in centimeters and record the measurements. Students will answer questions: Did you expect the data for 4 turns to be double the distance for 2 turns? Did you expect the data for 8 turns to be double the distance for 4 turns? Does your data reflect that? Why do you think your data was different from your expectations? (See Appendix C.)

For the speed question, students will also complete 5 trials with each different amount of rubber band rotations. They will also stretch out a roll of narrow paper for measuring, such as adding machine paper. Next they will mark a line at the beginning of the paper. They will line up the front wheels of the vehicle on the starting line, still holding the back axle so the rubber band does not unwind, and then release the vehicle. Another student will time the vehicle from starting to stopping. Students will record the times and distances together. They will calculate the speed by dividing the distance in centimeters by the seconds elapsed, 5 times each for 2 rubber band winds, 4 rubber band winds, and 8 rubber band winds.

Lesson 2

Essential Question: What do I want to find out?

Students will generate questions in their cooperative groups that they would like to answer about the data they gathered. Sample questions could be the following:

Will the vehicles travel farther with more rubber band turns, (energy)?
How much farther will the vehicles travel with more rubber band turns, (energy)?
Will the vehicles travel faster with more rubber band turns, (energy)?
Why do the vehicles travel faster with more rubber band turns, (energy)?
Will the vehicles continue to travel faster with more rubber band turns, (energy)?
How much faster will the vehicles travel with more rubber band turns, (energy)?
What is the greatest distance the vehicle could travel with the rubber band energy?
Can I predict the distances the vehicle will travel with selected rubber band turns?

You will conduct a whole class discussion to share the questions that cooperative groups generate. Not all of the above questions could be answered with a mathematical model. You can easily generate a mathematical model with the data collected to predict the additional distances that vehicles travel with increased rubber band turns and to predict the increase in speed for the vehicle with increased rubber band turns.

Lesson 3

Essential Question: What mathematical model can I create to answer my question?

Student cooperative groups will create point graphs of their data using the Excel spreadsheet. (You will need to show the students how to create a graph on Excel if they have never done this before.) Students will attempt to fit a line to the data. (Fifth grade students will likely try to fit a straight line to the data, since they have not had experience with any other type of data trendline.)

Lesson 4

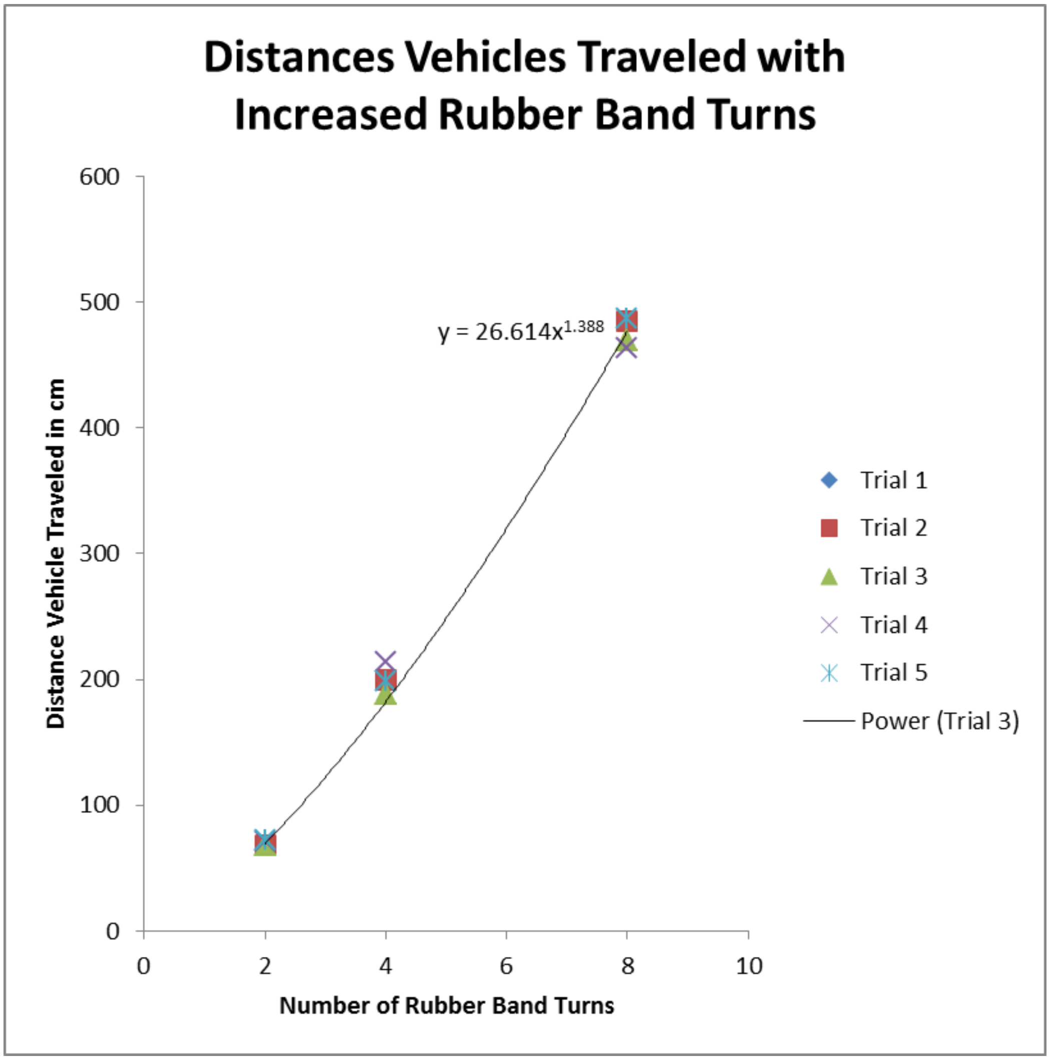
Essential Question: Can we create a mathematical model to reflect the data?

In whole class discussion, using a Smart Board, you will conduct a discussion with the class about fitting a straight line to fit the data. Since a straight line will not fit this data, there should be some discussion about how it only fits part of the data, but not all of the data. You will show the students how to get the Excel program to fit a curve to the data. Then you will show them how the program will create an equation for the line. It is likely that the group data will vary from group to group since the vehicles differ, even though they are built the same way, however, the curves that fit the data should be similar. From this point the class can have an evaluative conversation to determine if the curve, which is the mathematical model here, is indeed a good model for the data and if it is indeed predictive. Students could try a trendline and see how predictive it is for 5 rubber band turns.

In the conversation, you should instruct the students that if the model were not evaluated to be a good model, they would return to the modeling area of the cycle to find a new model which they would then test and evaluate.

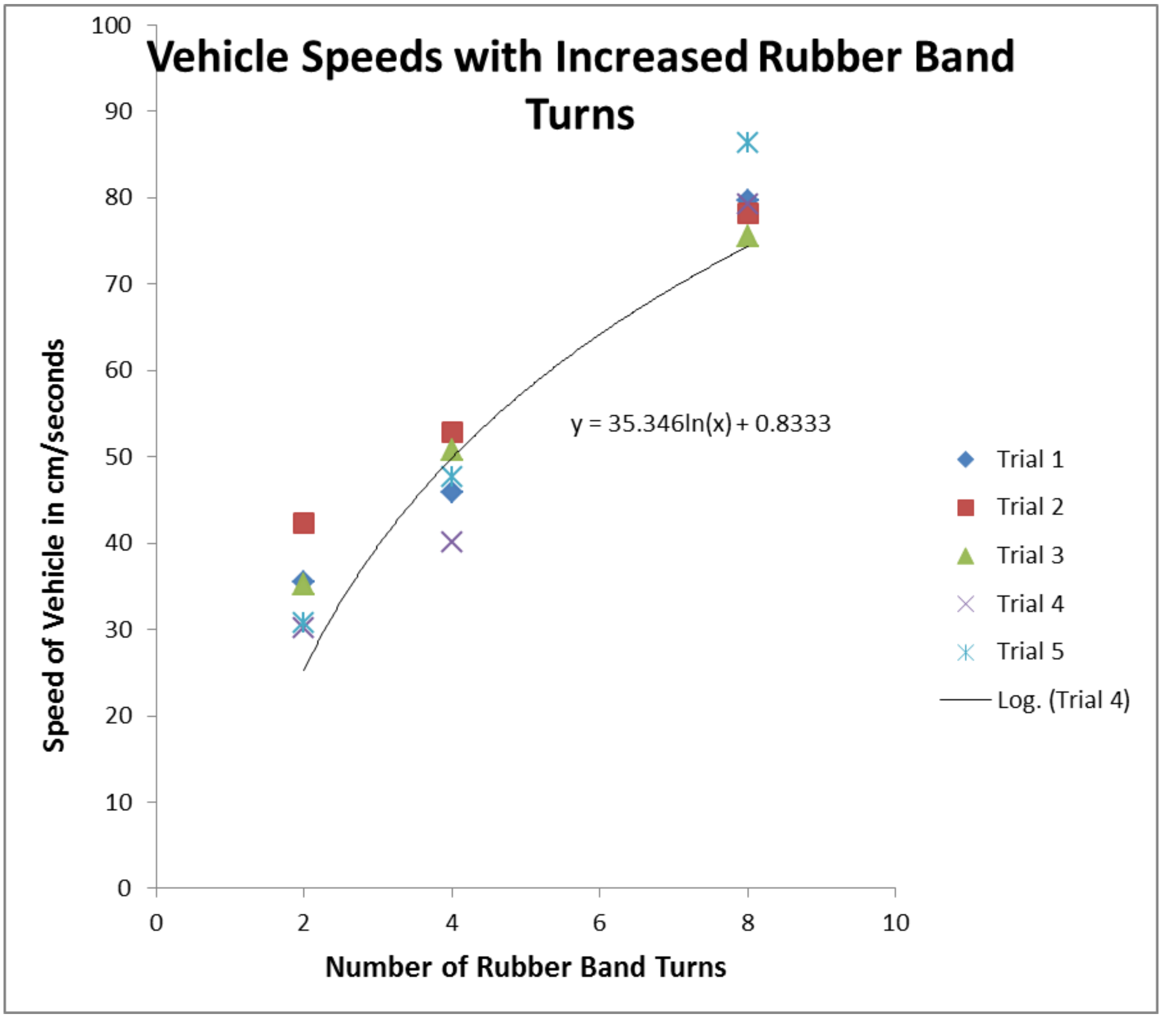
I have included sample graphs with trendlines formatted by the Excel program for both of the questions discussed above. I used powers for the curve for the graph that shows distances of the vehicle with increasing rubber band turns, see Figure 6. And I used a logarithmic curve for the graph that shows speeds of the vehicle with increasing rubber band turns, see Figure 7. You do not need to go into detail about the line equations since it involves higher mathematics than 5th grade, but you can tell the students that they are the formulas for the curves.

Figure 6

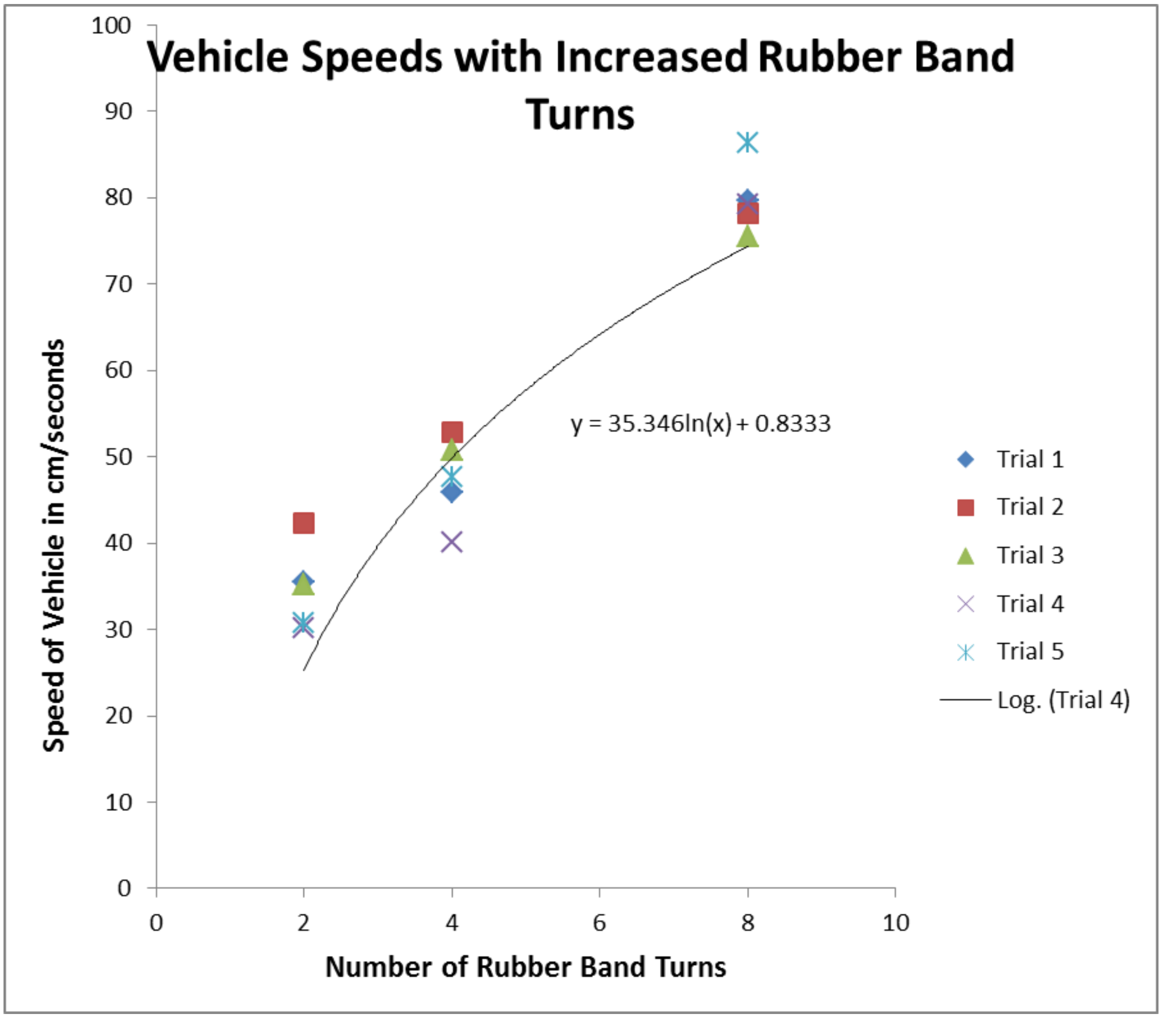


Distances Vehicles Traveled with Increased Rubber Band Turns
y = 26.614x1.388
Distance Vehicle Traveled in cm
Number of Rubber Band Turns
0
100
200
300
400
500
600
0
2
4
6
8
10
Trial 1
Trial 2
Trial 3
Trial 4
Trial 5
Power (Trial 3)

Figure 7



Vehicle Speeds with Increased Rubber Band Turns
Speed of Vehicle in cm/seconds
Number of Rubber Band Turns
y = 35.346ln(x) + 0.8333
Trial 1
Trial 2
Trial 3
Trial 4
Trial 5
Log. (Trial 4)
0
10
20
30
40
50
60
70
80
90
100
0
2
4
6
8
10



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Appendix A

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Appendix B

Math Standards

CC5M.2. Reason abstractly and quantitatively

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

CC5MP.3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

CC5MP.4. Model with mathematics
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

CC5MP.5. Use appropriate tools strategically
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CC.5.G.2 – Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Delaware Science Standards

Standard 3: Energy and Its Effects

The flow of energy drives processes of change in all biological, chemical, physical, and geological systems. Energy stored in a variety of sources can be transformed into other energy forms, which influence many facets of our daily lives. The forms of energy involved and the properties of the materials involved influence the nature of the energy transformations and the mechanisms by which energy is transferred. The conservation of energy is a law that can be used to analyze and build understandings of diverse physical and biological systems.

Strand 1: The Forms and Sources of Energy

3.1.B. The energy of a moving object depends on its speed. Faster moving objects have more energy than slower moving objects.

3.1.C. Energy can be stored in an elastic material when it is stretched.

Strand 2: Forces and the Transfer of Energy

3.2.A. Force is any push or pull exerted by one object on another. Some forces (e.g., magnetic forces and gravity) can make things move without touching them.

3.2.B. The speeds of two or more objects can be compared (i.e., faster, slower) by measuring the distance traveled in a given unit of time, or by measuring the time needed to travel a fixed distance.

3.2.C. A force must be applied to change the speed of a moving object or change its direction of motion. Larger forces will create greater changes in an object's speed in a given unit of time.

Appendix C

Vehicle Distance Data

Number of Rubber Band Turns	Trial 1 In cm	Trial 2 In cm	Trial 3 In cm	Trial 4 In cm	Trial 5 In cm
2 turns					

4 turns					
8 turns					

Questions:

1. **Were the distances for 4 turns double the distances for 2 turns?**
2. **Were the distances for 8 turns double the distances for 4 turns?**
3. **Why do you think the data did not double when the turns doubled?**

Curriculum Unit Title: Mathematical Modeling Meets Fifth Grade

Author: Ellen Shackelford

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Mathematical Modeling is a process that can be used to create a tool to predict future outcomes.

ESSENTIAL QUESTION(S) for the UNIT

How can we use mathematical modeling to answer questions we have about problems?

CONCEPT A	CONCEPT B	CONCEPT C
Students need to determine a question to answer.	Students need to design a mathematical model.	Students need to evaluate their models.
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
What questions do I have about the system or problem that I have?	What mathematical model would fit the question I chose?	Was my model predictive? Did it answer my question?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
	Best fit line, Best fit curve, Equation	Predictive

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

1.Abrams, Joshua Paul. “Mathematical Modeling: Teaching the Open-ended Application of Mathematics.” *2001 Yearbook* NCTM (2001): 269.

2. “Energy and kids.” Energy and kids. http://www.kids.esdb.bg/principles.html (accessed January 29, 2013).

3. Ibid

4. Ibid

5. English, Lyn D.. "Mathematical Modeling in the Primary School: Children's Construction of a Consumer Guide."*Educational Studies in Mathematics* 63 (2006): 303-323.

6. Mousoulides, Nicholas G., Christou, Constantinos, Sriraman, Bharath. “A Modeling Perspective on the Teaching and Learning of Mathematical Problem Solving.” Mathematial Thinking and Learning. 10: 293-304 (2008).

8. Ibid

9. DeCorte, Erik, Lieven Verschaffel, and Brian Greer. "Connecting Mathematics Problem Solving to the Real World."*Afdeling Didactiek* 1, no. 1 (2000).

10. Ibid

11. DeCorte, Erik, Verschaffel, Lieven, and Greer, Brian. “Connecting Mathematics Problem Solvingto the Real World. “*Afdeling Didactiek* 1, no. 1 (2000).

Appendix A

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Appendix B

Math Standards

CC5M.2. Reason abstractly and quantitatively
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

CC5MP.3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

CC5MP.4. Model with mathematics
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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