Mathematical Modeling and Weather

Gretchen Wolfe

Introduction

Each August when the new school year begins, I ask my students to draw and write what they hope to learn in first grade. Some write about wanting to read chapter books, others write about wanting to write and publish stories. Every year the majority of children write about wanting to learn mathematics and science. Each August I ask the children to complete a questionnaire in an effort to get to know them. One of the questions asks the children to indicate which subjects are their favorites in school. Each year the majority of children tell me that math and science are their favorite subjects in school. First grade children are naturally curious about the world; they have so much natural enthusiasm for mathematics and science waiting to be nurtured and refined! This is why I decided to create this unit for the first grade children in my school Brader Elementary.

Henry M. Brader Elementary School in the Christina School District of Delaware has been traditionally a well performing elementary school. Every year since Delaware has been ranking each public school's performance, Brader has earned a superior school status for its students' overall testing performance. However, the mathematics scores at Brader have frequently fallen below the school's performance scores in reading. Brader Elementary has used the Trailblazers mathematics program published by Kendall Hunt, for ten years. During that time the students' performance levels in state testing situations increased, then reached a plateau and remained relatively stagnant in the last four years of using Trailblazers as our core curriculum. To revamp the mathematics program last year, our district and school adopted the Math Connects program published by Macmillan McGraw-Hill, to serve as the core mathematics curriculum. As a result student performance on the state tests has improved slightly in our school. Mathematics is also an area where very little professional development time has been devoted in order to improve performance. Most of our professional development focused on reading standards. When the implementation of Response to Intervention requirements in reading was imposed, more time had been taken from mathematics instruction and dedicated to reading instruction and intervention. Now RtI requirements include implementing mathematics intervention daily. This has required teachers to rework schedules to accommodate an additional 30 minute mathematics intervention, often decreasing the daily science and social studies instructional time allotment.

When teaching mathematics to first grade students, using Math Connects as our core mathematics program I find there are very few instances in which children are guided through the mathematical modeling cycle. I do, however, recognize that modeling is a necessary component of a strong, well-rounded mathematics program. The Common Core State Standards (CCSS) include a mathematical practice standard to model with mathematics. The standard describes mathematically proficient students as: "students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose."[1] Before participating in the seminar my understanding of modeling in mathematics was confined to drawing models in the process of solving word problems. Through my participation in the Delaware Teachers Institute "Modeling with Mathematics" seminar, I was able to develop my understanding of the mathematical modeling cycle and began to form a better understanding of how very young children can be guided through this process.

As strong as the children's enthusiasm is for learning mathematics and science, misconceptions sometimes become apparent during our study of weather. When we begin experimenting with elements of the water cycle the abstract concept of water evaporation becomes challenging for some children to grasp. Each year this becomes a stumbling block on the road to understanding larger science concepts. I have often wrestled with creating different ways to approach introducing water evaporation that would help move the children into a better understanding of the concept. When I saw that the Delaware Teachers Institute was offering a seminar in modeling with mathematics I thought it could help me connect science and mathematics in a way that would allow me to supplement this area of the science curriculum. Participating in DTI allows me to have input from teachers of other grade levels as we discuss our ideas for building our units in seminar. Through the discussion I was presented with the misconceptions and misunderstanding that children can sometimes carry with them into the upper grades. Being able to speak with teachers of high school children has been valuable to help me determine how to focus my unit to build the conceptual understandings and content knowledge that will help my students succeed through subsequent grade levels.

If mathematics is the science of patterns and order, and patterns are everywhere in the world around us, melding mathematics and science together makes a logical connection for students to learn about the world[2]. There are many instances where mathematical modeling can be integrated into science content in first grade. I chose to integrate it into the weather unit that is traditionally taught in the fall months of the school year.

Throughout the weather unit the children observe weather features. First the children observe weather features using their senses then gradually observe, measure, and record weather features using tools such as thermometers, anemometers, and rain gauges. The children use different scales to measure temperatures, wind speed, and rainfall amounts. Different types of clouds are observed, sorted, and classified. Then the children do an experiment that simulates a puddle that they observe and record the changes that occur as the water evaporates. This is the experiment where I chose to begin implementing the mathematical modeling cycle with my students.

The goal of this unit is to help my students participate in the mathematical modeling cycle as a means of better understanding concepts in science. The children for whom this unit has been constructed are my 20 current first-grade students in my self-contained classroom at Henry M. Brader Elementary School in the Christina School District. Christina School District is located in New Castle County, Delaware and serves both the urban and suburban populations of Wilmington and the surrounding areas. There are approximately 17,000 pre-kindergarten through high school students in Christina's twenty-nine schools. Brader is one of eighteen elementary schools and serves its surrounding community in the city of Newark. Brader has approximately 534 kindergarten through fifth-grade students. Forty-eight percent of Brader's student population qualifies for free or reduced lunch. There are four first grade classrooms at Brader Elementary, I am part of a team of five highly skilled first-grade teachers.

Overview

Mathematical Models

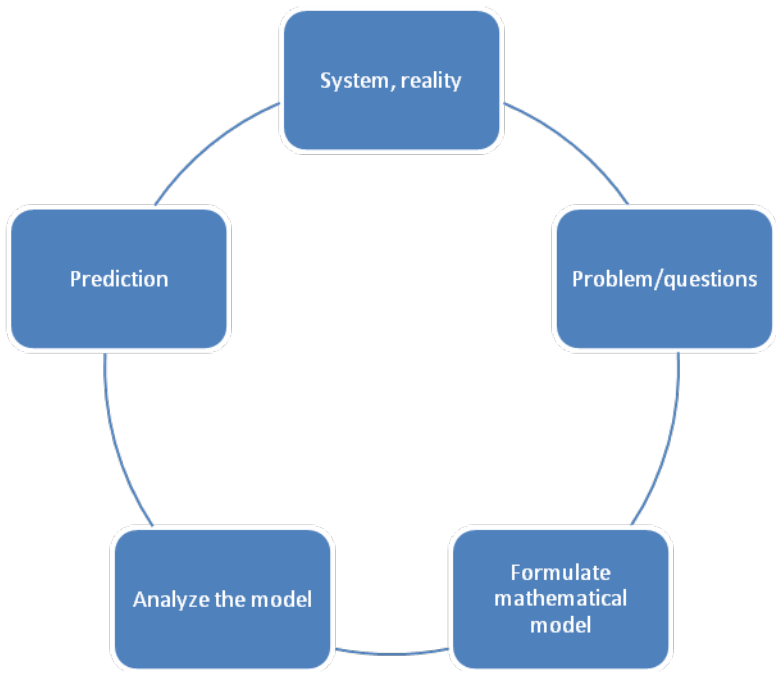
Abrams defines mathematical modeling as the, "process of using mathematics to study questions from outside our discipline[3]." Through the readings and the discussions in the Mathematical Modeling seminar, I learned about types of mathematical models. Zalman Usiskin, director of the University of Chicago School Mathematics Project, has identified three types of mathematical models: exact models, almost-exact theory-based models, and impressionistic models[4]. Exact models correspond to real situations where the model is created from the mathematics, such as compound interest or airplane route networks. Almost-exact theory-based models, the kind most frequently represented in textbooks, are models that give close representations of the actual real world situation but disregard some aspects, such as using a parabola to represent the motion of a kicked football. Impressionistic models are models that are not theory resultant but seem to fit a situation, such as predicting a new world record for running a mile using data from previous trials.

As almost-exact theory-based models are used most commonly by teachers, having an understanding of other models is useful for teachers to allow them to incorporate the use of different models into their curriculum. Usiskin suggested that instead of using the current practice of teaching children to attack word problems by looking for key phrases as signals to perform a particular operation, a better practice would be to start with a real situation that allows students to translate it into mathematics[5]. In my unit the real situations that the children will be exploring are weather related: measuring weather features and water evaporation rates.

The Mathematical Modeling Cycle

The mathematical modeling cycle is recursive and begins with the system or reality.
System, reality: the physical system to be explored, the setting.
Problem/questions: ask questions about the system, state a problem to be solved, identify the variables, determine the form in which the answer will be stated.
Formulate mathematical model: create the model, test the model's behavior, collect data, organize and represent the data through graphs, equations, or algorithms.
Analyze the model: look at the meaning of the outcomes, determine new questions or variables.
Prediction: use the model to look forward, predict other similar situations.

The mathematical modeling cycle that we studied in seminar is represented as follows:



First grade students will need guidance and direction in creating models and collecting and representing data, especially since this unit will be implemented early in the school year. My intent is to guide my students through this cycle, modeling each step with the students, then gradually releasing responsibility to the students to complete another cycle.

One way my first grade students will organize and represent the data they collect is graphing. A graph is a way to present data that makes it easier to interpret visually[6]. As first grade children are very concrete I will begin by creating a real graph with the children. I will ask them if they are wearing sandals (including flip-flops) or shoes. Each child will take off one of their shoes and we will create a concrete graph on the floor. Then we can recreate the same graph with pictures. Each child will draw a picture of their footwear on a Post-It note and add them to the graph. Finally, we will recreate the same graph as a symbolic graph using squares to symbolize a type of shoe. After we create our graphs we will spend time analyzing them, determining which column has most, how many more, etc. The next step is to continue to graph with the children then have them process the graphs. When we graph our favorite season of the year (one of the lessons in the weather unit) I will have the children write generalizations such as: most children like the winter season or more children like summer than fall. Then the children will write equations that show the relationships represented on the graphs[7]. By creating and analyzing graphs with the children I will have created the scaffolding needed for the children to begin the mathematical modeling cycle in science.

Weather Unit

One of the experiments included in the weather unit I currently teach is designed to help the children answer the question: where do puddles go? This allows the children to observe water evaporate. The children simulate a puddle in a nine inch aluminum pie plate. The children fill each puddle plate with 4 ounces of water that we color blue with food coloring. The puddles are all housed by the window of the classroom where the temperature and humidity variables are the same for all the puddles. Each day the children observe their puddles and record their observations, noting the changes in the amount of water in their puddle. After four or five days, when all the water has evaporated from their puddles, we engage in a very interesting discussion about where the water went over the observation time. We test the pie plates for holes or leaks and find none, we look for spills on the window sill and find none, ruling out those possibilities the discussion slows down and the idea of evaporation begins to take hold. One misconception that the children develop from this experiment is that all puddles take the same amount of time to evaporate. Water evaporation rates are not currently a part of the weather unit but present an excellent opportunity for incorporating the mathematical modeling cycle.

In my unit the children will complete another modeling cycle changing some of the variables to investigate the system or reality of water evaporation in the water cycle to answer the question: Do all puddles evaporate in the same amount of time? The children will create the model using different size aluminum pans as the simulated puddles. I will ask the children to predict which puddle will evaporate first. Most children will predict the puddle plate with the smallest surface area will evaporate first due to its seemingly smaller size. We will either affix laminated square inch grid paper to the bottom of the pans with hot glue or draw a grid with a permanent marker on the pan, to help us measure the area of the puddle. Each puddle will have four ounces of water, tinted blue with food coloring for maximum visibility, added to it. Then the puddles will be placed in the same area by the window to evaporate. The children will observe and record the evaporation using the grid to approximate the area of the puddle each day. Once the data is collected the children will organize and graph the data. Then we will look at the relationships reported and answer our original question. The children can then predict the water evaporation rates of puddles of different sizes and surface areas than those we used in our experiment.

Another activity that is currently part of the weather unit is creating a tool to measure wind and developing a wind scale. We spend some time observing the signs of wind: tree movement, sounds of the leaves rustling, flag movement. The children create a simple tool, a flag, to measure the wind. Using a simple wind scale or 0,1,2, no wind, some wind, strong wind. The children measure and record the wind each day for a month. Then we look across the data to determine how many days had no wind, some wind, and strong wind. We use this data to make predictions about the windy days in the next month. In the unit I will use this activity as an opportunity to model the process and have the children repeat the mathematical modeling cycle by creating a different tool for measuring wind speed.

Wind and Water Background

The water cycle includes water in the atmosphere becoming precipitation which takes the form or rain, snow, sleet, or hail. The precipitation collects in ground surfaces and as the water heats it turns to vapor - evaporation. The vapor gets cold and forms clouds - condensation. When the clouds become heavy precipitation falls and the cycle begins again. The rate that water evaporates depends on many factors: surface area, water temperature, air temperature, airflow at the surface, and the amount of water vapor in the air. The idea that larger the surface area of the water the faster the rate of evaporation is what I would like the children to demonstrate in their puddle models.

The Beaufort Scale was created to measure the force of the wind at sea by looking at the effects of wind on the surrounding area. The scale was later adapted for land. The scale ranged from 0 to 12, 0 being no wind and 12 being hurricane force winds. Anemometers also measure wind speed. Many operate by having cup shaped devices on arms that spin in the wind. The rotations are counted in time intervals and the wind speed is calculated. Today there are many different designs of anemometers used to measure wind speed. In this unit of study I will ask the children to design and create an anemometer and create a scale to measure wind speed.

Objectives

The science objectives for this unit are as follows. The students will:

- recognize that different tools are used to measure different features of weather (wind scales, thermometers, and rain gauges)
- recognize that meteorologists are scientists who observe and record weather information to forecast weather
- organize weather data on graphs and long-term data collection charts
- interpret and summarize long-term weather data

The mathematics objectives for this unit are as follows. The students will:

- create a model
- measure surface area
- collect and organize data
- create and interpret graphs
- make predictions
- create a wind scale
- measure wind speed

Standards

CCSS describes mathematically proficient students as being able to model with mathematics. Keeping this in mind, in the creation of this unit I used the following essential questions and standards:

Common Core Mathematics Standards for Grade 1

CC. 1, NO, 4
Use place value understanding and properties of operations to add and subtract.
4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

CC. 1, MD, 4
Represent and interpret data.
4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many

in each category, and how many more or less are in one category than in another.

State of Delaware Science Standard 1, Nature and Application of Science and Technology
Science is a human endeavor involving knowledge learned through inquiring about the natural world. Scientific claims are evaluated and knowledge changes as a result of using the abilities and understandings of inquiry. The pursuit of scientific knowledge is a continuous process involving diverse people throughout history. The practice of science and the development of technology are critical pursuits of our society.
C. Understand that: The purpose of accurate observations and data collection is to provide evidence. Scientists use tools to enhance their senses in order to obtain more evidence. Be able to: Collect data using observations, simple tools and equipment. Record data in tables, charts, and bar graphs. Compare data with others to examine and question results.

Science Standard 5, Earth's Dynamic Systems
Earth's dynamic systems are made up of the solid earth (geosphere), the oceans, lakes, rivers, glaciers and ice sheets (hydrosphere), the atmosphere, and organisms (biosphere). Interactions among these spheres have resulted in ongoing changes to the system. Some of these changes can be measured on a human time scale, but others occur so slowly, that they must be inferred from geological evidence.
Strand: *Components of Earth*, B. Water can exist as a solid, liquid or gas and in different forms such as rain, snow and ice.
Strand: Technology and Application B. Weather can be observed, measured and described through the use of simple tools such as a thermometer, rain gauge and wind vane.
Strand: *Interactions Throughout Earth's Systems* A. Weather influences plants, animals and human activity. D. Clouds are shaped by winds and are made of small water droplets or ice crystals. Cloud shapes can be used to help forecast weather.

Essential Questions:

- How do we measure weather?
- Why is measuring weather necessary?
- How do we organize and use data in our everyday lives?
- How does understanding the properties of Earth materials and the physical laws that govern their behavior lead to prediction of Earth events?
- How can making a model help me predict behavior?

Teaching Strategies

The teaching approaches that are imbedded in this unit include:

Cooperative learning: in which the children work collaboratively, in pairs or groups to complete tasks and activities. Working collaboratively allows children to work as a team and assume different roles while learning.

Indirect instruction: in which students are presented with a question to answer or problem to solve. The children observe, investigate, and draw conclusions while the teacher supports their investigation.

Learning Centers: children work collaboratively to complete activities or play games that reinforce content previously taught.

Classroom Activities

This unit is designed to be conducted any time of the year in first grade. The activities can be completed over two to three weeks. Introducing the experiment should be done during two whole class periods of 30-45 minutes. Collecting and recording data can be done using 5-10 minutes of either the science or mathematics class periods. After completing the experiment “Where do puddles go?” described above, introduce the mathematical modeling cycle to the children. Explain that they will repeat the cycle as they look more deeply at the system of water evaporation. In the first activity the children will work in small groups to answer the question: Do differently sized puddles evaporate at the same time? In order to answer this question the children will need aluminum pans of various sizes (rectangular pans are preferable so complete grids can be drawn on the bottom with a permanent marker), water, blue food coloring, measuring cups, recording sheets, pencils, chart paper and an area where the pans can be stored together.

Next, draw a square inch grid on the bottom of each pan with permanent marker or glue laminated square inch grid paper into the bottom of each pan. This grid will allow the children to measure the surface area of the water in their puddles. In order to measure the area of the water, the children will count the grid squares that are covered with water. As the water evaporates some discussion will have to be prompted about how to measure fractional parts of the square inches on the grid. Students could choose to count any fraction of a square inch that is covered with water as a whole inch or make estimations about the amount of the square inch that is covered. The children can estimate if one half or one quarter of a square inch is covered when they measure. This could be an excellent real-world way to introduce fractional parts to your first grade students.

Once the pans are prepared the students should measure out a predetermined amount of water (you will have to determine this before the experiment based on the amount of water needed to cover the largest pan but not to overflow the smallest pan) and pour it into each puddle. It is very important that you stress to the children that the same amount of water is to be poured into each puddle. Then the children should count all the squares that the water has covered and record this data. Next, ask the children to predict which "puddle" will evaporate fastest. I record student predictions on a T-chart (see appendix A) by writing the student's initials on the chart in the appropriate column.

The puddles should be stored next to each other in an area of the classroom where the children can observe them and the evaporation. The conditions for the evaporation, temperature, and air movement, should be identical for all the puddles. Each day the students should observe their puddles, count the squares that are covered by the water while the puddles are still, and record this data for future use. You can either end the experiment when the first of the puddles has evaporated or continue until all the puddles are gone. Discussion about why the puddle plate with the largest surface area evaporated before the smallest will arise as students observe their puddles. Lead the children to the conclusion that the largest surface area evaporated first even though it held the same amount of water as the smallest pan.

Finally, encourage the students to graph the data for each puddle. Have each small group create a graphic representation of the puddle data to compare and analyze or have the whole group work to create a large graph of the data for each puddle (appendix B). Have the students make predictions as the last step of the mathematical modeling cycle. Ask them to predict the evaporation rates of other sizes of puddles based on the results of the experiment.

The next activity in this unit that will utilize the mathematical modeling cycle is looking at the system of measuring the wind. In the weather science unit I teach the children create a simple tool for measuring the wind speed (a flag) and use a simple wind scale of 0 - 2 to record their measurements. I challenge the children to create another device to measure the weather and create a scale for the device they create (appendix C). The children begin by discussing the problems with measuring wind speed. From this discussion questions should arise about the different methods in which wind can be measured. I assign the creation of a device to measure wind speed and a wind scale as a homework project. I allow a couple of weeks for the children to create their anemometers. Once the children bring their anemometers and wind scales to school we analyze the devices for effectiveness. I set up a box fan with 3 speeds and we set each anemometer up in front of the fan. We test the anemometer at all three speeds. Then we compare the results to the wind scale created for the anemometer. Corrections are made when necessary and devices are retested. As the last step in the mathematical modeling cycle, after each anemometer is tested, the children make predictions about its effectiveness in measuring winds in extreme weather situations such as tropical storms, hurricanes, and blizzards.

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van de Walle, John A.. "Exploring What it Means to Do Mathematics." In *Elementary and middle school mathematics: teaching developmentally*. 6th ed. Boston: Allyn and Bacon, 2004. 13.

Appendix A

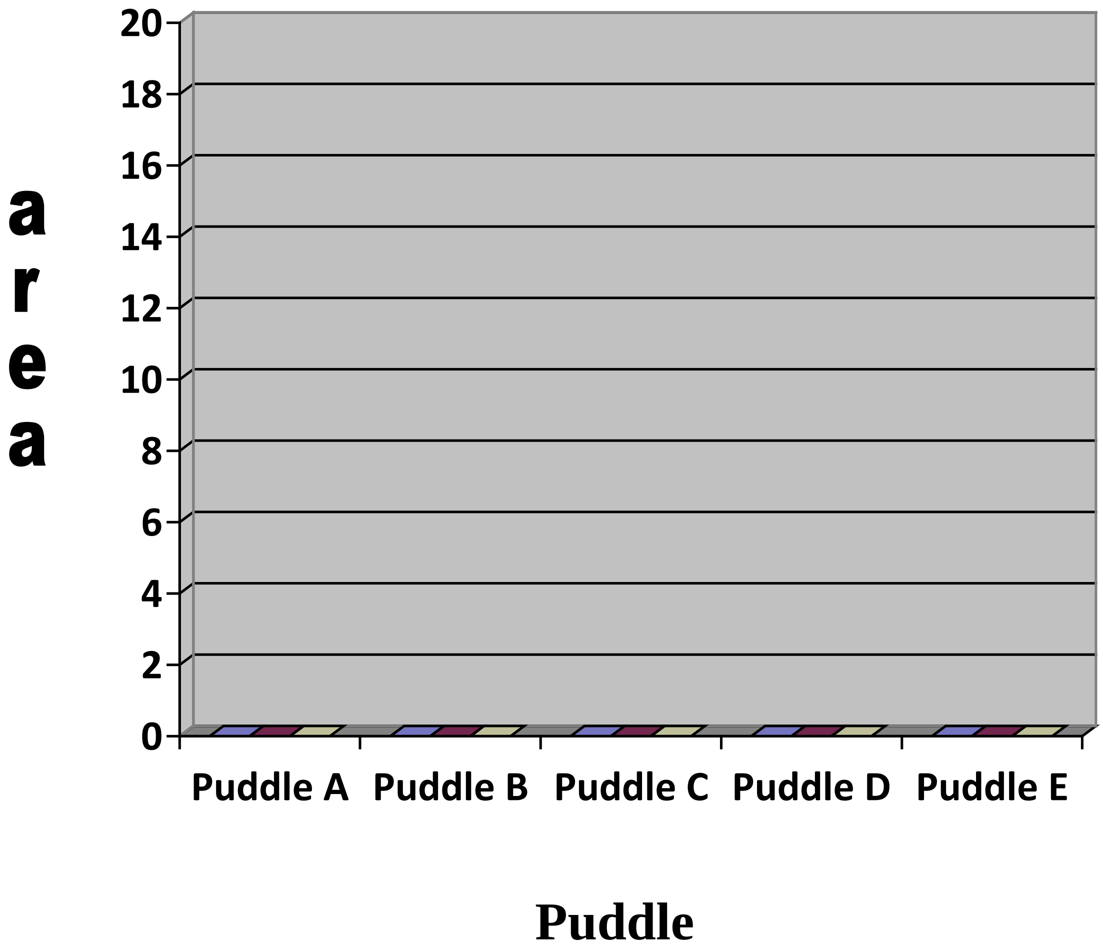
Puddle Predictions

Which puddle will evaporate first?

Puddle A	Puddle B	Puddle C	Puddle D	Puddle E
				

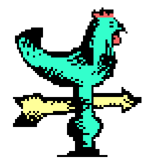
Appendix B

Graphing Puddle Data



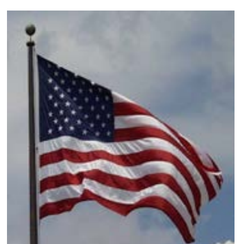
Appendix C

Design an Anemometer Project



Dear Families:

An anemometer is also called **wind gauge:** an instrument for recording the speed of winds. In the classroom we created a flag and measured the wind using the following wind scale:

0	1	2
No wind 	**Some wind** 	**Strong wind** 

Other than a flag there are many other ways to measure wind speed. My challenge to my students is to **design an anemometer other than a <u>flag</u>** to measure the wind speed **and create a wind scale** to report the measurement. Design and create your anemometer and wind scale at home and bring it to school on _____________________. We will test out each device and measure the wind speed! Enjoy your engineering experience!

Some things to think about while you design:

- Which object moves well in the wind?
- How do I measure that object's movement?
- What are the different speeds that wind can move?

Appendix D

This unit addresses the following Common Core State Standards in Mathematics: The students will be adding data - Numbers and Operations in Base Ten (#4) Use place value understanding and properties of operations to add and subtract; organizing, representing, and interpreting data - Measurement and Data (#4) Represent and interpret data.

This unit also addresses Delaware State Science Standards 1 and 5:
The students will be observing experiments, collecting and recording data (#1) modeling and observing the water cycle and measuring and reporting wind speed (#5).

Curriculum Unit Title	Mathematical Modeling and Weather	Author	Gretchen Wolfe

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Weather has patterns that change over time and that can be measured, observed, and predicted.

ESSENTIAL QUESTION(S) for the UNIT

How do we measure weather? Why is measuring weather necessary? How do we organize and use data in our everyday lives? How does understanding the properties of Earth materials and the physical laws that govern their behavior lead to prediction of Earth events? How can making a model help me predict behavior?

CONCEPT A	CONCEPT B	CONCEPT C
Mathematical modeling	Weather measurement	Graphically representing data
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How can making a model help me predict behavior?	How do we measure weather? Why is measuring weather necessary?	How do we organize and use data in our everyday lives?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
question analyze predict	anemometer scale	graph

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Endnotes

[1] National Governors Association Center for Best Practices and Council of Chief State School Officers. "Common Core State Standards Initiative | Home." Common Core State Standards Initiative | Home. http://www.corestandards.org/ (accessed December 3, 2012).

[2] van de Walle, John A.. "Exploring What it Means to Do Mathematics." In *Elementary and middle school mathematics: teaching developmentally*. 6th ed. Boston: Allyn and Bacon, 2004. 13.

[3] Abrams, Joshua P.. "Teaching Mathematical Modeling and the Skills of Representation." In *The roles of representation in school mathematics*. Reston, Va.: National Council of Teachers of Mathematics, 2001. 269.

[4] Usiskin, Zalman. "Mathematical Modeling in the School Curriculum." Lecture, UCSMP Conference Talks from University of Chicago School Mathematics Project, New York, September 26, 2011.

[5] Ibid

[6] Burns, Marilyn. "Probability and Statistics." In *About teaching mathematics: a K-8 resource*. Sausalito, Calif.: Marilyn Burns Education Associates ;, 1992. 76-77.

[7] Ibid.