Getting to the Heart of Multiplication: Understanding its Properties and Patterns

Kristin Becker

Rationale

After reflecting on the mathematical needs of my students over the past few years, it is evident that they lack number sense. Few enter third grade with true number sense. In my view, it cannot be taught like an algorithm. Instead, it takes exposure, experience, and practice working with numbers and problem solving situations over time to make sense of numbers. An avenue for facilitating number sense in my classroom is through analyzing and understanding multiplication, a concept around which most of the third grade curriculum revolves. Specifically, I would like to look at the properties and patterns of multiplication, as outlined in these two Common Core Math State Standards 3.OA.B.5 and 3.OA.D.9. (Appendix A)

The demographics of my class this year encompass a total of twenty students, 11 girls and 9 boys. The school's feeder pattern is an upper-middle class area. Only one student is new to the school this year, transferring from a parochial school. The other nineteen have attended our elementary school since kindergarten, experiencing our math program for three years prior to third grade. Despite the consistency of a math program since kindergarten, I have found that only a small number of my students have a true sense of numbers, including the number line. Of those students, only a few can clearly communicate their mathematical thinking.

So, it seems as though I must begin at the beginning in regards to multiplication. While all of my students can answer 2 x 4 = ?, many struggle with 2 x 4 = 4 x ?. Perhaps it's because students see the equals sign as an operator symbol instead of a relational symbol.[1] Though this study was years ago, I find this misconception to be true in today's classroom as well.

After understanding the true meaning of the equals sign, one can move on to properties of multiplication. The commutative and associative properties will lay the groundwork for understanding multiplication, but the distributive property will enhance a true understanding of the number sense in multiplication. As noted above, "*Knowing that* $8 \times 5 = 40$ *and* $8 \times 2 = 16$, *one can find* 8×7 *as* $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$*"* is an expectation of a third grader. But does the student know WHY? Can the student explain it to someone else? Can the student apply this concept to new situations? This is the level of number sense and understanding for which I will strive.

With the knowledge of the properties of multiplication in hand, one can analyze additional patterns in multiplication. Again, we will not just notice that "*4 times a number is always even"*, but we will be able to tell why and back it up with evidence. Also, we can analyze sets of numbers (as suggested in seminar), such as multiples of 2, 5, and 10. Using a Venn diagram to plot the multiples of 5 and 10, then another to plot the multiples of 2 and 5, will allow students to see patterns and develop a concept of sets. A three-ring diagram plotting multiples of 2, 5, and 10 will help the students "see" the more sophisticated relationships between all the sets of multiples.

Students will need to not only understand and explain the properties and patterns of multiplication, but also be able to represent them in various forms, such as number sentences and sketches.

My goal for this unit is to employ best practices in teaching mathematics while instructing students on the properties and patterns of multiplication. Students will understand multiplication inside and out, backward and forward, literally.

The following content is intended to provide teachers with the necessary background information to teach students to work with multiplication situations in an effective manner that makes sense. This unit is not intended to teach the properties, and certainly not intended to assess the students' knowledge of properties, but rather to help students use their understanding of the properties and patterns in multiplication to think about numbers abstractly, analyze expressions, solve equations, and decipher real-world problems.

Content

Number Sense

Number sense essentially refers to a student's "fluidity and flexibility with numbers." [2] Marilyn Burns describes students with sophisticated number sense in this way: "[They] can think and reason flexibly with numbers, use numbers to solve problems, spot unreasonable answers, understand how numbers can be taken apart and put together in different ways, see connections among operations, figure mentally, and make reasonable estimates."

The National Council of Teachers has determined that the following components encompass number sense: number meaning, number relationships, number magnitude, operations involving numbers and referents for number, and referents for numbers and quantities. Teaching strategies that help build number sense include: model different methods for computing, ask students regularly to calculate mentally, have class discussions about strategies for computing, make estimation an integral part of computing, question students about how they reason numerically, and pose numerical problems that have more than one possible answer. [3]

Commutative Property of Multiplication

"The commutative property of multiplication states that a x b gives the same result as b x a for any values of a and b, making the order of the factors irrelevant to the product." [4] This property is true of addition as well, but not subtraction or division.

Devlin states, "… the mathematician's concept of integer or real number multiplication is commutative: M x N = N x M. …The order of the numbers does not matter. Nor are there any units involved: the M and the N are pure numbers. But the non-abstract, real-world operation of multiplication is very definitely not commutative and units are a major issue. Three bags of four apples are not the same as four bags of three apples. And taking an elastic band of length 7.5 inches and stretching it by a factor of 3.8 is not the same as taking a band of length 3.8 inches and stretching it by a factor of 7.5." Thus, it is important that students can distinguish between abstract thinking of multiplication of numbers and real-world application.

Of great interest to me when reading various articles and research studies on the topic of the commutative property of multiplication was this excerpt from a paper published by Thomas and Keung of the Hong Kong Institute of Education. It points out how an understanding of the commutative property helps students understand how many fact they truly have to memorize – much less than what most students think. The multiplication chart may seem overwhelming at first, but a solid understanding of the commutative property of multiplication can ease a child's mind. The authors explain the content in this manner: "If we look at the whole multiplication table from 2 to 9, there are a total of 64 cells students have to memorize… However, because multiplication is commutative, the cells are symmetric along the diagonal. For example, 3 x 6 = 6 x 3, 4 x 7 = 7 x 4, 5 x 8 = 8 x 5, etc… Thus, by using the commutative property of multiplication, the actual number of cells needed to be memorized can be reduced to 36… This is a major breakthrough psychologically for many students. Their eyes would glow when you tell them how much memory hard work can be reduced by knowing and understanding the commutative property!" [5]

Associative Property of Multiplication

The Associative property of numbers states that the addition or multiplication of a set of numbers gives the same output regardless of how they are grouped. When students start the set should have a minimum of three different numbers. Later on, in other examples, some or all of the three numbers may be equal, for example (1+2)+1=1+(2+1). The property of associativity is not applicable for subtraction and division. (a-b)-c is not equal to a-(b-c). Instead, (a-b)-c = a-(b+c). Examples of the associative property of multiplication include: (a x b) x c = a x (b x c), for any numbers a, b, and c, (1 x 2) x 3 = 1 x (2 x 3), and 4 x (5 x 6) = (4 x 5) x 6.

Identity Property of Multiplication

The multiplication of any number and the identity value gives the same number as the given number. This is called the identity property of multiplication. In multiplication, 1 is the identity element, which means that a x 1 = 1 x a = a for any number a. For the operation of addition, the identity element is 0. This should not be confused with multiplication. a+0=0+a=a, for any number a. Examples of the identity property of multiplication include: 4 x 1 = 4, 17 x 1 = 17, and 4,345,999 x 1 = 4,345,999.

We can remember the three above properties of multiplication just as we can remember the corresponding properties of addition. With the Commutative Property of Multiplication, when only multiplication is involved, numbers can move ("commute") to anywhere in the expression. With the Associative Property of Multiplication, any numbers that are being multiplied together can "associate" with each other. Also, multiplying by 1 does not change the value of a number.

Zero Property of Multiplication

The zero property of multiplication is also called the zero product property. It states that there exists a unique number, zero, such that the product of any real number x and 0 is always equal to 0.[6] a x 0 = 0, for any number a.

The following excerpt clarifies and extends this explanation of multiplication’s zero property: The Zero property of multiplication says that any number multiplied by 0 is equal to 0. For any number a, the following are always true: a x 0 = 0 and 0 x a = 0. For example, 3×0 = 0 and 4,567,892,435×0 = 0.

Because multiplication commutes, if you are multiplying a long string of numbers that contains 0, you can move 0 to the beginning of the expression:

$$4\times234\times7\times9\times16\times0\times54 = 0\times4\times234\times7\times9\times16\times54$$

Because multiplication associates, this expression is equal to:

$$0\times(4\times234\times7\times9\times16\times54) = 0.$$

Thus, when multiplying any string of numbers, if 0 is one of the numbers, then the answer is always 0.

Distributive Property of Multiplication

The distributive property is a property of some binary mathematical operations, which are operations that affect two elements. Multiplication distributes over addition. That is, $a \times (b + c) = a \times b + a \times c$ and $(b + c) \times a = b \times a + c \times a$ for all real or complex numbers a, b, and c.

The distributive property is implicit in the common multiplication algorithm. For example, 27×4 means $4 \times$ (2 tens + 7 ones). To complete the multiplication, you use the distributive property: $4 \times (20 + 7) = (4 \times 20) + (4 \times 7) = 80 + 28 = 108$.

We use the distributive property more than once in carrying out such computations as $(3x + 4)(x + 2)$. Thus $(3x + 4)(x + 2) = (3x + 4)x + (3x + 4)2$ where $3x + 4$ is "distributed over" $x + 2$ and then $(3x + 4)x + (3x + 4)2 = 3x^2 + 4x) + (6x + 8) = 3x^2 + 10x + 8$ where x and 2 are distributed over $3x + 4$. [7]

Additional Patterns in Multiplication

An article titled *Meaning, Memory, and Multiplication – Integrating Patterns and Properties with Basic Facts*, by Don Ploger and Steven Hecht, provides us with both contextual background information and effective teaching strategies for a variety of patterns found in multiplication. I have evidenced a few scenarios below that I believe will benefit the implementation of this unit.

"I had been to school…and could say the multiplication table up to six times seven is thirty-five, and I don't reckon I could ever get any further than that if I was to live forever. I don't take no stock in mathematics, anyway." This quote, from Mark Twain's *Huckleberry Finn*, is used by Don Ploger and Steven Hecht to illustrate an important point. Although it is important for children to know that 6 x7 = 42, that fact is only part of the knowledge of mathematical operations. When multiplication is mastered, a student knows that 6 times 7 cannot possibly equal 35, because (among many other reasons) the product of an even number times an odd number is always an even number. [8]

Ploger and Hecht used the multiplication table as a springboard for a number of patterns students may identify and use in multiplication situations. They believe that it is more likely that children will form a richer understanding of arithmetic when explicitly taught the variety of meaningful patterns that can be seen within a multiplication table. They used a software program titled Chartworld, created by Ploger. But the same process they used in their study could be done on a Smartboard and/or paper and (colored) pencil.

In order to address the accuracy, or inaccuracy, of the Huck Finn quote, a student can study various patterns in an arithmetic chart, in this case, a hundred chart. All the multiples of 6 were highlighted in yellow. It was noted that all of these multiples were even. Then all the multiples of 7 were highlighted in blue. It was noted that these multiples had a pattern of odd, even, odd, even, and so on. The student noted that 42 was blue and yellow because 6 x7 = 42. It was also noted that 84 was highlighted in both blue and yellow. Further discussion could ensue on this finding as well. A further observation by the student included that fact that 6 x 7 cannot be 35 is an odd number and all multiples of 6 are even. Ploger and Hecht point out that while it is useful to point out that "no odd number can be a multiple of 6, it is much more effective when children actually see that fact illustrated." [9]

The multiplicative claims in the Huck Finn quote were investigated further by highlighting the multiples of 5 and 7 on a shared hundred chart. This process showed that 35 is the 7th multiple of 5 and the 5th multiple of 7. It also indicated that 70 is a common multiple of 5 and 7, as well. Again, discussion may ensue on the mathematical reasoning behind this and how it relates to the earlier finding of 42 and 84 being multiples of 6 and 7. According to Ploger and Hecht, "the visual approach does far more than show students that 6 x 7 = 42 and 5 x 7 = 35; it also helps students understand the underlying mathematical concepts."[10]

These authors suggest a side-by-side comparison of odd numbers on a hundred chart and a multiplication chart to further clarify and understand the concept of even x even = even, odd x odd = even, and even x odd = odd. In their case study, the students were asked, "How many odd numbers are in the 100-chart?" The students correctly answered, "50." When asked, "How many odd numbers are in the multiplication chart?, the students were surprised to discover there are only 25. After the initial surprise, students began to study the multiplication chart more deeply. Unlike the 100-chart, which has 100 separate numbers, the multiplication chart far fewer numbers, since many numbers are repeated, even though both charts have 100 numbers in total (when using a multiplication chart up to 10 x 10 = 100). Each square containing an odd number was highlighted in blue. Then the table can be analyzed. Findings would include: odd numbers are only in rows and columns that start with an odd number, and in these rows and columns, only every other number is odd; all rows and columns starting with an even numbers are filled completely with even-numbered products; so, five columns (the odd numbers from 1 to 10) contain 5 odd numbers each, so the number of odd numbers in the multiplication table is 5 x 5 = 25. This can be down by the rows, as well. 5 rows times 5 odd-numbered products in each row is also 5 x 5 = 25.[11] This visual representation further enhances the properties and patterns in multiplication.

The students in this study also discovered that the square numbers on the multiplication chart form a diagonal. I noted personally that if you were to shade the row and column for each square number, it forms a square with the outer edge of the chart. This may be a helpful way for students to visual square numbers, thus grasp the concept of square numbers.

In addition, one student discovered a connection between multiplication and division. She highlighted all the multiples of 6. She did not highlight 100. Thus, she knew that 100 was NOT a multiple of 6. Furthermore, since the last highlighted number was 96, she knew that if we divided 100 by 6, the answer would have a remainder of 4.[12] While this unit focuses on multiplication, students with astute number sense will undoubtedly make connections to division, which should be encouraged and celebrated.

According to Ploger and Hecht, "This approach to learning mathematics led children to a deeper understanding of the meaning of multiplication and its connection to other mathematical ideas… This study demonstrates that children can learn the facts of multiplication, while also learning to explain the meaning of their answers." [13]

Ploger and Hecht also noted, "Mathematical errors are interesting to children. In some cases, examining those errors can lead to rich learning experiences. Because we appreciate the errors of others more than our own, this study provided such an example. However, students can learn to recognize any incorrect answer not simply because it does not match the answer in the back of the book, but based upon sound mathematical reasoning." [14]

Learning Activities

I used a combination of content theories and instructional approaches from Thomas, Keung, Burn, Ploger, and Hecht to create learning activities for this unit. I melded a number of ideas to create engaging, hands-on mathematical activities and real-world problem solving opportunities for my students.

Activity One

The Array Game is designed to reinforce and apply students' understanding of multiplication, the commutative property, and area. Students work in pairs. Each pair needs graph paper (either an 8.5 x 11 sheet or large chart paper works well) with a line drawn in marker splitting it exactly in half and decks of cards with the face cards removed. In round one, each player flips over two cards and determines the multiplication statement. For example, a 5 and a 7 could be 5 x 7 = 35 or 7 x 5 = 35. The student must then outline an array that matches his number sentence on his half of the graph paper and write the number sentence inside of the outline. Round two is played the same way, and round three, and so on. However, as the graph paper begins to fill up, the students must become strategic in their placement of the arrays. The object of the game is to fill up your side with the least amount of empty spaces. When a player takes three turns without being able to draw any new arrays, her game is over. When the same happens for the opponent, the whole game is over. Count up the empty squares. The player with the least number of empty spaces wins. Appendix B provides an example of a partially played game. This first version of the Array Game allows students to manipulate two factors to best suit their play. The next version offers more possibilities, and therefore more strategic thinking.

A second version would be that when you flip over two cards, you first find the product, then choose ANY two factors that would equal that product to play on your board. For example, if a student flipped over a 4 and a 6, with a product of 24, he could choose from 1 x 24, 2 x 12, 3 x 8, and 4 x 6 arrays and their commutative partners. This requires higher understanding of multiplication and factors. Students must be more strategic in their thinking in order to win.

I begin with version one of this activity. As students demonstrate proficiency, I move them to version two. The inspiration for this activity and its two versions came from a game in the Trailblazers math series. Additional variations include using the Jack and Queen in the deck of cards as 11 and 12, respectively and using dice instead of cards. Also, for version two, you could simply write the products on index cards, and students could pick one card at a time, though I find that playing cards and dice make these activities more fun for the students.

After completing this activity, students will understand area as well. I don't even use the term area during the Array Game, but introduce it the next day, and show them that they already know how to find area based on the activity. Some students will be familiar with finding area by counting squares. This can be done, but now students will understand that simply multiplying length times width will also give them the area, but in a much more efficient manner. And most importantly, they will not just be memorizing an algorithm for finding area, they will truly understand the meaning of area and why the algorithm works.

Activity Two

Activity two allows students to apply what they have learned, stretch their thinking, and improve their mathematical communication. This activity involves writing a different word problem at the top of four separate pieces of chart paper. Then, draw a horizontal and vertical line the size of the page to divide it into four equal sections. Students will work in small groups to answer the questions. Each group has its own unique marker color. The challenge comes with this: each time a group approaches a word problem they must find a way to solve it that has not already been demonstrated by the previous group(s). For example, one group may answer a particular problem with the equation 21/3 = 7. The next group may show they solved the problem by asking, "3 times what number equals 21?" The third groups must then come up with a different strategy to solve this same problem, such as 21 – 3 – 3 – 3 – 3 – 3 – 3 – 3 = 0 to determine there are 7 threes in 21. This activity requires students to be broken into groups of a maximum of four students. In my class, I have twenty students. I actually broke them into eight groups of 2 or 3 each because I wanted small groups with 100% participation. I also tell the students that they must take turns being the recorder to encourage that 100% participation as well. So I wrote a total of four questions (Appendix C); each one written on two pieces of chart paper for a total of four questions, but each written twice, so a total of 8 pieces of chart paper. Students were put in their groups and given a starting location. Every five minutes or so, students were told to rotate. There were four rotations working through one set of problems. The same set of problems was another rotation on the other side of the room. This allowed for maximum participation, but a manageable number of word problems. Perhaps the most important part of this activity is the discussion that takes place after the problem solving. I posted the chart papers with the matching word problems together. Then we looked at each word problem and discussed various strategies and solutions to the problems.

After conducting this activity in the classroom, I found a concept that the students had only partially mastered, and that was the distributive property. In a previous activity, they had demonstrated that they could break apart multiplication sentences into two multiplication sentences through working with an array. Students would take an array such as 8 x 5, then break it into two smaller arrays, such as 5 x 5 and 3 x 5 in order to make it a more manageable multiplication to solve. They were able to write 8 x 5 = (5 x 5) + (3 x 5) and draw a picture to represent their thinking. So I mistakenly believed they fully understood the distributive property. But in fact, I realized after analyzing their responses in this activity that although they were able to take apart number sentences in terms of the distributive property, they did not transfer that content to putting together number sentences in terms of the distributive property. As a result, I created another activity for my students.

Activity Three

The third activity was designed to help students better understand the distributive property from every angle. I call this activity Down on the Farm. You can set up the problems for this activity in a number of ways, including simply writing out a number of problems for the children to strategize, solve, and discuss. However, in the interest of making math fun and engaging, I chose to create three spinners for this game. The first spinner has 2-legged animals – chickens, geese, and ducks. The second spinner has 4-legged animals – pigs, horses, cows, and sheep. The third spinner has the numbers 1 through 10. Spinners can be made by copying the spinner on paper, then holding a paper clip in place in the center of the spinner with a pencil point, then spinning the paper clip to see where it lands. Depending on the teacher's directions, students will spin spinner one and/or two and always spinner three. This will give the students the information needed to solve the problem. Students must complete the response sheet (Appendix D) for each problem. Specifically, they must record their data, write a question, write a number sentence that represents the parts being put together, draw a graphic representation of the problem, draw an array that incorporates all parts of the problem in a color-coded fashion, then write a final number sentence that represents the groups being taken apart or as separate groups.

To clarify, the teacher may say, “Spin the four-legged animal spinner and the number spinner. Record that information in the “Data” box. OK, now spin the four-legged animal spinner and the number spinner one more time. Record that information too.” The students would then complete the response form based on this data. Appendix E is an example.

This activity is easily differentiated. In order to cater to students’ interests, you may want to change the theme to Life in a Zoo. To address achievement differences, students in need remediation might only work with the two-legged animals so they are multiplying lower numbers, or they may only spin one animal spinner or the other to keep things in only groups of two or groups of four. Students grasping the concept can increase the difficulty level by spinning a given spinner three or more times. And those in need of even more challenging problems can spin each of the spinners a number of times. This will get very interesting and challenging when students attempt to draw arrays…a wonderful opportunity for stronger mathematicians to really analyze, and perhaps challenge, their own thinking.

Conclusion

This unit serves to provide its readers with the research that drove my learning activities, some insight into how the activities were implemented in my classroom and/or may be implemented in others’ classrooms, and some hands-on ways to help students dig deeper than just learning the properties of multiplication, but to apply their deeper understanding to new problem-solving situations.

Notes

[1] Baroody, Arthur J., and Herbert P. Ginsberg. "The Effects of Instruction on Children's Understanding of the "Equals" Sign." *The Elementary School Journal* 84, no. 2 (1983): 198-212.

[2] Gersten, R., and D. Chard. “Number Sense: Rethinking Arithmetic Instruction for Students with Mathematical Disabilities.” *The Journal of Special Education* 33.1 (1999): 18-28.

[3] Burns, Marilyn. *About Teaching Mathematics: A K-8 Resource*. 3rd ed. Sausalito, CA: Math Solutions, 2007.

[4] Lerner, Ed. K. Lee, and Brenda Wilmoth Lerner. "Distributive Property." *The Gale Encyclopedia of Science* 5th Edition (2014).

[5] Leung, Thomas Kim Wai, and Leung Hing Keung. "New Ideas in Teaching the Multiplication Table in Primary Mathematics Education." *Hong Kong Institute of Education Maths Department*: 133-41. http://math.unipa.it/~grim/AKeung&Leung.PDF.

[6] "Math Dictionary." Math Dictionary. Accessed December 9, 2014. http://www.mathematicsdictionary.com/math-vocabulary.htm.

[7] Lerner, Ed. K. Lee, and Brenda Wilmoth Lerner. "Distributive Property." *The Gale Encyclopedia of Science* 5th Edition (2014).

[8] Ploger, Don and Steven Hecht. "Meaning, Memory, and Multiplication: Integrating Patterns and Properties with Basic Facts." *Childhood Education* 88.3 (2012): 169.

[9] Ibid, 170.

[10] Ibid, 173.

[11] Ibid, 173.

[12] Ibid, 173.

[13] Ibid, 175.

[14] Ibid, 175.

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Baroody, Arthur J., and Herbert P. Ginsberg. "The Effects of Instruction on Children's Understanding of the "Equals" Sign." *The Elementary School Journal* 84, no. 2 (1983): 198-212. This article addresses students' typical understanding of the equals sign and the desired understanding for our students.

Burns, Marilyn. *About Teaching Mathematics: A K-8 Resource*. 3rd ed. Sausalito, CA: Math Solutions, 2007. This is a text that provides strategies for teaching mathematics.

"Devlin's Angle." What Exactly Is Multiplication? January 1, 2011. Accessed December 8, 2014. http://www.maa.org/external_archive/devlin/devlin_01_11.html. This article discusses the meaning of multiplication versus repeated addition.

Gersten, R., and D. Chard. "Number Sense: Rethinking Arithmetic Instruction for Students with Mathematical Disabilities." *The Journal of Special Education* 33.1 (1999): 18-28. This article addresses the needs of all students in the mathematics classroom.

Lerner, Ed. K. Lee, and Brenda Wilmoth Lerner. "Distributive Property." *The Gale Encyclopedia of Science* 5th Edition (2014). This entry provides a mathematical definition and explanation of the distributive property.

Lerner, K. Lee, and Brenda Wilmoth Lerner. *Real-Life Math*. Farmington Hills: Gale, 2006. This is a text that helps teachers make math meaningful to students.

Leung, Thomas Kim Wai, and Leung Hing Keung. "New Ideas in Teaching the Multiplication Table in Primary Mathematics Education." *Hong Kong Institute of Education Maths Department*: 133-41. http://math.unipa.it/~grim/AKeung&Leung.PDF. This article shares insight and strategies for mathematics instruction.

"Math Dictionary." Math Dictionary. Accessed December 9, 2014. http://www.mathematicsdictionary.com/math-vocabulary.htm. This website provides accurate definitions and explanations of mathematics vocabulary.

Ploger, Don and Steven Hecht. "Meaning, Memory, and Multiplication: Integrating Patterns and Properties with Basic Facts." *Childhood Education* 88.3 (2012): 169. This article relates multiplication to the hundred chart in order to identify various patterns and properties.

"Preparing America's Students for Success." Common Core State Standards Initiative. January 1, 2014. Accessed December 8, 2014. http://www.corestandards.org/. This website provides a detailed listing of the Common Core State Standards.

Appendix A

Standards

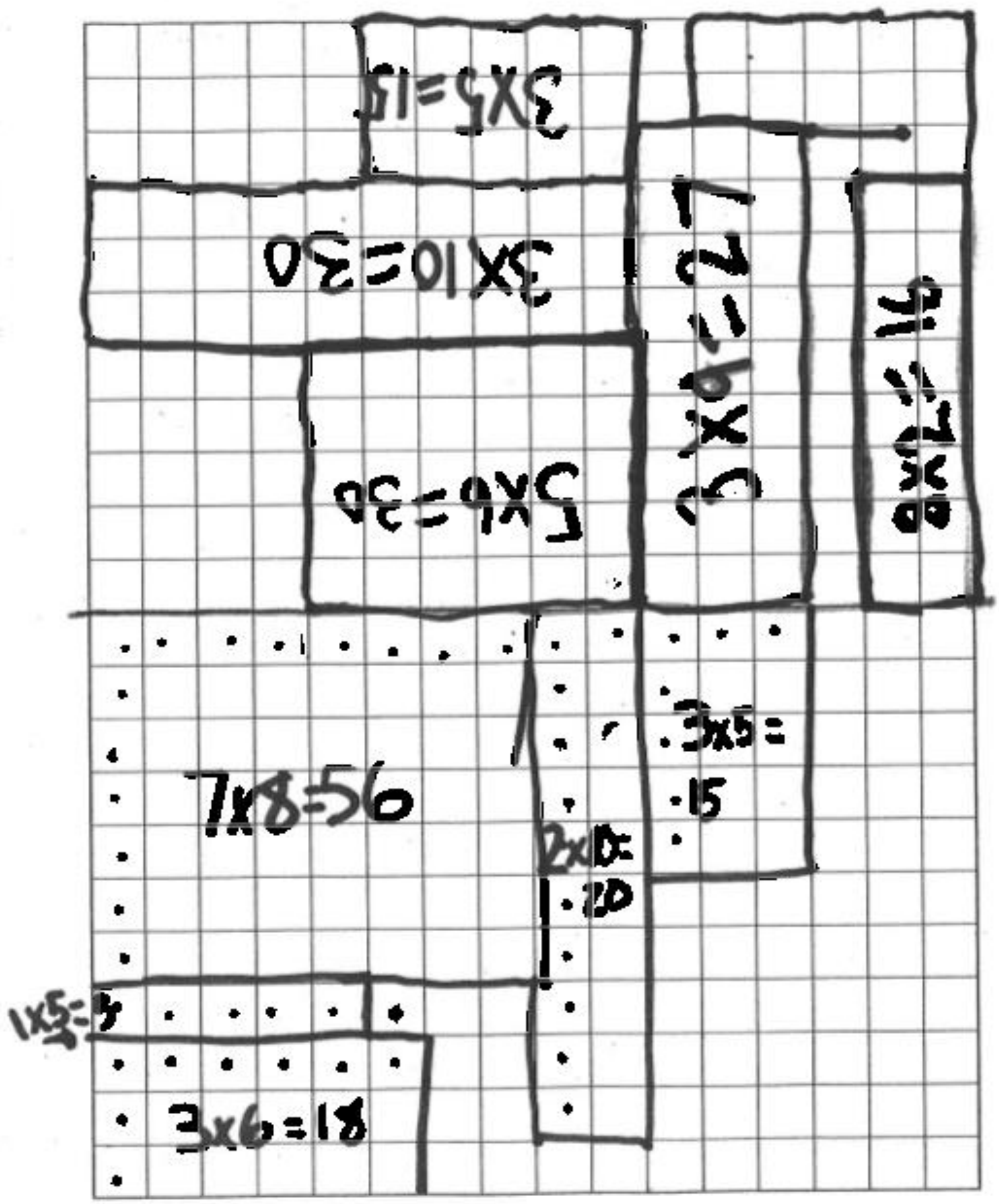
CCSS.Math.Content.3.OA.B.5 Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

CCSS.Math.Content.3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*. (Common Core Standards Initiative 2014)

Standards Alignment with Unit Lessons

Activity	Standard(s) Addressed
One – The Array Game	CCSS.Math.Content.3.OA.B.5 Apply properties of operations as strategies to multiply and divide. CCSS.Math.Content.3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.
Two – Problem Solving	CCSS.Math.Content.3.OA.B.5 Apply properties of operations as strategies to multiply and divide.
Three – Down on the Farm	CCSS.Math.Content.3.OA.B.5 Apply properties of operations as strategies to multiply and divide. CCSS.Math.Content.3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

Appendix B


3X5=15
3X10=30
5X6=30
3X9=27
8X2=16
7X8=56
3x5=
15
2x10=
20
1x5=5
3x6=18

Appendix C

Questions for Activity Two

1. 28 students are in a relay race. They must split up into teams of 4 people. How many teams will there be?

2. Mrs. Becker's class went to Old MacDonald's farm. They saw 2 horses, 5 chickens, and 4 pigs. How many legs did they see?

3. There are 9 baskets of eggs. Each basket has 3 yellow eggs and 5 blue eggs. How many eggs in all?

4. Farmer Brown is planting 2 rectangular gardens. The first one is 5 feet by 9 feet. The second one is twice that size. He has one bag of fertilizer, which covers 125 square feet of garden. Does he have enough fertilizer to cover both of his gardens?

Appendix D

Down on the Farm

Data:

Question:
Number Sentence:
Graphic:
Array:
Number Sentence:

Appendix E

Down on the Farm

Data:

4 pigs 5 sheep

Question:

How many legs?

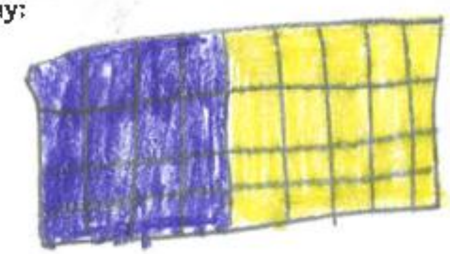
Number Sentence:

$(4\times4)+(5\times4)=16+20=36$

Graphic:

pigs sheep

Array:



pig

Number Sentence:

$36=4\times9=(4\times4)+(4\times5)$

Curriculum Unit Title

Getting to the Heart of Multiplication:

Understanding its Properties and Patterns

Author Kristin Becker

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

CCSS.Math.Content.3.OA.B.5 Apply properties of operations as strategies to multiply and divide.

CCSS.Math.Content.3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

ESSENTIAL QUESTION(S) for the UNIT

How does understanding the properties of multiplication help you solve mathematical problems?
How can identifying patterns help you solve mathematical problems?

	CONCEPT A	CONCEPT B	CONCEPT C
	Commutative Property of Multiplication	Finding Area	Distributive Property of Multiplication
ESSENTIAL QUESTIONS	How does the commutative property of multiplication apply to arrays?	How are arrays and area related? How does understanding the commutative property help you to understand and solve for area?	What is the distributive property of multiplication? How does understanding the distributive property of multiplication help you solve mathematical problems?
VOCABULARY	commutative property of multiplication array	area length width	distributive property of multiplication “put together” versus “take apart”

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

These lessons may be taught in succession, or throughout a unit of study.