Investigating Sequences and Series

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Rationale

The Precalculus curriculum at Conrad covers a plethora of math topics! Units in our book, Precalculus with Limits: A Graphing Approach, review previously learned mathematical concepts and then extend student thinking further or deeper into those concepts. Many students – and the math teachers – think the curriculum bounces around from topic to topic. However, there are connections from one unit to the next and students begin to see those connections as we move through the textbook as the school year progresses. Currently sequences and series are a chapter in the Precalculus text that is not covered in the typical Precalculus course at Conrad. However, upon reflection of the Common Core State Standards students need to know prior to graduation in order to be college and career ready, this chapter needs to be incorporated into the Precalculus curriculum as it does not appear to be taught in the other high school mathematics courses at Conrad prior to Precalculus. Since this course is a senior course for many students at my school, this unit is necessary. Thus begins my adventure into the world of sequences and series for my students!

Prior to Precalculus, students may or may not have been exposed to the vocabulary of sequences and series, depending on whether or not their Algebra 2 teacher had time to address these concepts. Students in Algebra 1 have been exposed to patterns of change – both arithmetic and geometric – but have probably not generated formulas or expressions to generate the nth term of sequences beyond simple linear or quadratic patterns. The Precalculus textbook chapter consists of the following sections: an introduction to sequences and series, arithmetic sequences and partial sums, geometric sequences and series, mathematical induction, the Binomial theorem, counting principles, and probability. Students have some prior knowledge on these topics – specifically counting principles and probability. The intent of teaching this section in Precalculus will be to introduce some students to the concept of sequences and series, enrich other students understanding of sequences and series, deepen each student's understanding of these concepts, and enable students to apply these concepts in context empowering students to utilize these concepts in real world applications. Requiring students to think critically about various problem solving scenarios involving sequences and series will prepare them for the new state assessment. A unit covering these topics will also enable students to understand how concepts surrounding sequences and series are utilized in real life scenarios. I think students will be surprised to learn how some of those pesky word problems are actually sequence and series problems. It is my intention that when this chapter is finished, students will be able to understand and conquer those pesky problems experiencing a sense of empowerment in their achievements.

Student Population

The Precalculus classes at Conrad Schools of Science consist of students who are both diverse in both age and ability levels. They come from urban and suburban backgrounds. Students must choice into the school which requires both the student and parent to desire entrance into the school. Once students have met the minimum criteria – living in district, interest in science, prior attendance and grades – students are drawn from a lottery system and invited to attend our school. The school population tends to be approximately 40% Caucasian, 30% African American, and 30% Hispanic. The Precalculus Honors students are in grades nine through twelve and have chosen this honors math class with or without meeting stated math prerequisites of an A or B in the previous honors course. They represent ability levels from mediocre to gifted. Students have completed the Algebra 2 curriculum in either eighth grade – for my ninth and tenth grade students – or the previous year for the eleventh and twelfth grade students. The Precalculus College Prep students consist mostly of a diverse mix of eleventh and twelfth grade students who have completed Algebra 2 the previous year with a passing grade of a D or higher. The Precalculus CP classes also have a small number of tenth grade students who have been encouraged to take the honors track, but may believe they are not ready for the rigor of an honors course. When the school year began, I encouraged those students to switch into the honors course in order to prepare them for a Calculus course the following year but had no success.

My students are very social. They enjoy working in groups and being able to ask their peers for possible methods of problem solving. If I can present scenarios that engage students and allow them to conquer problems together, I believe all my students can benefit from the lesson. Utilizing peer sharing and cooperative learning opportunities allow struggling students an opportunity to ask/receive assistance as well as engage all students in mathematical communication. I believe most students are more successful in persevering on a given problem if they have others with which to share their thoughts and ideas.

My classroom is set up in cooperative learning groups and each group has access to a laptop and monitor. This is the first year for my classroom to have this set-up and I would like to incorporate more group learning activities into my curriculum. I believe that as students analyze concepts and discuss their understandings, students can share inferences, predictions, and pose questions to their peers. Discussions and debates about the conclusions drawn by students can strengthen their conceptual knowledge as students learn to justify their answers to their peers. Building critical thinking skills is an essential part of the state assessment and a necessary skill for success in real life. Hopefully this unit will encourage these skills in my classroom.

Following a block schedule allows me to have my students for ninety minutes every other day. This class length will let me offer problem solving scenarios that take a little more time for the typical classroom activity. I intend to utilize activities involving sequences and series scenarios that students can complete and then share with their classmates in presentation type style on their monitors, whether a poster, PowerPoint, or other creative media. This should encourage my students to persevere in the mathematics and concept problem solving as well since they will be presenting their solutions to their peers. I intend to teach this unit in the early spring of the 2014/2015 school year.

Unit Objectives

- Students will be able to use sequence notation to write the terms of sequences
- Students will be able to find sums of infinite series
- Students will be able to use factorial notation
- Students will be able to use summation notation to write sums
- Students will be able to use sequences and series to model and solve real-life problems
- Students will be able to recognize, find, and write the *n*th terms of arithmetic and geometric sequences
- Students will be able to find *n*th partial sums of arithmetic and geometric sequences

Common Core State Standards

CCSS: CC.9-12.F.IF.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.[1]

Student Prior Knowledge for this standard: Students have been exposed to functions – linear, quadratic, and exponential. They have used function notation, and evaluated functions using graphs and input values. Prior to this unit, students will be exposed to higher degree polynomials and rational functions in their study of Precalculus units in this text.

CC.9-12F.BF.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.[2]

Student Prior Knowledge for this standard: Students have used a variety of strategies to write expressions that generate arithmetic and geometric patterns. They have used tables, graphs, and symbols to model and solve linear and exponential functions. They may not realize that they have generated expressions for the nth term and this will need to be incorporated into the lesson and connected to their prior knowledge.

CC.9-12.F.LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output parts (include reading these from a table).[3]

Student Prior Knowledge for this standard: Students have used a variety of strategies (patterns and change) to write expressions that generate linear, quadratic, and exponential patterns. They have used tables and graphs for generating those expressions in previous Algebra courses.

Research of Yale Databases

I found and read two different units involving sequences and series. The first, "Modeling Popular Culture,"[4] in which the intent was to encourage students to apply nth terms and sums of sequences in modern day applications using Facebook and/or Twitter accounts. This unit included one interesting activity for encouraging students to develop the idea of a geometric series. Using popular movie titles and their gross one-day ticket sales at the box office, this teacher has students estimate the number of tickets sold and then calculate how to make a potential mega-hit become a flop using social networking. Students generate the formula for one person bad mouthing the movie to two friends, who then spread the word to two friends – the doubling model. Students find how much iteration it may take for the movie to flop or the number of levels of sharing for the critical mass of people to hear the movie is bad when only one person sees the movie. I think this scenario would be interesting to students in this age of social networking.

The second unit I read, "Know Your Position – How?"[5] jumps off from the position that students assume they know their position on a standardized test based upon how well they think they performed. This unit begins with the probability of tossing coins and moves through scenarios with dice to SAT scores. The teacher utilizes Pascal's Triangle to encourage students to make connections between the combination formula and Pascal's Triangle as well as to be able to solve problems on the binomial theorem. The concepts in this unit are covered in the later part of my Precalculus chapter on sequences and series.

Historical Background

Even though I am not a history buff, many of my students are intrigued by where the mathematical concepts they are studying came from as well as the "when will I ever use this" question students typically ask! So, I endeavor to answer those questions and find that introducing the history behind the mathematics draws my students into the classroom activities.

Throughout history mathematicians have used series to perform calculations. The history behind arithmetic sequences appears to be credited to a scenario involving a young student by the name of Karl Gauss whose teacher asked him to find the sum of the integers from 1 to 100. Apparently his teacher believed this would keep the class of students busy for a while, but his young student Gauss quickly proved him wrong. Whether he was ten years old at the time or not, he was considered to be a math prodigy as a child and later grew to be a well-known mathematician making contributions to the fields of mathematics of geometry and number theory.[6]

Leonardo Pisano is another historical figure connected to the mathematics of sequences and series. He is better known as Fibonacci – which means "son of Bonacci" – and is remembered for the famous Fibonacci sequence. This sequence was the result of solving a mathematical problem calculating the reproduction of rabbits published in his book in 1202 called *Liber Abaci* (The Book of Calculating). Leonardo is reported to also be a child math prodigy who began at an early age to help his father and other Italian merchants keep track of their commercial transactions. His sequence is used in artwork around the globe and found in the designs of plants and animals in nature.[7]

Another famous mathematician is also known for his contributions to the world of sequences and series. The Swiss mathematician Jacob Bernoulli contributed work on infinite series, geometry, and probability – the law of large numbers. He is known for his sequence of rational numbers called Bernoulli numbers.[8]

Mathematical Content

The goal of this unit is for my students to understand how to represent a sequence of numbers, find the sum of the terms of a sequence – both arithmetic and geometric sequences, finite and infinite, and understand the difference between convergent and divergent series. In collaboration with my seminar leaders, we discussed and concluded that an excellent beginning point for this unit would be to present multiple scenarios of patterns and allow students to investigate those patterns – to "play" with the sequences and make discoveries. This would allow them to tap into their prior knowledge of patterns and draw conclusions about the formula describing the pattern. Students would generate the formula from the terms as well as use the formula to generate the terms of a sequence. In order for students to understand and connect the algebraic pattern to the graphical perspective of the sequence, activities will include using points on a graph and requiring students to generate the formula from the graph.

The mathematical definition of a sequence is not vastly different from the everyday understanding that students may have of what a sequence is. Having students discuss how they describe the pattern would help them understand how to define a sequence in mathematics and could produce an excellent starting place for this unit. As students hone their ideas, they tap into the patterns and expressions they have used in previous courses of study. Having them arrive at the mathematical definition of a collection of numbers listed in a particular order or an ordered list of numbers from which they can generate an expression from the pattern will enable them to jump into the unit on sequences. Since a sequence is a list of numbers in a specific order, I think my students will be happy to find that they will be defining an infinite sequence as a function whose domain is the set of positive integers and a finite sequence as a function with a domain of a finite number of positive consecutive integers.

I believe some confusion will happen when the notation for sequences is introduced. Students will need to understand how the subscript notation and summation notation are used in this chapter by connecting them to the function notation. For example, *a* with the subscript 10 means the value of the function that describes the sequence corresponding to the input n=10. I have used some subscript notation in the function notation of higher degree polynomials and my students have struggled with understanding why the *a* has a subscript when the *x* had a superscript – exponent. Students will need to understand that the subscript of $n + 1$ denotes the next term in the sequence. Many students will think this is addition instead of naming the next term in the sequence. It will be necessary for me to help them understand that the subscripts of the sequence make up the domain of that sequence. The subscripts will be used to identify the location of a particular term within the sequence. As my students work with more examples I believe the notation will make sense and they will be able to grasp how the notation is necessary to the unit.

As we progress further into the unit, students will begin to understand how a sequence must be defined using an *n*th term. Simply giving a list of numbers will not define a unique sequence. Allowing students to see how just listing the first three terms of different sequences does not clearly identify the unique sequence will enable students to understand why the *n*th term is useful and necessary. This should also help them understand that when they are only given the first few terms, they are generating an apparent *n*th term for the sequence as other *n*th terms could be possible.

Another possible misconception in this content area is that my students may think that all sequences begin with the number 1. It will be my intention to give them sequences that begin with other numbers. They will need to understand that a sequence begins with whatever number it needs to begin.

As students become familiar with writing the terms of a sequence, looking at the graph and generating the terms, and finding the *n*th term of a sequence or writing the expression for the apparent sequence, we will look at the famous Fibonacci Sequence, a recursive sequence, and sequences defined with factorials. When exploring recursive sequences, students are given the first few terms and define the sequence using previous terms, in other words defining a sequence by a relation between its terms. For example, the Fibonacci sequence is defined by $a(1)=1$, $a(2)=1$, and $a(n+2)=a(n)+a(n+1)$ for all n greater than 1. I think a teaching strategy for this section is to point out to students that the subscripts of the sequence actually make up the domain of the sequence as well as identify the location of a term within the sequence. Students may be initially confused by the addition of k but will hopefully see how it is incorporated into generating the sequence.

Following the progression of lessons in the book, students will continue their study of sequences and series moving into summation notation. Also called sigma notation, summation notation is a convenient method of representing the sums of the terms of finite or infinite sequences. It gives the upper and lower limits of the sequence and is represented by the formula given in the definitions and formulas section below. Summation notation instructs students to add the terms of a sequence and provides them with the appropriate terms to generate prior to finding the sum. Once students are familiar with using this notation and have reviewed the properties of sums, they are given the definition of a series – the sum of a finite series or partial sum and the sum of an infinite sequence or infinite series.

All of these concepts are covered in the first section of the chapter on sequences and series. By incorporating these concepts with the activities mentioned above – looking at patterns, graphing points, and generating terms, students will be prepared for focusing on arithmetic sequences – the next section of their textbook. The following section covers arithmetic sequences and how to use the common difference between consecutive terms to generate the nth term. Students are given the formula for the sum of a finite arithmetic sequence and then given plenty of practice using the formula to calculate sums of arithmetic sequences. Students use this new formula to solve real-life application problems involving total sales and auditorium seating.

Geometric sequences and series follow the arithmetic sequences. Students discover that while consecutive terms in an arithmetic sequence have a common difference, consecutive terms in a geometric sequence have a common ratio. Students are given opportunities to calculate that common ratio and then using the formula, write the first terms of the sequence or find a particular term given the first term and the common ratio. Using the formula for the sum of a finite geometric sequence, students find the sums of particular sequences. This exercise is followed by extending the formula for finite sums to producing a formula for the sum of an infinite geometric series. Application problems involving calculating annuity balances are included in this section to allow students to apply their understanding of geometric sequences and series.

It will be imperative for me to remind students that not all sequences are either arithmetic or geometric. Students will need to remember that many sequences do not meet the conditions for either of these types of sequences and that to determine whether a sequence is arithmetic or geometric, students will need to check for the common difference or common ratio.

Algebraic Structures

It is interesting to note that in our seminar class, we discussed algebraic structures. While I have never given much thought to sequences and series being a part of set theory, we quickly discovered how to look at our math concepts in the abstract world of abstract algebra and algebraic structures.

I) If we consider the set of all sequences of the same length, and the operation of addition of sequences as functions, then the set of such sequences with respect to addition has an abelian group structure. Since the addition of sequences is defined as component by component addition, the set of sequences has the following properties:
   1) There is an identity element which is a sequence of zeros.
   2) Every sequence {a(n)} has an additive inverse which is the sequence of opposite terms of the terms of a(n), namely, the sequence {a(n)}.
   3) The operation of addition is commutative, associative, and closed.

II) On the set of sequences we can define the scalar multiplication operation as the multiplication of every term by the same real number (the scalar). The properties are:
   1) 1*{a(n)}={a(n)} for any sequence {a(n)}
   2) k*(p*{a(n)})=(k*p)*{a(n)} for any sequence {a(n)} and any real numbers k and p
   3) k*({a(n)}+{b(n)})=k*{a(n)}+k*{b(n)}
   4) (k + p)*{a(n)}=k*{a(n)}+p*{a(n)}

The set of sequences together with the operations of addition and scalar multiplication as defined above has a vector space algebraic structure. The algebraic structure properties are reflected in the properties of sums that the students learn in this unit. This perspective has broadened my understanding of how operations give structure to sets and has encouraged me to evaluate the mathematics I teach in deeper and more thought provoking ways.

Definitions and Formulas

The following are taken from the student textbook and include the necessary definitions and formulas for success in this chapter. Students will be working with these throughout the course of the sequences and series activities.[9]

Definition of Sequence – An infinite sequence is a function whose domain is the set of positive integers. The function values $a_1, a_2, a_3, a_4, \ldots a_n, \ldots$ are terms of the sequence. If the domain of a function consists of the first *n* positive integers only, the sequence is a finite sequence.

Definition of Summation Notation – The sum of the first n terms of a sequence is represented by $\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \ldots + a_n$ where i is called the index of summation, n is the upper limit of summation, and 1 is the lower limit of summation.

Definition of a Series – Consider the infinite sequence $a_1, a_2, a_3, a_4, \ldots a_i \ldots$

1. The sum of the first n terms of the sequence is called a finite series or the partial sum of the sequence and is denoted by $a_1 + a_2 + a_3 + a_4 + \ldots + a_n = \sum_{i=1}^{n} a_i$.
2. The sum of all the terms of the infinite sequence is called an infinite series and is denoted by $a_1 + a_2 + a_3 + a_4 + \ldots + a_i + \ldots = \sum_{i=1}^{\infty} a_i$.

Definition of Arithmetic Sequence – A sequence is arithmetic if the differences between consecutive terms are the same. So, the sequence $a_1, a_2, a_3, a_4, \ldots a_n, \ldots$ is arithmetic if there is a number d such that $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots d$. The number d is the common difference of the arithmetic sequence.

The nth term of an arithmetic sequence has the form $a_n = dn + c$ where d is the common difference between consecutive terms of the sequence and $c = a_1 - d$.

The sum of a finite Arithmetic Sequence – The sum of a finite arithmetic sequence with n terms is given by $S_n = \frac{n}{2}(a_1 + a_n)$.

Definition of Geometric Sequence – A sequence is geometric if the ratios of consecutive terms are the same. So, the sequence $a_1, a_2, a_3, a_4, \ldots a_n, \ldots$ is geometric if there is a number r such that $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = \ldots = r$ where $r \neq 0$. The number r is the common ratio of the sequence.

The nth term of a Geometric Sequence – the nth term of a geometric sequence has the form $a_n = a_1 r^{n-1}$ where r is the common ratio of consecutive terms of the sequence. So, every geometric sequence $a_1, a_2, a_3, a_4, \ldots a_n, \ldots$ can be written in the following form. $a_1, a_1r, a_1r^2, a_1r^3, a_1r^4, \ldots a_1r^{n-1}, \ldots$

The sum of a Finite Geometric Sequence – the sum of the finite geometric sequence $a_1r, a_1r^2, a_1r^3, a_1r^4, \ldots a_1r^{n-1}, \ldots$ with the common ratio $r \neq 1$ is given by $S_n = \sum_{i=1}^{n} a_1 r^{i-1} = a_1\left(\frac{1-r^n}{1-r}\right)$.

The Sum of an Infinite Geometric Series – if $|r|$, 1, then the infinite geometric series $a_1 + a_1r + a_1r^2 + a_1r^3 + \ldots + a_1r^{n-1} + \ldots$ has the sum $S = \sum_{i=0}^{\infty} a_1 r^i = \frac{a_1}{1-r}$.

Classroom Activities

As stated earlier, I am planning to incorporate activities that encourage students to work cooperatively and think critically about these concepts. My textbook offers few actual activities, but the book has quite a few real life problems. As this will be my first year teaching these concepts, I plan to incorporate activities from interactive websites that will encourage students to delve into these concepts. I also like to use the show "Numb3rs" to draw the students into the activities and help them see how mathematical concepts are utilized in real life problem solving scenarios. Three of the activities I plan to utilize are included in the Appendix section. These activities will be incorporated into the sections in the text not only to build student understanding and conceptual knowledge of sequences and series, but to add a little "fun" to the mathematics of this chapter.

The first activity includes graphing arithmetic sequences, graphing terms, and graphing geometric sequences. This activity requires students to reflect upon how the graphs appear, what they can understand about the graphs, and how they can utilize the graphs to aid them in writing the formula. I believe students will more fully understand the connections between the terms, the formula, and the graphs after completing this activity. I have included two examples for each section but will have students complete at least four of each type in a classroom activity. This activity can also be used to solidify student understanding of divergent and convergent series. By looking at the graphs they create, students can quickly determine whether or not the series approaches a specific value or increases/ decreases without bound. This concept will prepare students for their study of limits later in the school year.

I am calling the second activity a round robin for lack of a better name. This activity includes a handout of ten cards per cooperative group of four students. Students must calculate each card in the activity and place the cards in the correct order – matching the card to the correct answer – in order to complete the activity. This activity will strengthen students understanding of arithmetic sequences as they work together to complete the answers and order the cards. Each card has an answer for a different card in the upper right corner and cards must be placed so that the clue and answer are side by side. I believe that making it into a game will encourage participation in the activity by individuals and groups. Allowing students to work together strengthens conceptual understanding and supports struggling students.

The third activity in the Appendix is a math number search. It is similar to a word search in that students will be looking for specific sequences as one typically looks for particular words. However, students will generate the sequences prior to finding the terms in the number search. Given the n^{th} terms, students will work together to generate the first five terms of each sequence. Finding their answers in the number search will guarantee that they were successful in generating the terms. I believe this activity will encourage students to persevere in generating the terms more so than just a worksheet as they will be able to check their work using the number search.

These three activities will be utilized in addition to the activities and word problems in the student textbook. My plan is to complete the first three sections – the introduction, arithmetic sequences and series, and geometric sequences and series – prior to the end of the school year. If time allows, we will venture into the remaining sections on mathematical induction, the Binomial theorem, counting principles, and probability. However, the study of sequences and series will prepare students for their study of limits which is necessary for students taking Calculus the following year.

Conclusion

As my students finish the textbook activities and the classroom activities in the appendices, it is my hope that their understanding of sequences and series will be strengthened beyond the depth of previous years of teaching these concepts. Implementing these activities will afford students a deeper understanding of the graphs of these functions and the connections between the graphs and the formulas as well as offer students a solid understanding of real life scenarios involving sequences and series.

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Appendix A
Graphing Sequences

Arithmetic Sequences
For each sequence, calculate the first 6 terms. Then plot the sequence on the grid.

a) 3n + 2

__ __ __ __ __ __

What is the common difference of the sequence?

____

Where will this graph intersect with the y axis?

____

How does this connect back to the formula for the sequence?

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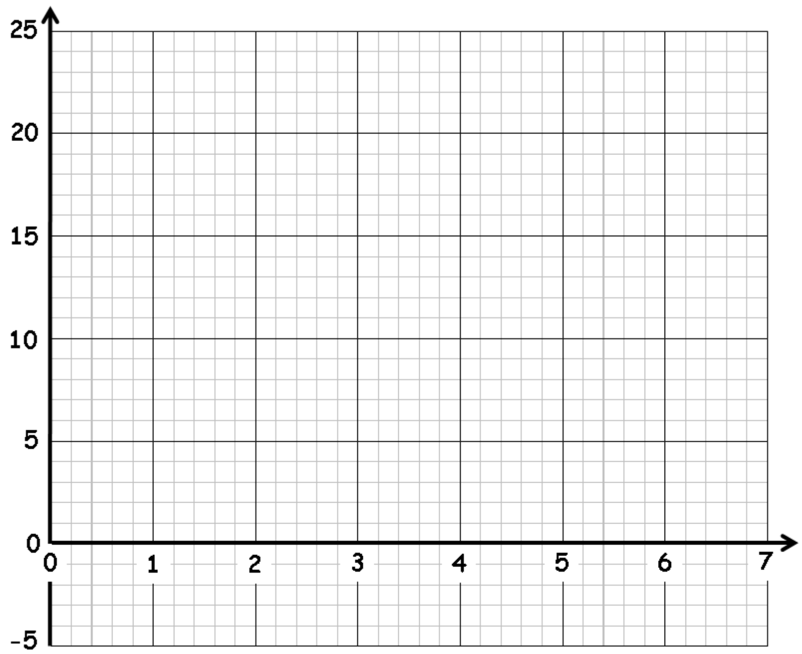


b) 4n - 3

__ __ __ __ __ __

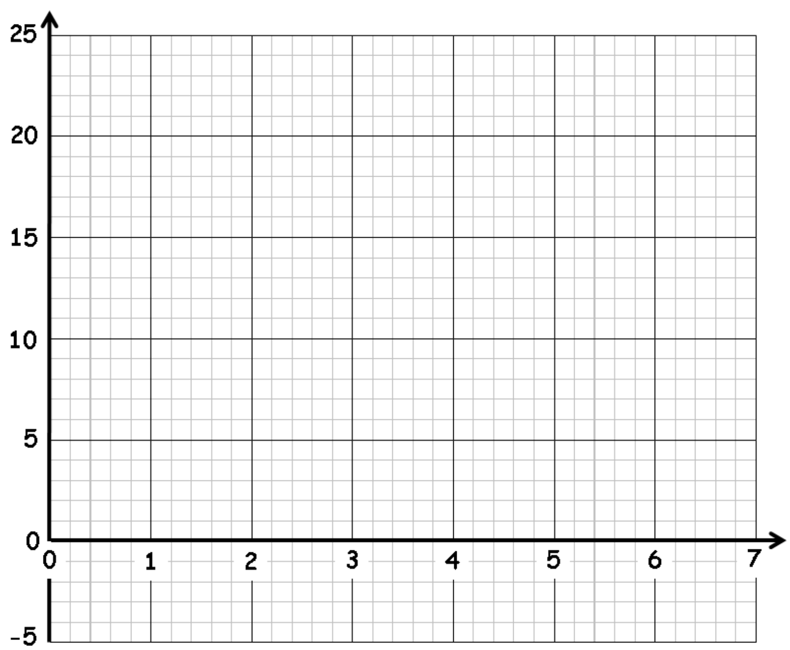
What is the common difference of the sequence?

____

Where will this graph intersect with the y axis?

____

How does this connect back to the formula for the sequence?



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Graph the Terms

Graph the terms and determine whether or not the sequence is arithmetic. If it is, find the common difference.

c) 4, 9, 14, 19, 24

Is the sequence arithmetic?

____

If so, what is the common difference of the sequence?

____

Where will this graph intersect with the y-axis?

____

What is the formula for the sequence?

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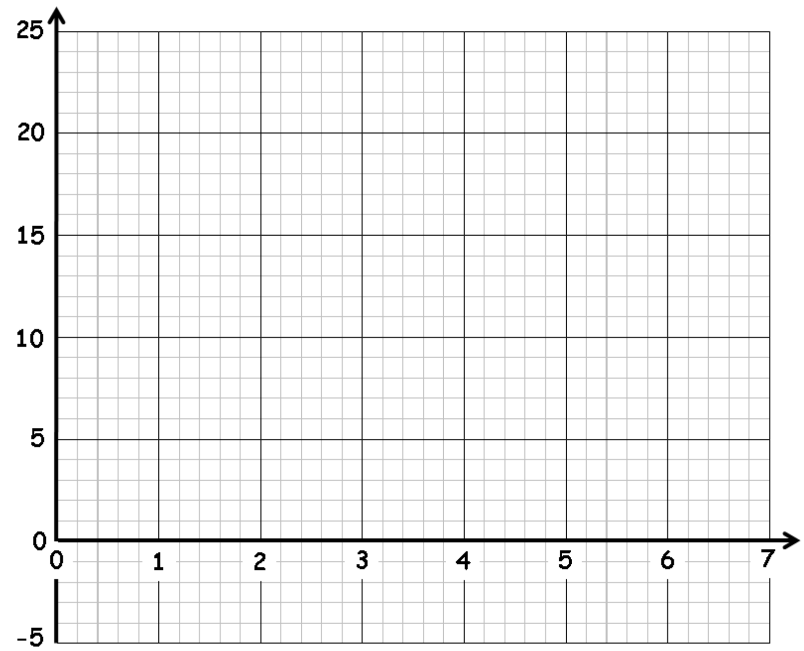


d) -5, 1, 7, 13, 19

Is the sequence arithmetic?

____

If so, what is the common difference of the sequence?

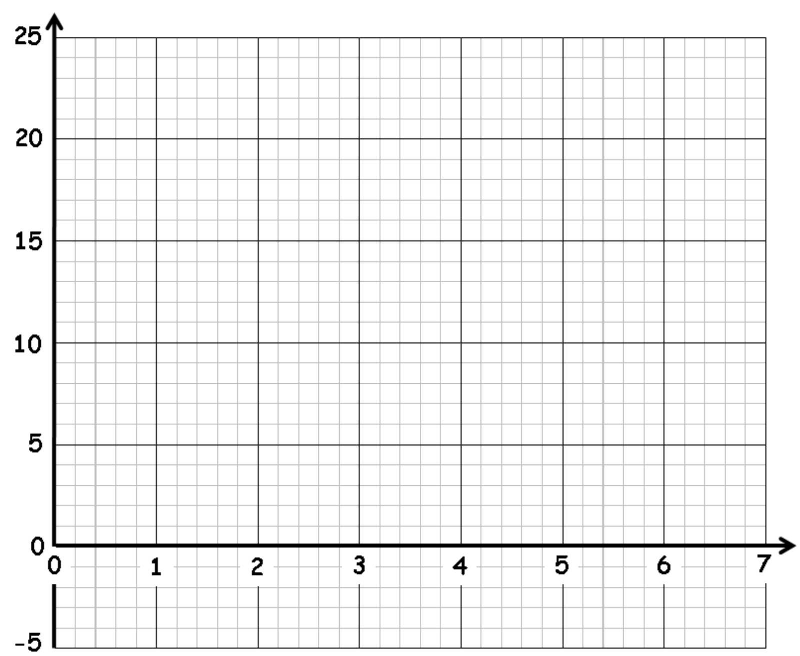
____

Where will this graph intersect with the y-axis?

____

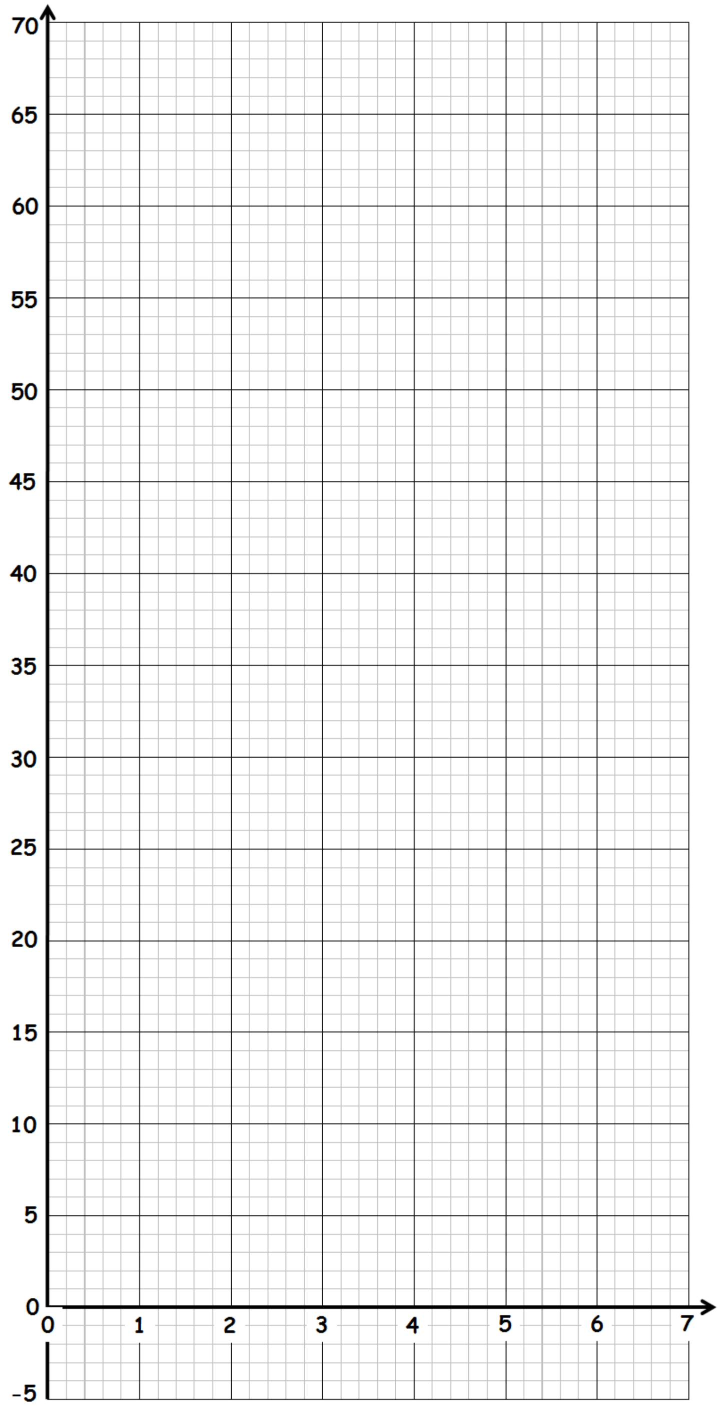
What is the formula for the sequence?

____



Geometric Sequences

For each sequence, plot the terms on the graph and make an observation about the graph. How do these graphs differ from the linear ones? Can you calculate a formula to represent the sequence?



e) 2, 4, 8, 16, 32

How does this sequence differ from the arithmetic sequences?

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____

What is the 1st term of the sequence?

____

What is the common ratio for this sequence?

____

Calculate the formula for the sequence.

____

What would be the next 4 terms in the sequence?

__ __ __ __

f) 2, 6, 18, 54, 162

How does this sequence differ from the arithmetic sequences?

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____

What is the 1st term of the sequence?

____

What is the common ratio for this sequence?

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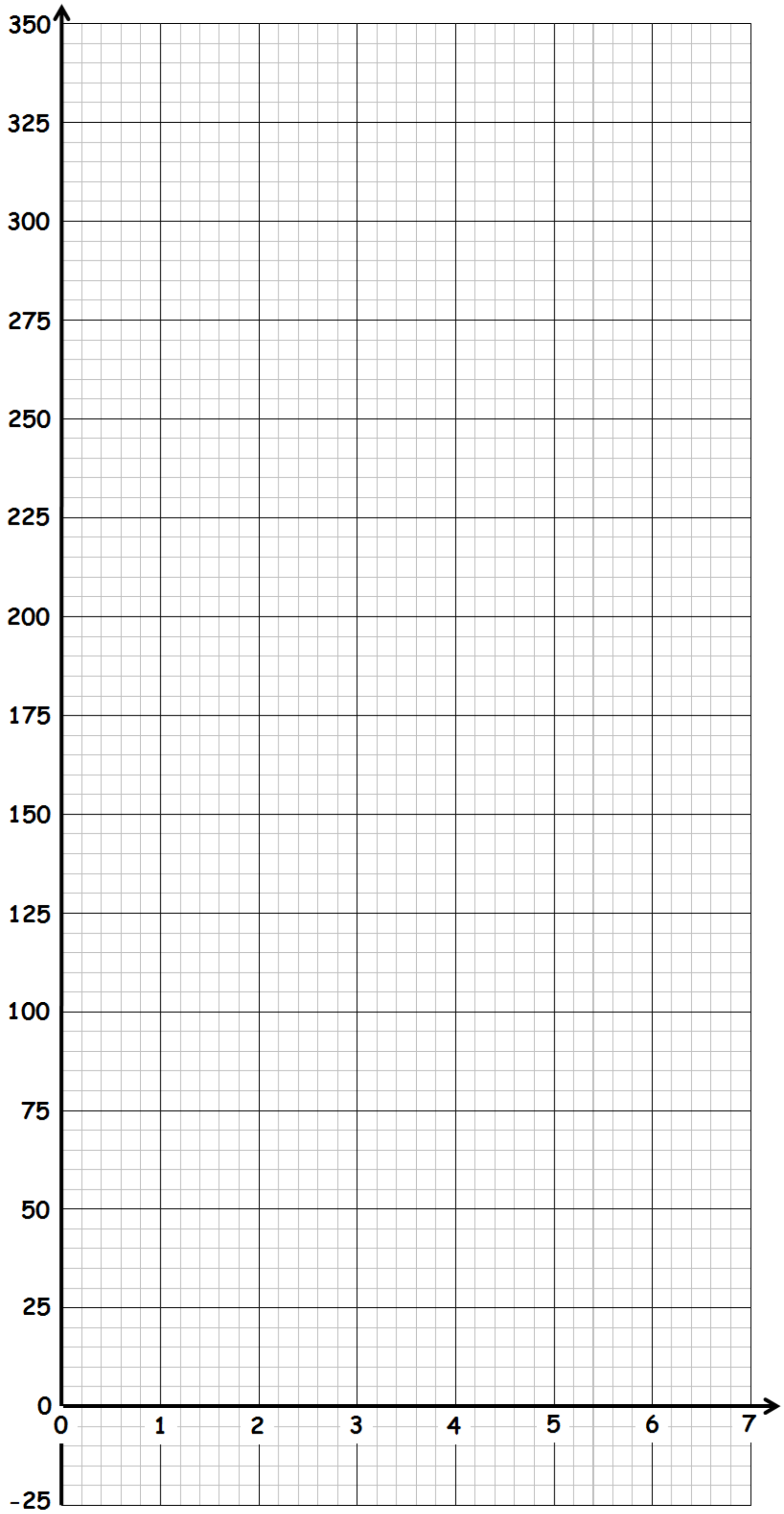
Calculate the formula for the sequence?

____

What would be the next 4 terms in the sequence?

__ __ __ __



Appendix B
Round Robin Math

Students: calculate the answer for each notecard. Then place your cards in order so that the clue and the correct answer are side by side around the circle.

<u>Card 1</u> Answer = 192

Calculate the sum of the 1st five terms of my arithmetic sequence that starts with 4 and has a fifth term of 6.

<u>Card 2</u> Answer = 5.5

Find a_{10} for my arithmetic sequence starting -17, -13, -9, ...

<u>Card 3</u> Answer = 13

What is the total sum of terms 1 to 12 if I am a sequence that starts with -6, -2, 2, 6, ...?

<u>Card 4</u> Answer = 19

I am an arithmetic sequence. My 1st term is -24 and my 5th term is 8. What is my common difference?

<u>Card 5</u> Answer = 25

I am a sequence but am I arithmetic? -3, -5.5, -8, -10.5, -13

Card 6 Answer = no

I am an arithmetic sequence. My 1st term is 21 and my 5th term is 73. What is my common difference?

Card 7 Answer = 15

I am an arithmetic sequence. The sum of my first 12 terms is -306. My 12th term is -53. What is my first term?

Card 8 Answer = 8

I am an arithmetic sequence. My 25th term is 87 and my common difference is 3. What is my 1st term?

Card 9 Answer = yes

What is the next term in my arithmetic sequence?

3.7, 4.3, 4.9, ...?

Card 10 Answer = 2

Am I an arithmetic sequence: n(n-3)?

**Round Robin Answer Key – Card Order: 1, 5, 9, 2, 4, 8, 7, 10, 6, 3, 1

Appendix C
Sequence Number Search
Calculate the first 5 terms of each sequence with the following n^{th} terms and then find and circle your answers within the grid.

1. n^{th} term is -5n + 1

2. n^{th} term is 4n – 3

3. n^{th} term is 3n – 5

4. n^{th} term is 10n + 4

5. n^{th} term is 11n – 6

6. n^{th} term is 2n +5

7. n^{th} term is 7n -3

8. n^{th} term is 58 – 3n

9. n^{th} term is 2.5 – 4.5n

10. n^{th} term is 20 – 3n

11. n^{th} term is 12n - 6

12. n^{th} term is -3n + 1

13. n^{th} term is 14 – 4n

14. n^{th} term is 29 + n

15. n^{th} term is 46 – 2n

16. n^{th} term is -8 – 4n

17. n^{th} term is 4 – 4n

18. n^{th} term is -9 + 3.5n

-1	-4	5	7	10	6	2	-2	-6	-3
3	0	-2	1	4	7	10	-5	-7	-12
-5.5	-2	1.5	5	8.5	0	-4	-8	-12	-16
-3.5	2	4	9	3.5	-2	-6.5	-11	-15.5	-20
7	9	11	13	15	-4	-9	-14	-19	-24
14	23	18	17	14	11	8	5	0	-28
25	10	25	15	24	8	13	16	18	27
30	31	32	33	34	39	45	27	24	22
-3	-8	-9	-11	44	42	40	38	36	42
6	18	30	42	54	55	52	49	46	43

Notes

[1]"Common Core State Standards for Mathematics," http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf, 69. (accessed September 7, 2014).

[2]Idib, 70.

[3]Ibid, 71.

[4]Magee, Maryellen. "Modeling Popular Culture." Charlotte Teachers Institute. http://charlotteteachers.org/2012/06/modeling-popular-culture/ (accessed October 26, 2014).

[5]Cherian, Anne. "Vol. 1: Oats, Kings, Proofs, and Climate Change: How Do You Know?." http://tip.drupalgardens.com/seminar-number/vol-1-oats-kings-proofs-and-climate-change-how-do-you-know (accessed September 26, 2014).

[6]Weisstein, Eric. "Gauss, Karl Friedrich (1777-1855) -- from Eric Weisstein's World of Scientific Biography." http://scienceworld.wolfram.com/biography/Gauss.html (accessed October 23, 2014).

[7]"Fibonacci Sequence - History." - Month, Rabbits, Pairs, and System. http://science.jrank.org/pages/2705/Fibonacci-Sequence-History.html Fibonacci Sequence – History (accessed October 26, 2014).

[8]O'Conner, J.J. "Jacob (Jacques) Bernoulli." Bernoulli_Jacob biography. http://www-history.mcs.st-and.ac.uk/Biographies/Bernoulli_Jacob.html (accessed October 26, 2014).

[9]Larson, Ron et al., *Precalculus with limits a graphing approach*, 580.

Curriculum Unit Title: ***Investigating Sequences and Series***

Author: Karen C Brown

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Students will analyze sequences and series using various methods of representing sequences and series – graphs, terms, and formulas. They will use summation notation and then model and find sums of arithmetic and geometric sequences. Students will understand how to represent sequences of numbers and sums of sequences and differientiate between arithmetic and geometric sequences.

ESSENTIAL QUESTION(S) for the UNIT

1. How do you represent a sequence of numbers or the sum of a sequence?
2. How do you find the *n*th term or partial sum of an arithmetic sequence?
3. How do you find terms and sums of geometric sequences?

CONCEPT A	CONCEPT B	CONCEPT C
Sequences and Series	Arithmetic Sequences and Series	Geometric Sequences and Series
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How do you calculate the apparent *n*th term of a sequence? How can you use summation notation to represent the sum of a sequence?	How do you find the *n*th term or partial sum of an arithmetic sequence? How do you calculate the common difference of a sequence?	How does knowing the first term and the common ratio help you write the formula? How do you find terms and sums of geometric sequences?
VOCABULARY A	**VOCABULARY B**	**VOCABULARY C**
infinite and finite sequence series summation notation *n*th term sigma notation partial sum	arithmetic sequence common difference recursion formula	geometric sequence common ratio geometric series

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Precalculus textbook is Larson's Precalculus with Limits: A Graphing Approach 5th edition
3 activities printed for student handouts
calculators