Understanding Functions

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My unit for the Delaware Teachers Institute will focus on understanding and interpreting the characteristics of functions and function notation. In our curriculum, functions and function notation is not given the time it requires to truly understand it. I have noticed that students seem to misinterpret the meaning of $f(x)$ and always feel more comfortable writing a function as $y=$ instead of $f(x)=$. They do not grasp what separates a function from a non-function, or relation, or why it is even necessary to define something as a function. I see students struggle to determine the domain and range of a function mostly because there is no algebraic algorithm they can follow in order to determine it. In our Core Plus Book 2, there is only a single investigation that teaches function notation, functions versus non-functions, and domain and range. This is simply too much content to put in an investigation that is supposed to take no more than two school days. It is supposed to serve as an introduction into a quadratic functions unit, but it does not connect to the concepts of quadratic functions. I have created a unit that will introduce the idea of functions and connect specifically to quadratic functions.

School Background

I teach at Hodgson Vocational Technical High School in Newark, Delaware. Because we are a vocational school, using application in our teaching is very important. When teaching functions, I want to use as many real world examples as possible in order to reinforce the content. Real world examples will be important when teaching domain and range in order to contrast practical domain and range versus theoretical domain and range. One challenge I may face with 9^{th} graders is, in our school, we get students who come from many different middle school districts, both public and private. Therefore, their math backgrounds and comfort levels can often vary. I will make sure that I do not assume any prerequisite knowledge and I will try to offer extension activities for students who have a stronger background.

The course that I will be teaching which covers functions is Integrated Math II. This is a course that the majority of 9^{th} graders take during the second semester. In Unit 5 of book 2 of the Core Plus Mathematics Series, entitled Non-Linear Functions and Systems, students learn about solving, graphing, and rewriting polynomial and logarithmic function. In the first investigation, function notation is introduced through a couple examples, including tables, graphs, equations, and real world examples. The investigation feels disjointed from the rest of the unit and often we don't give it the time it deserves because we want to go to the more algebraic topics.

Objectives and Standards

It is important as math teachers that we constantly align to the Common Core State Mathematics Standards. Therefore, all of my objectives will align directly to these standards and specifically the ones under the functions strand. My unit will have three main objectives. My first objective will be that students will understand function notation and will be able to interpret the meaning of function in terms of a graph, table or context. One of the Common Core functions standards states that students should "understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.[1]" This standard defines a function and explains the meaning of the function notation. Another part of the Common Core functions standards says that students should "use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.[2]" In other words, students must be able to evaluate a function and explain the meaning of specific values of a function in terms of the context given. This will be the basis of my first objective. It is important that students understand this before we move on to more complex ideas related to functions.

My second objective will be for students to interpret the domain and range of a function in terms of an equation, graph, or real world situation. For example, students should be able to look at a quadratic function, use what they know about that function, and determine that its domain is all real numbers and its range is some interval based off of its maximum or minimum point. For a real world context, they should understand that even though a function may have a specific domain, that domain might not make sense in terms of the real world context of the situation. The Common Core State Standards states that students should be able to "relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.[3]" Therefore, students must connect a function's domain to its actual context to give it some meaning.

My third objective will require students to characterize key features of specific functions, focusing on linear, quadratic, and general polynomial functions. Those are the main types of functions that the 9th grade students will have been exposed to prior to my unit. Often students can recognize these functions, but to be able to characterize them using precise mathematical language is difficult. In the Common Core Functions Standard, it states that students should be able to "interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.[4]" The only one of these features that I will not cover is periodicity since that is usually related to trigonometric functions. Students must be able to use correct interval notation when describing where a function is increasing, decreasing, positive, or negative. Students will learn to describe symmetries by writing equations for a line of symmetry. I will also have students discover ways to generalize end behavior of polynomial functions based on leading coefficients and even and odd powers.

Content

My unit will start with an introduction to functions, including what defines a function and examples of non-functions. I will include function notation because this is a concept students very often misunderstand and interpret as multiplication. Students will learn about the domain and range and characteristics of special functions and domain and range using a real world context. Finally, I will perform operations on linear, quadratic functions, and polynomial functions in order to investigate special properties of these types of functions.

Introduction to Functions

A function is defined as a relationship between two sets where to each input corresponds exactly one output. We define the set of all inputs as our domain, and the set of all outputs as our range. Often in mathematics, we think of domains and ranges as numerical sets, but they do not have to be. For example, a function's domain could be the set of all polygons. The function could be to find the sum of the angles in that polygon. Therefore, any triangle would have an output of 180 degrees. This is an example of a function where there are different inputs that have the same output. For example, if you take two distinct quadrilaterals that are not the same, they would have the same output of 360 degrees as the sum of their angle measures, so this function gives the same output to two different inputs. In my unit, I will focus on linear and quadratic functions in my unit, a small amount on general polynomial functions, which have very specific characteristics, but it is important for students to understand that there are other different types of functions as well.

I have often noticed that students struggle with understanding function notation. They see the term *f(x)* and they think multiplication of f and x. In order to help students understand function notation, it is helpful to start with non-numeric examples of functions. So we can use the above example where the function, f, represents the sum of the angles in a polygon. If we pick some examples of common polygons, we can have the domain given by the set {square, rectangle, triangle, hexagon, pentagon}. Using this set of inputs, we will have the following outputs f(square) = 360, f(rectangle) = 360, f(triangle) = 180, f(hexagon) = 720, and f(pentagon) = 540. So the range of this function is the set {180, 360, 540, 720}. This example can help to introduce function notation as well as the concepts of domain and range.

Non-Functions

In order for students to have strong understanding of what defines a function, it helps to understand examples of relations that are not functions. Students are often taught the vertical line test as a way to test for functions. The vertical line test states that if you draw a vertical line on a coordinate grid that passes through two points on that relation, then the relation is not defined by only one function. It is important for students to understand why this is true. Let's look at the equation $x^2 + y^2 = 25$ which represents a circle centered at the origin with a radius of 5. My students will have learned how to represents circles with equations on a coordinate grid but they have not related this concept to the idea of functions. It is pretty clear that you can draw a vertical line that intersects this circle in more than one point (two points), such as the vertical line x=3. We can also connect this to the definition of a function where each input must have only one output. When we have the input of x=3, we have two distinct outputs y=4 and y=-4. This is because $3^2 + 4^2 = 25$ and $3^2 + (-4)^2 = 25$, meaning that (3,4) and (3,-4) are both pairs in this relation which is not a function.

In order to reiterate the idea of non-functions, we can look at non-numerical examples. In an article entitled "Introducing Function and Its Notation," the author using a family tree as an example to set up relations between different family members.[5] So we can use an example where we have a father, Bob, mother, Margaret, and three children, Samantha, Diane, and Thomas. We can define the relation F(x) = the father of the person x. This is function because for any input, which would be the set (Samantha, Diane, and Thomas), they would all have one distinct output. In this special case, they each have the same output. F(Samantha)=Bob, F(Diane)=Bob, and F(Thomas)=Bob. This is also a way to reiterate function notation. Now we can define the relation S(x) = the sister of person x. Students should understand that this is not a function because S(Thomas) = Samantha, but also S(Thomas) = Diane. So the input, Thomas, has two different outputs and therefore the relation of "being a sister" is not a function. Students could think of their own family tree to come up with similar examples of functions and non-functions.

Domain and Range

The domain and range of a function give all possible inputs (domain) and outputs (range) of that function. When students learn linear functions, they learn that all linear functions have a theoretical domain and range of $(-\infty, \infty)$, meaning all real numbers. This is because there are no restrictions on your inputs in the real number system in a linear equation. And all real numbers are possible outputs for all linear equations. The only exception to this is constant linear functions such as *f(x) = 5*. Here, for any input, the output is always 5. For example, f(3) = 5, f(-1) = 5, etc. The graph of this function is a horizontal line. This function has a domain of $(-\infty, \infty)$ and a range of {5}.

However, this is an example with a real world context. When linear functions are used to model a real world situation, their domain and range may be different. Often students completely ignore the domain and range when trying to answer questions that have a context associated with it[6]. For example, if a dog walker is paid $10 per day and $5 for every dog he walks, then his pay can be modeled by the equation P(d)=10+5d, where d represents the number of dogs he walks, and P represents his total pay for the day. In this situation, since the function P(d) is a linear equation, it has a theoretical domain of all real numbers. However, in terms of the context of the situation, all real numbers does not make sense. Since d represents the number of dogs that the man walks, the domain that makes sense for the situation would be all positive integers including 0, {0,1,2,3,…}. We call this domain the actual domain. This domain is a discrete set of numbers as opposed to an interval. I think it is important to include examples with different types of domains, because studies have shown that students struggle with domains that are not simple intervals.[7]

Quadratic functions are the first type of function that students see that don't have the range of $(-\infty, \infty)$. We can see this by looking at a graph of a quadratic function. The function f(x)=x^2 is located completely in the first and second quadrants, which means there are no negative number outputs. Therefore, the domain of this function is $(-\infty, \infty)$ and the range of this function is $[0, \infty)$. You can also think about the range being restricted because the outputs come from squaring a real number. Since all real numbers when squared are positive real numbers, then it is impossible to get a negative real number in your range. When we get to more complicated versions of quadratic functions in standard form $f(x) = ax^2 + bx + c$, where a, b, and c are in the real number system, then finding the range of this function gets more complicated. My students will not be finding this algebraically, but they should be able to find this graphically by locating the vertex of the parabola. For example, if you take the quadratic function $f(x) = x^2 + 4x - 12$ and you graph it on a coordinate grid, you will see it has a vertex of (-2,-16). The vertex of a parabola is also its minimum or maximum point and therefore affects the range of the function. In this case, (-2,-16) is the function's minimum point so there is no output value that is less than -16. This function has a range of $[-16, \infty)$, with its domain still being $(-\infty, \infty)$. The vertex of a parabola is also the point on a quadratic function where the function changes from increasing to decreasing or from decreasing to increasing. The table below explains using the point (h,k) as the quadratic function's vertex.

(h,k) is a min. or max.	**Increasing interval**	**Decreasing interval**	**Range**
Minimum	(h, ∞)	$(-\infty, h)$	(k, ∞)
Maximum	$(-\infty, h)$	(h, ∞)	$(-\infty, k)$

The determining factor of whether a quadratic function has a maximum point or a minimum point is the leading coefficient of the function, *a*, where $f(x) = ax^2 + bx + c$. If $a>0$ then the quadratic function has a minimum point and if $a<0$, the quadratic function has a maximum point. Whether a quadratic function has a minimum or maximum affects its range as shown in the table above.

In the Common Core State Standards, students are also supposed to understand where a function has positive or negative outputs in their range. If we use a different form of the algebraic rule for a quadratic function, we can see where the function is positive and negative. We will define a quadratic functions as $f(x) = a(x-b)(x-c)$, let $b \leq c$, where $x=b$ and $x=c$ are the zeroes of the function. Zeroes are important for determining where the function is positive and negative because the zeroes show where the function crosses between positive and negative values. It is important to note that some quadratic functions do not have zeroes. These quadratic functions are either all positive (a>0) or or all negative (a<0) values. Given the function above for f(x), we have the following intervals of positive and negative values.

a positive or negative	*f* is positive intervals	*f* is negative intervals
+	$(-\infty, b)$ and (c, ∞)	(b, c)
-	(b, c)	$(-\infty, b)$ and (c, ∞)

Operations on Linear Functions

My students in this course will have a strong background on linear functions already. They will understand the characteristics of linear functions, such as a constant rate of change, a straight line graph, and an algebraic rule in the form $f(x) = mx + b$, where *m* and *b* are real numbers.

When doing operations on functions, I will start with the set of all linear functions where $f(x) = mx + b$, where *m*, defined as the constant rate of change of the function and *b*, defined as the function's y-intercept, are real numbers. For the set of linear functions, we will keep the domain and range as all real numbers. We can first look at the operation of addition. So if we have any two linear functions $f_1(x) = m_1x + b_1$ and $f_2(x) = m_2x + b_2$, we will define addition of these two functions as $(f_1 + f_2)(x) = m_1x + b_1 + m_2x + b_2$. Since $(f_1 + f_2)(x) = (m_1 + m_2)x + (b_1 + b_2)$ and $m_1 + m_2$ and $b_1 + b_2$ are real numbers, then linear functions are closed under addition. The addition of linear is associative because for any three linear functions $f_1(x)$, $f_2(x)$, and $f_3(x)$, we have $(f_1 + f_2) + f_3 = f_1 + (f_2 + f_3)$. Under addition of linear functions, there is an inverse which is $f_0(x) = 0x + 0 = 0$, meaning *m* and *b* are both equal to *0*, because $f + f_0(x) = mx + b + 0x + 0 = mx + b = f(x)$. We also have an inverse of linear functions under addition because if $f_1(x) = mx + b$ for any m and b in the set of real numbers, then there exists –*m* and –*b* in the set of real numbers such that there is a linear function $f_2(x) = -mx + -b$ and $(f_1 + f_2)(x) = (m + -m)x + b + -b = 0x + 0 = f_0$ which is the identity of the set of linear functions. Lastly, the addition of linear functions is commutative under addition because $(f_1 + f_2)(x) = (f_2 + f_1)(x)$ for any two linear functions $f_1(x) = m_1x + b_1$ and $f_2(x) = m_2x + b_2$. Therefore because linear functions under the operation of addition have closure, associativity, identity, inverse, and commutativity, then the set of linear functions has an abelian group structure with respect to the operation of addition.

After my unit is finished, my students will be investigating the properties of quadratic functions. So as an introduction to these functions, we will look at the set of linear functions on the operation of multiplication. When you multiply two linear functions, you will not necessarily get a linear function. As long as your m_1 and m_2 are non-zero then the multiplication of two linear functions will result in a quadratic function. As a way to see this, we can take *f(x) = x – 1* and *g(x) = x + 2* and look at $(fg)(x) = f(x)g(x)$. We can start by looking at a table of values for f(x), g(x), and (fg)(x).

x	*f(x)*	*g(x)*	*fg(x)*
-3	*-4*	*-1*	*4*
-2	*-3*	*0*	*0*
-1	*-2*	*1*	*-2*
0	*-1*	*2*	*-2*
1	*0*	*3*	*0*
2	*1*	*4*	*4*
3	*2*	*5*	*10*

From this, you should notice that *(fg)(x)* is not only non-linear, it is also not one-to-one, which is a characteristic of quadratic functions. The reason this function is not one-to-one is because *(fg)(-1)=(fg)(0)=-2* and *(fg)(-3)=(fg)(2)=4*, along with many other examples. The key characteristic that students look for to identify linear functions is that the difference between the successive terms is constant. Looking at the above function, this is clearly no longer true with the function *(fg)(x)*, but the difference between the differences of two successive terms is constant, as shown below. This is another characteristic of quadratic functions and can be used to identify them.

x	*(fg)(x)*	*Difference between term and previous*	*2nd Difference*
-3	*4*		
-2	*0*	*-4*	
-1	*-2*	*-2*	*2*
0	*-2*	*0*	*2*
1	*0*	*2*	*2*

2	*4*	*4*	*2*
3	*10*	*6*	*2*

Operations on Quadratic Functions

We can define quadratic functions in the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$, since if *a=0*, then *f(x)* is a linear function. At this level, the only operations I will be focusing on are addition and subtraction of these functions. The addition of two quadratic functions $f_1(x) = a_1x^2 + b_1x + c_1$ and $f_2(x) = a_2x^2 + b_2x + c_2$ is defined by

$(f_1 + f_2)(x) = a_1x^2 + b_1x + c_1 + a_2x^2 + b_2x + c_2 = (a_1 + a_2)x^2 + (b_1 + b_2)x + (c_1 + c_2)$

. The set of all quadratic functions is not closed under addition because if we take two quadratic functions such that $a_1 = -a_2$, then $a_1 + a_2 = 0$, which would be the coefficient with x^2, and by my definition of quadratic functions, the coefficient associated with the x^2 term must be non-zero. The set of quadratic functions is not closed with respect to addition and neither with respect to multiplication.

Operations on Polynomial Functions with Degree Less Than or Equal to 2

The set of quadratic functions did not have closure under addition, which limited what we define about that set, but we can alter the set slightly to form an abelian group. If we change our set to be defined as all polynomial functions with degree less than or equal to 2, then we can establish some more algebraic properties. This will be defined as $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and a can be equal to 0. This set incorporates all quadratic (a is non-zero), linear (a=0, b is non-zero), and constant functions (a=0 and b=0). The addition of two of these functions is defined as

$(f_1 + f_2)(x) = a_1x^2 + b_1x + c_1 + a_2x^2 + b_2x + c_2 = (a_1 + a_2)x^2 + (b_1 + b_2)x + (c_1 + c_2)$

. Under this operation, we will have closure since the addition of any two real numbers is still in the real number system, so $a_1 + a_2 \in \Re$, $b_1 + b_2 \in \Re$, and $c_1 + c_2 \in \Re$. Addition on this set will also have associativity since for any three functions in the set, f_1, f_2, f_2, we will have$(f_1 + f_2) + f_3 = f_1 + (f_2 + f_3)$. This set under the operation of addition has an identity defined as the function $f_0(x) = 0x^2 + 0x + 0 = 0$ because for any function, $f(x) + f_0(x) = ax^2 + bx + c + 0x^2 + 0x + 0 = ax^2 + bx + c = f(x)$. This set also has an inverse for any $f(x) = ax^2 + bx + c$, with $a, b, c \in \Re$, there exists $-a, -b, -c \in \Re$ and therefore a function $f_{-1}(x) = -ax^2 + -bx + -c$ such that $f(x) + f_{-1}(x) = (a + -a)x^2 + (b + -b)x + (c + -c) = 0x^2 + 0x + 0 = f_0(x)$, which is the identity function. Lastly, addition will be commutative since for any two functions in the set, $f_1 + f_2(x) = f_2 + f_1(x)$. Therefore, the set of polynomial functions with degree less than or equal to 2, has an abelian structure.

Another operation on the set of quadratic functions is multiplication by a scalar. I can take the set of quadratic functions and multiply them by a constant. I will call the constant, *k*, where *k* is a non-zero real number and our quadratic function is $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. We can now perform scalar multiplication by doing $k \cdot f(x) = k \cdot (ax^2 + bx + c) = (ka)x^2 + (kb)x + (kc)$. Since $ka, kb, kc \in \Re$, because the real numbers is closed under multiplication, and $ka \neq 0$, then kf is also a quadratic function Also, we have associativity because if we take any two scalars, $k_1 \; and \; k_2$, then $k_1 \cdot (k_2 \cdot f(x)) = (k_1 \cdot k_2) \cdot f(x)$, for all quadratic functions $f(x) = ax^2 + bx + c$ in our set. In this set under scalar multiplication, we have an identity of *k = 1*, because for any quadratic function, we will have $1 \cdot f(x) = 1ax^2 + 1bx + 1c = ax^2 + bx + c = f(x)$. Also, the following properties hold: $(k_1 + k_2)f = (k_1 f + k_2 f)$ and $k(f_1 + f_2) = kf_1 + kf_2$ for any real numbers *k*, k_1, k_2, and any polynomial function *f*, f_1, and f_2, of degrees at most 2. Thus, the set of polynomial functions of degree at most 2 together with the operations of addition of functions and the scalar multiplication of functions has a vector space algebraic structure.

My unit will cover this content through activities that stress the common core mathematics standards. By the end of the unit, students should understand and be able to use function notation. They should understand what defines a function and what makes a relationship a non-function. Students should be able to determine the domain and range of any function, both in numerical and real world examples. In particular, students should be able to determine the domain and range of quadratic functions and linear functions using a real world context. Lastly, students will perform operations on functions to understand properties of these functions. They will multiply two linear functions to see that it produces a quadratic function. These concepts will hopefully provide a good basis for students going forward when learning about other algebraic functions.

Activity #1

This first activity will serve as an introduction to functions and function notation. I will be starting with some non-traditional functions so that students do not narrow their focus too quickly to only think of algebraic functions with numerical inputs and outputs. This activity also deals with functions versus non-functions. In the second part of the activity, I show an example of a function that is not one-to-one and then students are given an opportunity to restrict the domain to make the function one-to-one. This can be used as an extension activity or perhaps as a whole class activity since the idea of one-to-one functions will be new to the students.

<u>Families and Polygons- Special Cases of Relations and Functions</u>

Directions: Use the following family history to answer the questions below.

PART 1- Family History: Bob and Margaret had 4 children, Samuel, Thomas, Sarah, and Katherine. Each of those kids got married and had children as well. Samuel married Lisa and had one kid, Mark. Thomas married Faye and had two kids, Jack and Diane. Sarah married John and had two kids Ryan and Richard. Katherine married Scott and had three kids, Claude, Michael, and Alexis. **HINT:** It might help you to put this information into a tree diagram.

We are going to define relations based on this family tree. The function F, is defined as the father of the input so F(Samuel) = Bob. In this situation, Samuel is your input (part of the domain) and Bob is your output (part of the range). Other relations will be

S- the sister of the person
B- the brother of the person
M- the mother of the person
H- the husband of the person
W- the wife of the person

Find the following:

1. B(Ryan) =
2. W(Bob) =
3. M(Jack) =
4. S(Thomas) =

What was special about #4?

Using each of the following relations, determine the largest possible domain and range. Then list the domain and range as ordered pairs. The first example is done for you.

1. Relation: H
   a. Domain: {Margaret, Lisa, Faye, Sarah, Katherine}
   b. Range: {Bob, Samuel, Thomas, John, Scott}
   c. All possible ordered pairs: (Margaret, Bob), (Lisa, Samuel), (Faye, Thomas), (Sarah, John), (Katherine, Scott)

2. Relation: M
   a. Domain: ____
   b. Range: ____
   c. All possible ordered pairs:
      ____
      ____

3. Relation: W
    a. Domain: ____
    b. Range: ____
    c. All possible ordered pairs:
    ____
    ____

4. Relation: B
    a. Domain: ____
    b. Range: ____
    c. All possible ordered pairs:
    ____
    ____

5. Think about your answers to #1-#4 and about the definition of a function. Which of the following relations are functions and which are not? Explain your answer!

PART 2- Polygons and their angles: Our new domain will be the following set of special polygons:
$\{square, rectangle, right\ triangle, isosceles\ \ triangle, pentagon, hexagon\}$. Using this domain, we will define some relations as shown:

- S- the number of sides of the polygon
- A- the sum of the angles in the polygon

1. Using our given domain, determine the range of S and A, then list the ordered pairs of each relation.
    a. Range of S:
    ____
    b. Range of A:
    ____
    c. Ordered pairs of S:
    ____
    d. Ordered pairs of A:
    ____

2. Are the relations S and A functions? Explain why or why not?

3. There are certain functions that are called one-to-one functions. In these functions, each input has exactly one output and each output has exactly one input. Are the functions S and A one-to-one? Explain with examples

4. How could you restrict the given domain in order to make the functions S and A one-to-one? Show how your new domain makes these functions one-to-one.

PART 3- For each of the following relations and domains

- Determine its range
- Decide whether or not it is a function
- If it is a function, decide whether or not it would be one-to-one.

1. Relation- H: a person's current height in inches. Domain: the set of all students at Hodgson.

    a. Range:

    b. Function?

    c. One-to-one?

2. Relation- B: a person's birthdate. Domain: the set of all people in the United States.

    a. Range:

    b. Function?

    c. One-to-one?

3. Relation- b: a person's birthdate. Domain: the set of all people living in your house currently. (Each student will have a different answer for this question!)

    a. Range:

b. Function?

c. One-to-one?

Activity #2

In this activity, students will work with linear functions, which is something they should be quite familiar with, in order to see that multiplication of two of these functions can lead to quadratic functions. It is important that students see these relationships in tables, graphs, and through the equations themselves. I think the graphical representation is the key to this investigation so it may help to make a large copy of the graph on chart paper. Students will also explore properties of quadratic functions such as having intervals of both increasing and decreasing behavior. Again, the graph will be the key tool for students to use in exploring the characteristics of this function. It is not important yet that students are able to expand the multiplication of two linear functions to the equations into standard quadratic form. It also would help if students have a basic knowledge of quadratic functions in that they should be able to recognize one from a graph or table. Even without this understanding, students will be able to see that multiplication of two linear functions is clearly not linear.

Operations on Linear Functions

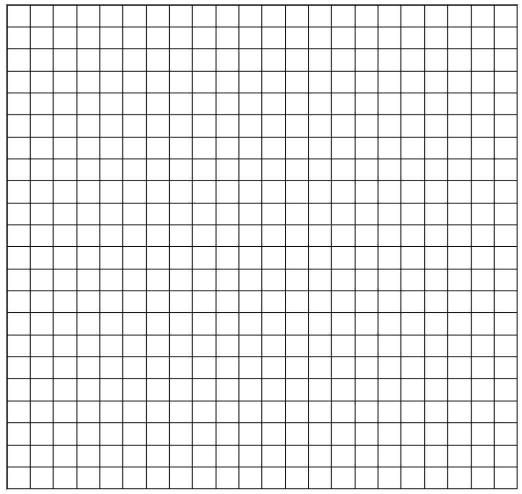
For the following activity, we will use the functions $f(x) = x + 3$ and $g(x) = 2x - 2$. We will be performing operations on these functions.

PART 1: We will start with the operation of addition. In order to add these two functions, for any given value of x, we will add the two outputs. For example, $(f+g)(1) = f(1) + g(1) = 4 + 0 = 4$. By adding these two functions, we are creating a new function $(f+g)(x)$.

1. Start by filling in the following table:

x	***f(x)***	***g(x)***	***(f + g)(x)***
-3			
-2			
-1			
0			
1			
2			

2. Now graph the function (f + g)(x) making sure to include the points from your table above.



3. What type of function resulted by adding these two linear functions? Why does this make sense?

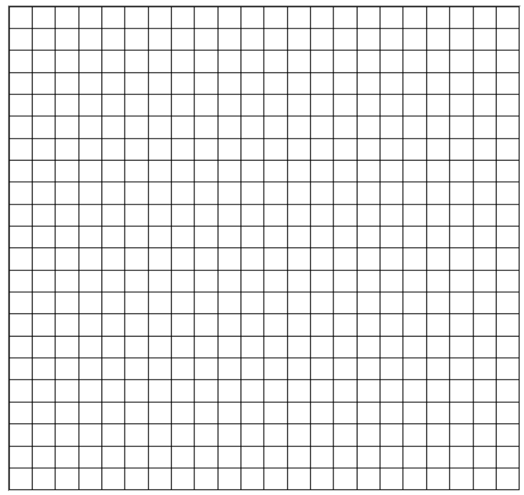
4. Write the equation for this function in its simplest form. (f + g)(x) = ___________

PART 2: Now we will perform multiplication on these same two functions. In order to multiply these two functions, for any given value of x, we will multiply the two outputs. For example, (fg)(2) = f(2) *g(2) = 5*2= 10. By multiplying these two functions, we are creating a new function (fg)(x).

1. Start by filling in the following table:

x	***f(x)***	***g(x)***	***(fg)(x)***
-4			
-3			
-2			
-1			
0			
1			
2			

2. Now graph the function (fg)(x) making sure to include the points from your table above as well as any other that help to determine its shape.



3. Did multiplying these two functions result in a linear function? Justify your answer.

4. Where is this function increasing?

5. Where is this function decreasing?

6. Where is this function positive?

7. Where is this function negative?

8. What are the x-intercepts of the functions? How are these related to the x-intercepts of the original functions, f(x) and g(x)?

9. What type of function does this seem to be? Explain.

10. The point (-1,-8) is called the vertex of this function. What is special about this point?
11. ***Possible Homework Assignment:*** Repeat part 2 of this activity with two new linear functions of your choice! Do the following steps:

    a. Create two linear functions and write them in slope-intercept form. HINT: To make it easy on yourself, make your slope and y-intercept whole numbers and between -5 and 5.
    b. Multiply your two linear functions together to create a table and graph like you did in #1 and #2.
    c. Answer questions #3 - #9
    d. For #10, determine the vertex of the function that you created and explain how you found this point.

12. ***Extension Activity:*** Try this activity using two of the **<u>same</u>** linear functions.

Bibliography

Graham, Karen G. and Ferrin-Mundy, Joan. "Functions and their Representations." *Mathematics Teacher* 83 (1990): 209-215.

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[1] "Mathematics > High School: Functions > Linear, Quadratic, & Exponential Models," Common Core State Standards Initiative, accessed November 1st, 2013, http://www.corestandards.org/Math/Content/HSF/LE
[2] Ibid
[3] Ibid
[4] Ibid
[5] Anita Johnson, "Introducing Functions andits Notation," *Mathematics Teacher* 80 (1987) 559.
[6] Karen Geuther Graham and Joan Ferrin-Mundy, "Functions and Their Representations," *Mathematics Teacher* 83 (1990) 210.
[7] Geuther and Ferrin-Mundy, "Functions and Their Representations," 211.

Curriculum Unit Title

UNDERSTANDING FUNCTIONS

Author Elizabeth Lancellotti

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Students will be able to determine values of functions using function notation. They will be able to distinguish between functions and non-functions. They will be able to determine the range of a function give its range. They will understand how two linear functions can be multiplied to create a quadratic function.

ESSENTIAL QUESTION(S) for the UNIT

What defines a function? What is the difference between a function and a non-function? How can we write functions using function notation? What is the relationship between linear functions and quadratic functions?

CONCEPT A

Function versus Non-Function

CONCEPT B

Function Notation

CONCEPT C

Linear and Quadratic Functions

ESSENTIAL QUESTIONS A

What are examples of functions and non-functions? What distinguishes the two?

ESSENTIAL QUESTIONS B

How can I write and interpret function notation?

ESSENTIAL QUESTIONS C

What is the result of multiplying two linear functions together? How is this result shown in a table, graph, and equation?

VOCABULARY A

Input
Output
One-to-one
Venn diagram

VOCABULARY B

Function notation
F(x): F of x
Input
output

VOCABULARY C

Linear function, slope, y-intercept

Quadratic function, parabola, zeroes, x-intercepts

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES