Exponential Functions

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Introduction

The exponential function is probably the most recognizable graph to many non-mathematicians. The typical graph of an exponential growth model shows that the values have a slow initial growth leading to a rapid increase in value. The simple doubling function 2, 4, 8, 16, 32... is probably the simplest case of an exponential but they can be used to model many situations: describing growth in populations, money investments, spread of disease and decay of radioactive substances or depreciation of value. Over the past few years, I have noticed that my students can recognize graphs and tables that model exponential growth and decay when contrasted with a linear model, but lack the understanding of the algebraic properties of the exponential functions. By discussing the basic concept of function, followed by a look at the algebraic structure of the exponential function, this unit will represent a different approach. We will examine the set of exponential functions and discuss their algebraic properties. By developing this unit, I hope to develop a student population with a sense of understanding for the equation of the exponential function. I will be presenting this unit to a first year algebra class.

Concept of Function in High School Mathematics Curriculum

This unit will be presented to students in an introductory algebra class; as such these students have had a few experiences with the concept of a function. In this course, I will have exposed my students to the linear function or $f(x) = mx + b$. They will have made many tables and graphs and discussed the concept of a constant rate of change $\frac{\Delta y}{\Delta x}$. They will have been introduced to the concept of a function as a machine that assigns one number usually called the input with a unique value known as the output.

An Overview of the High School and Students

My unit will be aimed at my algebra I class for the 2014 - 2015 school year. This is a college prep non honors introductory algebra course. At this time, my roster indicates 33 ninth grade students. Although Conrad is a sixth through twelfth grade building, a large number of our freshman class consists of students from other public middle schools, private schools and parochial schools. Some of these students may have been introduced to exponential growth problems through the integrated curriculum while other students will not have been exposed to functions at all. This unit will help develop a better understanding of the structure of exponential functions and their characteristics.

Conrad Schools of Science is a magnet school under the umbrella of the Red Clay Consolidated School District. It is located in the Wilmington area and draws students from New Castle to Newark. Students apply to Conrad by application, essay, interview, and science project. Each student receives a score on each admission measurement and all students meeting a certain percentage of a given total score have their name placed into a lottery system. As a result of the lottery system, we have a somewhat diverse group of academically performing students, meaning, that students with "A" averages have an equally likely chance in the lottery as students with a "C" average. Our school is operated on an A B block schedule, which means I will have approximately 85 minutes with each class every other day.

Each grade level houses approximately 160 students and I will have one section of college preparatory algebra I with approximately 33 students in the class. Conrad has approximate 35% minority population with 14.6% of our students qualifying for free or reduced lunch services. About 95% of our current juniors passed the state assessment in mathematics. This data was collected from School Profiles and I have provided a snapshot of the data in figure 1 and 2.[1]

Fall Enrollment

	2012-13	2013-14
Grade 6	163	173
Grade 7	167	169
Grade 8	166	173
Grade 9	176	167
Grade 10	155	169
Grade 11	139	159
Grade 12	116	143
Total	1,082	1,153

Enrollment by Race/Ethnicity

	2012-13	2013-14
African American	10.4%	11.2%
American Indian	0.1%	0.1%
Asian	2.7%	3.9%
Hispanic or Latino	20.3%	18.9%
White	66.0%	65.1%
Multi-Racial	0.6%	0.8%

Other Student Characteristics

	2012-13	2013-14
English Language Learner	7.9%	7.7%
Low Income	36.7%	14.6%
Special Education	3.0%	2.9%
Enrolled Full Year	98.9%	N/A

Figure 1

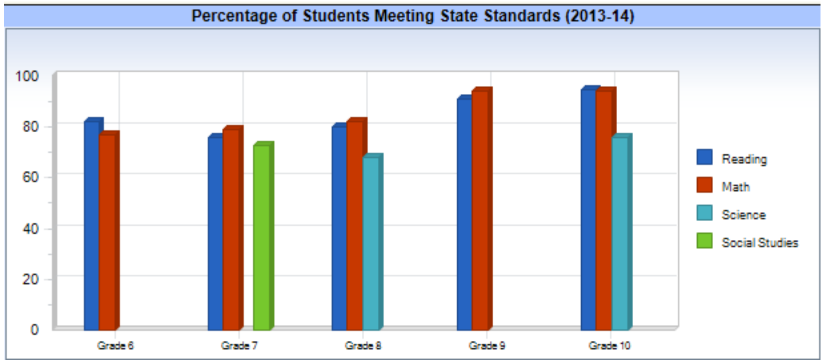


Figure 2

Unit Objectives

- Students will be able to define a function using inputs and outputs
- Students will be able to define the domain and range of a given function
- Students will be able to describe a function as either increasing, decreasing or constant
- Students will be able to determine if a given function is injective or 1-1, thus creating an inverse
- Students will be able to demonstrate their knowledge of the properties of the addition and multiplication operations with real numbers; commutativity, associativity, distributivity, identity element and inverse element.
- Students will be able to define an exponent and use exponential notation to evaluate expressions
- Students will be able to simplify expressions using the properties of exponents.
- Students will be able to define an exponential function with an initial value and a growth or decay rate
- Students will be able to observe using graphs, tables and equations the basic characteristics of an exponential function.

The Mathematics of the Unit

I will begin the unit with the review of the definition of a function. In almost every aspect of our lives, we find examples of situations where one quantity depends on another. For example, the wind chill depends on the speed of the wind; the area of a circle depends on its radius; and the amount earned by your investment depends on the interest rate. A function, *F*, from a set A to a set B is a relationship (rule) that associates each input value from set A a unique output value from set B. An example of a relation that is a function is
$F = \{(2,3), (5,7), (4,-2), (9,1), (8,4), (6,3)\}$.
A function can also be defined as a rule that assigns exactly one element in set B to each element in set A. Set A, the set of all first coordinates, or the input values, is called the domain of the function. Set B, the set of all second coordinates, or the output values, is called the range. In the

function F above, the domain is the set of first elements: Domain = {2, 5, 4, 9, 8, 6} and the range is the set of second elements: Range = {3, 7, −2, 1, 4}. The function F can be represented in a mapping diagram as demonstrated below in figure 3.

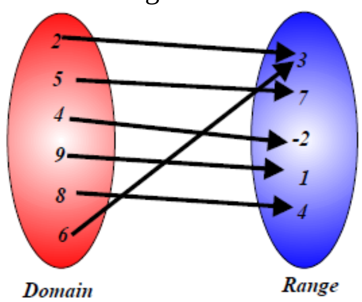


Figure 3

The characteristic that distinguishes a function from any other relation (rule) is that there is only one output (y-value) for each input (x-value). After reviewing the concept of a function, I will review the basic properties of exponents. My students will recall that the purpose of an exponent is to write repeated multiplication in short hand script. For example we know that $3^4 = 3 \times 3 \times 3 \times 3 = 81$ where 3 is defined to be the base and 4 is defined to be the exponent. This type of short hand script makes writing large numbers easier, for example it is easier to record 5^{20} than it is to record the numerical value 95,367,431,640,625. I can expand our definition of an exponent to be

$b^n = b \times b \times b \times b \times b \ldots \times b \ (n\ times)$

The basic properties of exponents are

Property	**Example**
$b^1 = b$	$5^1 = 5$
$b^0 = 1$	$125^0 = 1$
$b^{-1} = \frac{1}{b}$	$6^{-1} = \frac{1}{6}$
$b^m \times b^n = b^{(m+n)}$	$2^3 \times 2^4 = 2^{(3+4)} = 2^7 = 128$
$\frac{b^m}{b^n} = b^{(m-n)}$	$\frac{4^6}{4^4} = 4^{(6-4)} = 4^2 = 16$
$(b^m)^n = b^{(m \times n)}$	$(3^2)^3 = 3^{(2\times 3)} = 3^6 = 729$
$(ab)^m = a^m \times b^m$	$(3 \times 4)^2 = 3^2 \times 4^2 = 9 \times 16 = 144$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{3}{4}\right)^5 = \frac{3^5}{4^5} = \frac{243}{1024}$
$b^{-m} = \frac{1}{b^m}$	$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

After reviewing the concept of a function and the properties of exponents, I will show the video "The Greatest Shortcoming of the Human Race Is Our Inability to Understand the Exponential Function" by Al Barlett on Growth and Sustainability.[2] This video shows the importance of understanding mathematics in our daily lives. After showing the video, I would ask the students what information they could tell me relating to the video. I would create a list of things they know, things they do not know, and things they want to learn more about. After the video, I would define the exponential function. In this unit, my set will be defined as the set of all exponential functions of the form $f(x) = b^x$.

> *the exponential function can be written as* $f(x) = b^x$, *where* $b > 0$ *and* $b \neq 1$
> *where b is defined to be the base and x can represent any real number.*

In this definition, x, the variable is now the exponent and the base is a fixed number. This idea will be new to my students, up until now, the base has been the variable, *x* in most cases, and the exponent was a fixed number. Before I proceed into this topic, I would address the restrictions on *b*. I will place the restriction that b $\neq 1$ *and that* $b > 0$, for if b = 0 I would have $f(x) = 0^x = 0$ and if b = 1 I would have $f(x) = 1^x = 1$ and these are constant functions. They will not have all the same properties of general exponential functions. We define the domain of the exponential function to be the set of real numbers, and the range of the exponential function to be the set of positive numbers. Identifying the domain (set of inputs) and the range (set of outputs) can be shown by the graph of an exponential function as shown in figure 4.

For example $f(x) = 2^x$

***x* value (input)**	**$f(x)$ (output)**
-9	$f(-9) = 2^{-9} = \frac{1}{2^9} = \frac{1}{512} \approx .001953$
-3	$f(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8} = .125$
-2	$f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4} = .25$
-1	$f(-1) = 2^{-1} = \frac{1}{2^1} = \frac{1}{2} = .50$
0	$f(0) = 2^0 = 1$
1	$f(1) = 2^1 = 2$
2	$f(2) = 2^2 = 4$
3	$f(3) = 2^3 = 8$
4	$f(4) = 2^4 = 16$

Graph

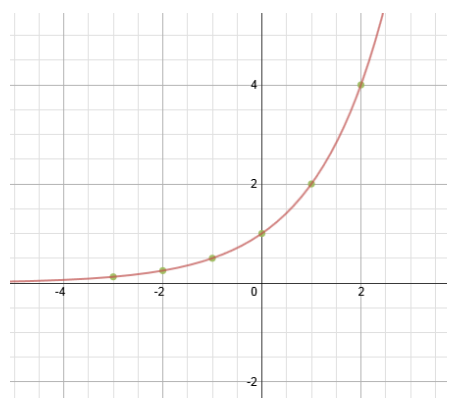


Figure 4

From the graph of $f(x) = 2^x$ in figure 4, I can identify the domain as (-∞, ∞) and the range as (0, ∞). I can also note that this graph starts with a slow initial growth leading to a rapid increase in value. This graph makes sense based on our inputs and our outputs. If we think about $f(x) = 5^x$ *or* $f(x) = 10^x$, I would expect the process to be the same with larger output values. I would expect the graph to have similarities to the graph of $f(x) = 2^x$. Thus I can conclude that I have a general idea of the shape of the exponential function for base values that are positive whole numbers. What would the graph look like if *b* <1? Remember the initial definition claimed that *b* must be greater than zero. Would the graph have similarities to the graph above? Let's look at $f(x) = \left(\frac{1}{2}\right)^x$, as pictured below in figure 5. Now that I have developed the first graph of the exponential I will look at a smaller table to generate the graph. After I generate the graph for an exponential with a base value less than one, I will look for any similarities and differences in the graphs.

***x* value (input)**	***f*(*x*) (output)**
-3	$f(-3) = \left(\frac{1}{2}\right)^{-3} = \left(\frac{1^{-3}}{2^{-3}}\right) = \frac{2^3}{1^3} = \frac{8}{1} = 8$
-2	$f(-2) = \left(\frac{1}{2}\right)^{-2} = \left(\frac{1^{-2}}{2^{-2}}\right) = \frac{2^2}{1^2} = \frac{4}{1} = 4$
-1	$f(-1) = \left(\frac{1}{2}\right)^{-1} = \left(\frac{1^{-1}}{2^{-1}}\right) = \frac{2^1}{1^1} = \frac{2}{1} = 2$
0	$f(0) = \left(\frac{1}{2}\right)^{0} = 1$
1	$f(1) = \left(\frac{1}{2}\right)^{1} = \frac{1^1}{2^1} = \frac{1}{2}$
2	$f(2) = \left(\frac{1}{2}\right)^{2} = \frac{1^2}{2^2} = \frac{1}{4}$
3	$f(3) = \left(\frac{1}{2}\right)^{3} = \frac{1^3}{2^3} = \frac{1}{8}$

Graph

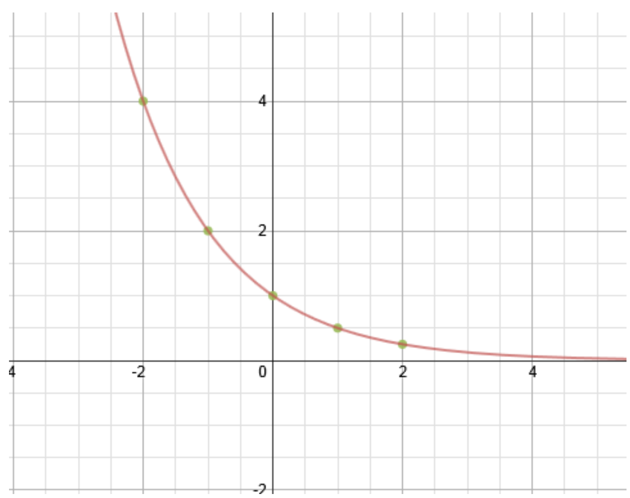


Figure 5

As I examine both graphs we can conclude the common characteristics:

- The graph crosses the *y*-axis at (0,1) or the y intercept is (0,1)
- The domain is the set of all reals numbers (-∞, ∞)
- The range is the set of positive real numbers (0, ∞)

- The graph represents a function, each input value is associated with a unique output value
- The graph is injective or 1-1, which indicates there is an inverse
- When b > 1, the graph increases from left to right
- When 0 < b < 1, the graph decreases from left to right
- The graph is asymptotic to the *x*-axis which means the graph gets very, very close to the *x*-axis but does not touch it or cross it.

Next I would want to examine what happens to the graph and equation if I multiply the function by a constant number, k, where k > 0. The new function would be defined as $(x) = k \times b^x$. Will the graph still retain the above characteristics? Let's examine the functions $f(x) = 5 \times 2^x$ and $f(x) = 5 \times \left(\frac{1}{2}\right)^x$ where k = 5 in figure 6 and 7.

$f(x) = 5 \times 2^x$	
x value (input)	$f(x)$ (output)
-3	$f(-3) = 5 \times 2^{-3} = 5 \times \frac{1}{8} = \frac{5}{8}$
-2	$f(-2) = 5 \times 2^{-2} = 5 \times \frac{1}{4} = \frac{5}{4}$
-1	$f(-1) = 5 \times 2^{-1} = 5 \times \frac{1}{2} = \frac{5}{2}$
0	$f(0) = 5 \times 2^{0} = 5 \times 1 = 5$
1	$f(1) = 5 \times 2^{1} = 5 \times 2 = 10$
2	$f(2) = 5 \times 2^{2} = 5 \times 4 = 20$
3	$f(3) = 5 \times 2^{3} = 5 \times 8 = 40$

$f(x) = 5 \times \left(\frac{1}{2}\right)^x$	
x value (input)	$f(x)$ (output)
-3	$f(-3) = 5 \times \left(\frac{1}{2}\right)^{-3} = 5 \times 8 = 40$
-2	$f(-2) = 5 \times \left(\frac{1}{2}\right)^{-2} = 5 \times 4 = 20$
-1	$f(-1) = 5 \times \left(\frac{1}{2}\right)^{-1} = 5 \times 2 = 10$
0	$f(0) = 5 \times \left(\frac{1}{2}\right)^{0} = 5 \times 1 = 5$
1	$f(1) = 5 \times \left(\frac{1}{2}\right)^{1} = 5 \times \frac{1}{2} = \frac{5}{2}$
2	$f(2) = 5 \times \left(\frac{1}{2}\right)^{2} = 5 \times \frac{1}{4} = \frac{5}{4}$
3	$f(3) = 5 \times \left(\frac{1}{2}\right)^{3} = 5 \times \frac{1}{8} = \frac{5}{8}$

Figure 6

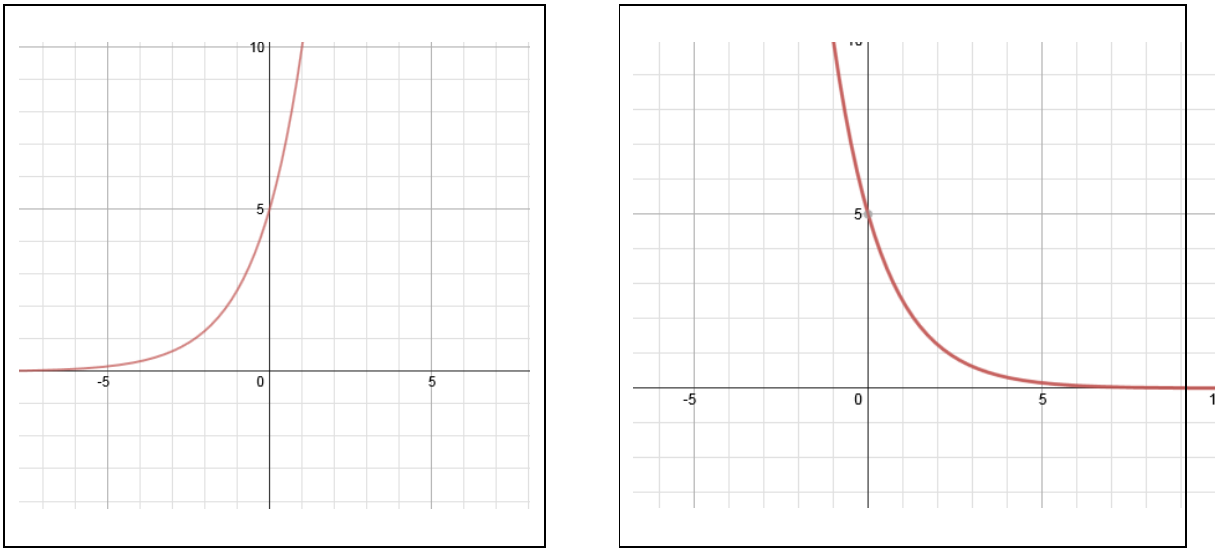


Figure 7

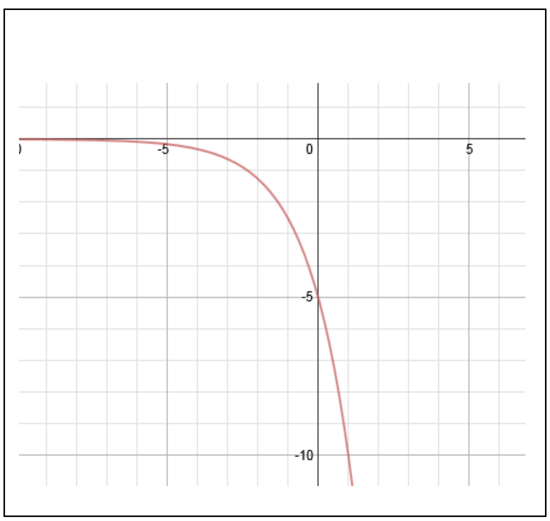
As I examine both graphs I can conclude the common characteristics are relatively the same except for the y intercept and our output values have been multiplied by 5.

- The graph crosses the *y*-axis at (0, 5) or the y intercept is (0, 5). I can conclude that the initial y value of 1 has been multiplied by the same constant value of 5, or that k has become our initial value
- The domain is still the set of all reals numbers (-∞, ∞)
- The range is still the set of positive real numbers (0, ∞)
- The graph still represents a function, where each input value is associated with a unique output value
- The graph is still injective or 1-1
- When $b > 1$, the graph increases from left to right at a faster rate (five times faster) thus the graph has been stretched
- When $0 < b < 1$, the graph decreases from left to right at a faster rate (five times faster) thus the graph has been stretched
- The graph is asymptotic to the *x*-axis which means the graph gets very, very close to the *x*-axis but does not touch it or cross it.

Thus, I can conclude that multiplying an exponential function by a constant, $k > 0$ does not change the basic characteristics of the graph. What happens if I multiply by a constant $k < 0$? A quick look at $f(x) = -5 \times 2^x$ and $f(x) = -5 \times \left(\frac{1}{2}\right)^x$ yields the following tables and graphs in figure 8.

$f(x) = -5 \times 2^x$	
x value (input)	$f(x)$ (output)
-2	$f(-2) = -5 \times 2^{-2} = -5 \times \frac{1}{4} = -\frac{5}{4}$
-1	$f(-1) = -5 \times 2^{-1} = -5 \times \frac{1}{2} = -\frac{5}{2}$
0	$f(0) = -5 \times 2^{0} = -5 \times 1 = -5$
1	$f(1) = -5 \times 2^{1} = -5 \times 2 = -10$
2	$f(2) = -5 \times 2^{2} = -5 \times 4 = -20$
3	$f(3) = -5 \times 2^{3} = -5 \times 8 = -40$

$f(x) = -5 \times \left(\frac{1}{2}\right)^x$	
x value (input)	$f(x)$ (output)
-2	$f(-2) = -5 \times \left(\frac{1}{2}\right)^{-2} = -5 \times 4 = -20$
-1	$f(-1) = -5 \times \left(\frac{1}{2}\right)^{-1} = -5 \times 2 = -10$
0	$f(0) = -5 \times \left(\frac{1}{2}\right)^{0} = -5 \times 1 = -5$
1	$f(1) = -5 \times \left(\frac{1}{2}\right)^{1} = -5 \times \frac{1}{2} = -\frac{5}{2}$
2	$f(2) = -5 \times \left(\frac{1}{2}\right)^{2} = -5 \times \frac{1}{4} = -\frac{5}{4}$
3	$f(3) = -5 \times \left(\frac{1}{2}\right)^{3} = -5 \times \frac{1}{8} = -\frac{5}{8}$



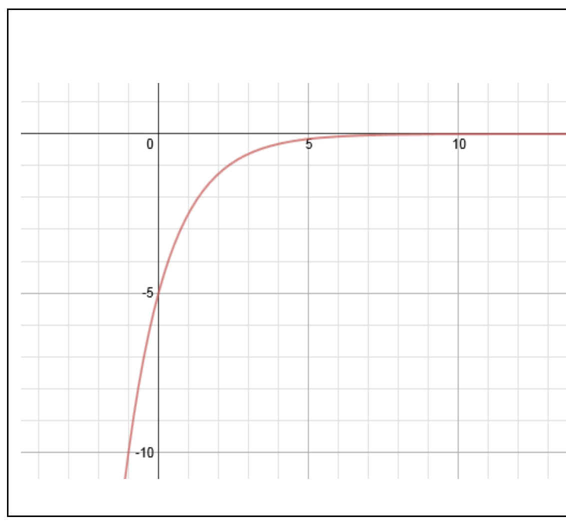


Figure 8

- Multiplying the exponential function by a constant, k < 0, changed some of the characteristics of the exponential function. The graph now crosses the *y*-axis at (0, -5) or the y intercept is (0, -5). I can conclude that the initial y value of 1 has been multiplied by the same constant value of -5, or that k has become the initial value

- The domain is still the set of all reals numbers (-∞, ∞)
- The range is now the set of all negative real numbers (-∞, 0)
- The graph still represents a function, where each input value is associated with a unique output value
- The graph is still injective or 1-1
- When b > 1, the graph decreases from left to right at a faster rate (five times faster) thus the graph has been stretched
- When 0 < b < 1, the graph increases from left to right at a faster rate (five times faster) thus the graph has been stretched
- The graph is still asymptotic to the *x*-axis which means the graph gets very, very close to the *x*-axis but does not touch it or cross it.

At this time, I can now expand my set to include all exponential functions of the form $f(x) = k \times b^x$ where *k* represents the initial value, or the multiplier. In the first two examples our multiplier, *k*, would be 1, because

$f(x) = 2^x$ *is the same as* $f(x) = 1 \times 2^x$

and $f(x) = 1 \times \left(\frac{1}{2}\right)^x$ *is the same as* $f(x) = \left(\frac{1}{2}\right)^x$

Now let's look at what happens if I try to multiply two exponential functions. Will the result still be an exponential function? To start I will look at multiplying exponential functions with no constant multiplier. Let's look at $f(x) = 3^x$ *and* $g(x) = 4^x$

$f(x) \times g(x)$

$= (3^x)(4^x)$

$= (3 \times 4)^x$ using the properties of exponents

$=12^x$ which represents an exponential function with base 12. So it will have all of the characteristics of a general exponential graph.

What happens if I include some constant multipliers? Let's look at

$f(x) = 2 \times 3^x$ *and* $g(x) = 5 \times 2^x$

$f(x) \times g(x)$

$=(2 \times 3^x)(5 \times 2^x)$

$=2 \times (3^x \times 5) \times 2^x$ by applying the associative property of multiplication

$=2 \times (5 \times 3^x) \times 2^x$ by applying the commutative property of multiplication

$= (2 \times 5) \times (3^x \times 2^x)$ by applying the associative property of multiplication

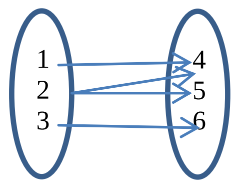
$= (10) \times (3 \times 2)^x$ by applying properties of exponents

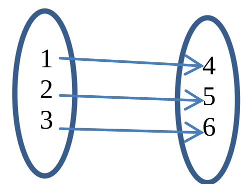
$=10 \times 6^x$ which represents an exponential function with base 6 and an initial value or y intercept of 10. Thus the set of exponential functions of the form $(x) = k \times b^x$ is closed under the operation of multiplication. In order to determine the algebraic structure of my set, I would need to check to see if my set has the property of associativity and since the operation is multiplication and multiplication is associative, then my set has associativity. I could make the same argument in regards to commutativity. Although I would like to define the identity function of my set to be $f(x) = 1 \times 1^x$, I realize that although this function looks like it has the same algebraic structure we see that its graph is merely the horizontal line y = 1 and does not have most of the characteristics of an exponential function. Hence my set of exponential functions does not have an identity function, and without an identity function I cannot discuss the concept of an inverse. So my set represents a commutative semigroup as defined in *Abstract Algebra* by Herstein.[3]

An Overview of the Activities

Over the past few years I have noticed that high school students enjoy making things, so I have read a lot about the interactive notebook and am using these approaches in my algebra class this year. It may be worth noting that my calculus students have commented "why don't we get to do some of these?" I find it interesting that the smallest shift in approach can lead to such peaked interest. The first item to be discussed in this unit is the concept of a function. Students will cut out and decorate a function versus non function foldable to be placed in their notebook. This foldable will contain information regarding the definition of a function, a mapping diagram, a table and a graph. It will have examples showing what a function would look like, contrasted with non-function examples. A sample is given below in figure 9.

Not Functions	***If it is not a function, that means that an input value is paired with more than one output value Input is the domain Output is the range***	***A function means that each input value has exactly one output value. Input is the domain Output is the range***	**Functions Domain/Range**

<table>
<tr>
<td>Mapping Diagram Domain/Range</td>
<td>input output


Domain 1,2,3
Range 4,5,6</td>
<td>input output


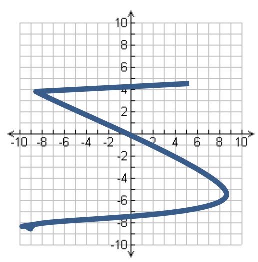
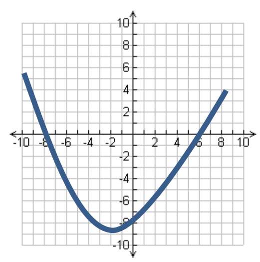
Domain 1,2,3
Range 4,5,6</td>
<td>Mapping Diagram Domain/Range</td>
</tr>
<tr>
<td>Table Domain/Range</td>
<td><table><tr><th>Input</th><th>Output</th></tr><tr><td>X</td><td>Y</td></tr><tr><td>4</td><td>2</td></tr><tr><td>4</td><td>7</td></tr><tr><td>9</td><td>5</td></tr></table>Domain 4, 9
Range 2,5,7</td>
<td><table><tr><th>Input</th><th>Output</th></tr><tr><td>X</td><td>Y</td></tr><tr><td>4</td><td>10</td></tr><tr><td>5</td><td>11</td></tr><tr><td>6</td><td>12</td></tr></table>Domain 4,5,6
Range 10,11,12</td>
<td>Table Domain/Range</td>
</tr>
<tr>
<td>Graph Domain/Range Vertical Line Test Fails</td>
<td>
Domain
Range</td>
<td>
Domain
Range</td>
<td>Graph Domain/Range Vertical Line Test Passes</td>
</tr>
</table>

Figure 9.

Next I would review the basic properties of exponents. Again I would have the kids make a foldable or chart to place in their notebook

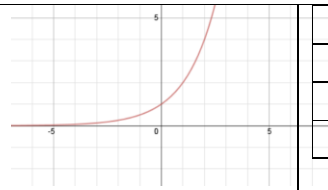
Exponents' rules and properties

Rule name	Rule	Example
Product rules	$a^n \cdot a^m = a^{n+m}$	$2^3 \cdot 2^4 = 2^{3+4} = 128$

	$a^n \cdot b^n = (a \cdot b)^n$	$3^2 \cdot 4^2 = (3 \cdot 4)^2 = 144$
Quotient rules	$a^n / a^m = a^{n-m}$	$2^5 / 2^3 = 2^{5-3} = 4$
	$a^n / b^n = (a / b)^n$	$4^3 / 2^3 = (4/2)^3 = 8$
Power rules	$(b^n)^m = b^{n \cdot m}$	$(2^3)^2 = 2^{3 \cdot 2} = 64$
	$b^{n^m} = b^{(n^m)}$	$2^{3^2} = 2^{(3^2)} = 512$
	$\sqrt[m]{(b^n)} = b^{n/m}$	$\sqrt[2]{(2^6)} = 2^{6/2} = 8$
	$b^{1/n} = \sqrt[n]{b}$	$8^{1/3} = \sqrt[3]{8} = 2$
Negative exponents	$b^{-n} = 1 / b^n$	$2^{-3} = 1/2^3 = 0.125$
Zero rules	$b^0 = 1$	$5^0 = 1$
	$0^n = 0$, for $n>0$	$0^5 = 0$
One rules	$b^1 = b$	$5^1 = 5$
	$1^n = 1$	$1^5 = 1$

After reviewing the concept of a function and the properties of exponents, I will define the exponential function. At this point I would have my students complete a typical calculator exploration of the function. We would graph a series of functions that represent growth at different rates, and a series of functions that represent decay at different rates. For each graph I would explore the characteristics of the graph, pointing out the y intercept, whether the function was increasing or decreasing, the domain, the range, and that the graph is asymptotic to the x-axis. We would complete a chart similar to the one below.

<table>
<tr><th>Equation</th><th>Graph</th><th>Table</th><th>Description</th></tr>
<tr><td>$y = 2^x$</td><td></td><td><table><tr><td>X</td><td>Y</td></tr><tr><td>0</td><td>1</td></tr><tr><td>2</td><td>4</td></tr></table></td><td>Increasing
y intercept: (0,1)
Domain (-∞,∞)</td></tr>
</table>

		3 8 5 32 -2 1/4 -3 1/8	Range $(0,\infty)$ Asymptotic to x-axis
$y = 5^x$	Student would complete	Student would complete	Student would complete
$y = 10^x$	Student would complete	Student would complete	Student would complete
$y = \left(\frac{1}{2}\right)^x$	Student would complete	Student would complete	Student would complete
$y = \left(\frac{3}{4}\right)^x$	Student would complete	Student would complete	Student would complete

Once the students had a chance to examine the graphs of each function, I would review the properties of operations on real numbers such as

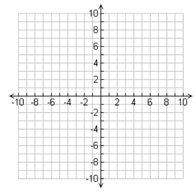
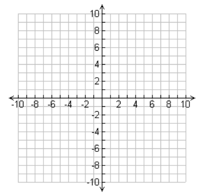
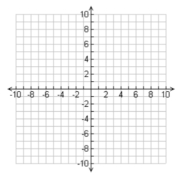
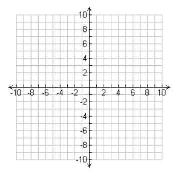
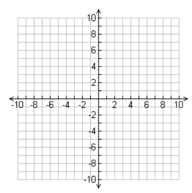
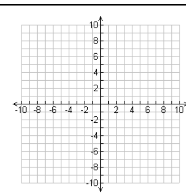
Commutative property: When two numbers are multiplied together, the product is the same regardless of the order of the multiplicands. For example 4 * 2 = 2 * 4

Associative Property: When three or more numbers are multiplied, the product is the same regardless of the grouping of the factors. For example (2 * 3) * 4 = 2 * (3 * 4)

And I would ask my students to find the product of $(2 \times 3^x)(5 \times 2^x)$using the above properties.

My last task for the students would be to cut and sort the activity cards below and to fill in the missing blanks. I am providing the answer key. The items that are in bold, would be the blanks that the students would need to add.

Equation	Table	Graph	Description

$y = 2^x$	X	Y		Increasing y intercept **(0,1)**
	-3	.125		
	0	**1**		
	1	2		
	3	8		
$y = \left(\frac{1}{3}\right)^x$	X	Y		**Decreasing** y intercept **(0,1)**
	-3	27		
	1	1/3		
	0	**1**		
	2	**1/9**		
$y = -2^x - 1$	X	Y		Decreasing y intercept **(0,-2)**
	0	**-2**		
	3	-9		
	2	-5		
	-2	-1.25		
$y = 5^x + 2$	X	Y		**Increasing** y intercept **(0,3)**
	-2	2.04		
	1	7		
	2	**27**		
	0	**3**		
$y = \left(\frac{1}{2}\right)^x - 1$	X	Y		Decreasing y intercept **(0,0)**
	-3	**7**		
	-1	1		
	0	0		
	2	**-.75**		
$y = -\left(\frac{1}{5}\right)^x + 2$	X	Y		**Decreasing** y intercept **(0,0)**
	0	**1**		
	2	1.96		
	-2	**-23**		
	1	1.8		

Appendix A: Standards

The following list of Common Core State Standards in mathematics will be directly addressed in this unit

Primary Content Standards

- CCSS.Math.Content.8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[4]
- CCSS.Math.Content.8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).[5]
- CCSS.Math.Content.8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear, for example exponential functions.[6]
- CCSS.Math.Content.8.EE.A.1
  Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.[7]
- CCSS.Math.Content.8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.[8]

Secondary Content Standards

- CCSS.Math.Content.HSA.SSE.A.1.a
  Interpret parts of an expression, such as terms, factors, and coefficients.[9]
- CCSS.Math.Content.HSA.SSE.A.1.b
  Interpret complicated expressions by viewing one or more of their parts as a single entity.[10]
- CCSS.Math.Content.HSA.SSE.A.2
  Use the structure of an expression to identify ways to rewrite it.[11]
- CCSS.Math.Content.HSA.SSE.B.3.c
  Use the properties of exponents to transform expressions for exponential functions.[12]
- CCSS.Math.Content.HSF.IF.B.4
  For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features.[13]
- CCSS.Math.Content.HSF.IF.B.5
  Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.[14]
- CCSS.Math.Content.HSF.IF.C.7.e
  Graph exponential functions, showing intercepts and end behavior.[15]
- CCSS.Math.Content.HSF-IF.5. Functions, Interpreting Functions: Interpret functions that arise in applications in terms of the context. Relate the domain of a

function to its graph and, where applicable, to the quantitative relationship it describes.[16]

Primary Mathematical Practice Standards

While I believe all eight mathematical practices are applicable to this unit, these five are highlighted throughout the unit:

- MP-1. Make sense of problems and persevere in solving them.[17]
- MP-2. Reason abstractly and quantitatively.[18]
- MP-4. Model with mathematics.[19]
- MP-6. Attend to precision.[20]
- MP-7. Look for and make use of structure.[21]

Bibliography

""The Greatest Shortcoming of the Human Race Is Our Inability to Understand the Exponential Function". Al Barlett on Growth and Sustainability • /r/Futurology." Reddit. Accessed January 16, 2015. http://www.reddit.com/r/Futurology/comments/2k6lcc/the_greatest_shortcoming_of_the_human_race_is_our/.
This site contains a video on exponential functions

Herstein, I. N. *Abstract Algebra*. New York: Macmillan, 1986.
A book on Abstract Alegbra useful for identifying different types of group structures

"Introduction To Functions." Webalg Pre-Calculus. Accessed January 16, 2015. http://www.math.tamu.edu/~scarboro/150TextChapter4.pdf.
A good reference for definitions and properties of functions

"School Profiles - School Profiles." School Profiles - School Profiles. Accessed January 16, 2015. http://profiles.doe.k12.de.us/SchoolProfiles/School/Default.aspx?checkSchool=284&districtCode=32.
This website provides data and statistics on Delaware High Schools relating to enrollment and test scores

"Standards for Mathematical Practice." Home. Accessed January 16, 2015. http://www.corestandards.org/Math/Practice/.
This website provides the common core state standards and mathematical processes

Wagner, Patty. "Exploring Exponential Functions." Exploring Exponential Functions. Accessed January 16, 2015. http://jwilson.coe.uga.edu/EMAT6680Fa09/Wagner/assignment1/assignment1.html.
This website provides many examples of working with exponential functions.

Weingarden, Michael. "Too Hot To Handle, Too Cold To Enjoy." Too Hot To Handle, Too Cold To Enjoy. Accessed January 16, 2015. http://illuminations.nctm.org/lesson.aspx?id=3180.
This website provides lessons on exponential functions through laboratory exercises

Notes

[1]http://profiles.doe.k12.de.us/SchoolProfiles/School/Default.aspx?checkSchool=284&districtCode=32.
[2]http://www.reddit.com/r/Futurology/comments/2k6lcc/the_greatest_shortcoming_of_the_human_race_is_our/.html
[3]Herstein, I. N. *Abstract Algebra*,4
[4]http://www.corestandards.org/Math/Practice/.
[5]Ibid,8
[6]Ibid,8
[7]Ibid,8
[8]Ibid,8
[9]Ibid.
[10]Ibid.
[11]Ibid.
[12]Ibid.
[13]Ibid.
[14]Ibid.
[15]Ibid.
[16]Ibid.
[17]Ibid.
[18]Ibid.
[19]Ibid.
[20]Ibid.

Curriculum Unit Title

Exponential Functions

Author Michelle Northshield

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Exponential functions and their characteristics

ESSENTIAL QUESTION(S) for the UNIT

What are the characteristics of an exponential function?

CONCEPT A	CONCEPT B	CONCEPT C
Functions	Exponential Functions	Algebraic Structure
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
What are the characteristics of a function?	What are the characteristics of an exponential function?	What does the product of two exponential functions look like?
VOCABULARY A	**VOCABULARY B**	**VOCABULARY C**
mapping diagram, input, output, domain, range, vertical line test	exponential, initial amount increasing, decreasing, domain, range, asymptotes	associative, commutative,

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

""The greatest shortcoming of the human race is our inability to understand the exponential function". Al Barlett on Growth and Sustainability

http://www.reddit.com/r/Futurology/comments/2k6lcc/the_greatest_shortcoming_of_the_human_race_is_our/ (accessed October 25, 2014).