Using Reasoning to Solve Problem Situations in Mathematics

Gretchen Wolfe

Introduction

I have taught first grade at Henry M. Brader Elementary School for the last twelve years. In the last four years I have watched the curriculum change and as each update is presented, the suggested activities are becoming more and more dry and lackluster and additional assessments are being required by the school district. So, along with my fellow first grade teachers, I looked for ways to make the content and the curriculum more engaging and captivating for my students. I always consulted the kindergarten teachers at my school, they were my go-to people for finding creative ways to present content and make it more fun and game-like. This year I decided to make a change. The last few years I have become intrigued with the idea of teaching mathematics and reading with children who are just beginning their school career and learning through play, so I switched from teaching first grade to full-day kindergarten at Brader Elementary School.

Brader is a suburban elementary school, in the Christina School District, educating kindergarten through fifth grade students. In first grade I taught in a self-contained inclusion classroom with push-in and pull-out support for my special education students. I am currently teaching kindergarten for the first time this school year. I am in a self contained classroom with pull-out support four days a week for reading intervention. In the Christina School District the Kindergarten through fifth grade core mathematics curriculum materials are MacMillan/McGraw Hill's Math Connects, used to support the teaching of the Common Core State Standards (CCSS). In kindergarten the mathematics block is 60 minutes for core instruction with an additional 30 minute block for daily intervention instruction. During the core instruction time I teach whole group lessons, breaking out into small needs-based groups, and mathematics learning center activities also take place during this time.

Rationale

This summer, as I was preparing to teach Kindergarten, I studied the CCSS and the Math Connects curriculum materials for Kindergarten. In the Math Connects program, problem solving is part of an addition and subtraction unit that is approximately four weeks in duration. There are also problem-of-the-day situations presented for daily practice opportunities. Each unit chapter also included two problem solving strategy lessons. There are few opportunities for the children to compose word problems in kindergarten. I see a need for additional work in analyzing a problem before solving it in Kindergarten.

During my years teaching first grade, I have watched children struggle with problem solving. Solving word problems (or story problems as they are often referred to in the primary grades), in particular seems to be daunting for the children. I have found that children who can perform addition and subtraction operations with proficiency and fluency often have difficulty solving word problems. Some children have difficulty with how to attack a word problem, they are confused by the information within the word problem and cannot determine which operation to use to find the solution. Some children look for key words to help them identify the operation. This strategy can backfire as word problems can be multi-step and contain several key words, which confuses the children more. I have found that children will sometimes simply combine all the numbers in a word problem when they are having difficulty determining the important information or the situation to be solved. I believe that many of these difficulties and misunderstanding could be avoided if the children would spend more time *reasoning* through a word problem and approaching it like a story to be analyzed rather than jumping to writing an equation and solving it. In writing this unit I wish to help children build their reasoning skills in kindergarten so that they do not move from reading a story problem to writing an addition equation because they are learning how to add at the time when the problem is presented or writing a subtraction equation because they are currently learning how to subtract. The children need to be able to reason through the problem to determine which operation will be used to solve the word problem, not just insert numbers into an equation without understanding why.

The Kindergarten Math Connects curriculum materials that we are currently using include opportunities for daily problem solving and two problem solving strategy lessons in each of the nine units. The children use manipulatives or picture counting to add or subtract most frequently in the beginning of the year. The story problems are often not set up as a problem to be solved. For example, one early problem solving activity directed the children to sort items into two baskets. There was no story, no importance or meaning implied - simply a task to be completed. Marilyn Burns suggests that word problems should have the following elements[1]:

1. a perplexing situation that the student understands
2. student interest in finding a solution
3. a situation where the student is unable to proceed directly toward a solution
4. a solution that requires use of mathematical ideas

I find that I must adjust or supplement the curriculum to give students an opportunity to find other ways to add, subtract, and work through word problems that have meaning and interest to prepare the children for real life problem solving. Preparation for the more intensive work of solving word problems in first grade and beyond should include working through word problems in enjoyable and meaningful ways in Kindergarten. Therefore, the need for additional practice acting out, drawing, composing, finger counting, and using objects to represent situations in real-life word problems will help the children to reason through problem solving situations. Working in groups will allow for social-collaborative learning, which is why I am developing a strategy for children to do more collaborative learning as part of this unit.

Problem Solving

In mathematics how should we define a "problem" to be solved? For this unit I will use the following definition: "any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific "correct" solution method."[2] I will not expect the children to use only one method for solving the problem. Working collaboratively, the children can try several methods for solving a problem and justifying their approach through conversation and questioning. Problem solving is a learning goal for students, teachers teach children how to solve problems in mathematics. Problem solving is a means for teaching mathematics as well.[3] I will focus on teaching addition and subtraction through problem-solving. Making the problem situations real and important for my group of students will help to foster their productive disposition.[4] First the children will act out real world problems and use manipulatives to represent quantities. Next the children will move to counting on fingers and drawings to represent quantities. Then the children will move to using words and pictures to solve problems presented verbally. In my unit I plan to provide opportunities for students to solve *and* compose word problems using a variety of strategies while working in small groups. The children will work with real-life, concrete experiences, moving to symbolic problems.[5]

Addition and Subtraction Situations

There are 14 types of addition and subtraction situations included in the CCSSM (see Appendix A).[6] They are categorized by situations that include change, comparison, or part-part whole.[7] It is recommended that in first grade, mathematics word problems should be presented in varied types: change, comparison, and part-part whole.[8] Kindergarten focuses on providing the change and part-part-whole types of problems. Looking through the practice problems in my math curriculum materials, I see that a majority of the practice problems are change - result unknown. It is important to include each type of change and part-part-whole problem in the students' practice activities so children do not become overly familiar with only one type of problem.

Objectives

My unit will address the following CCSS standards for processes and operations and algebraic thinking:

Reason abstractly and quantitatively.

> Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems

> involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.[9]

Understand addition, and understand subtraction.

CCSS.MATH.CONTENT.K.OA.A.1

Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

CCSS.MATH.CONTENT.K.OA.A.2

Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

CCSS.MATH.CONTENT.K.OA.A.3

Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

CCSS.MATH.CONTENT.K.OA.A.5

Fluently add and subtract within 5.

Enduring Understandings

Mathematical problem solvers apply a variety of strategies and methods to solve problem situations.

The language of mathematics is communicated through symbols used to represent and describe relationships.

Essential Questions

The essential questions addressed in this unit are:

- How do I determine the best method to solve the given situation?
- Why do I need mathematical operations?
- How do mathematical operations relate to each other?
- How do I know which mathematical operation to use?
- How do I know which materials to use to help me problem solve?

Content

Sets

During my participation in the seminar Using Abstract Reasoning: From Counting Fingers to Solving Challenging Puzzles, we began our discussions about better understanding how to teach children abstract reasoning by first defining number sets. An understanding of number sets will set the stage for understanding operations and functions. We defined the sets as follows:

- Whole Numbers = {0, 1, 2, 3, 4, 5, 6, 7…} these are the counting numbers and zero
- Integer Numbers = {… -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6…} these are the counting numbers, zero, and the negative of the counting numbers.
- Rational Numbers = these are any numbers made by diving one integer by another.
- Irrational Numbers = these are real numbers that cannot be expressed at a ratio between two integer numbers or written as a simple fraction because the numbers in the decimal go on forever without repeating, e.g., Pi.
- Real Numbers = this includes natural, integers, rational, and irrational numbers.
- Imaginary Numbers = are numbers that give a negative result when squared.
- Complex Numbers = (a+bi) where a and b are real numbers and i is the imaginary number, these are the combination of real numbers and the imaginary numbers.

In this unit my students will be working with the numbers in this set of Whole Numbers {0, 1, 2, …20}.

Operations

The operation is the process part of arithmetic. The operations that I will focus on in this unit are both addition and subtraction. The properties of addition are associativity, commutativity, and identity. These operations are defined as the following:

- The commutative property of addition tells us that order does not matter when you add: a+b=c then b+a=c
- The associative property of addition tells us that when we add more than two numbers, it does not matter how the numbers are grouped: (a+b)+c= a+(b+c)
- The identity property states that any number plus zero equals the original number: a+0=a = 0+a=a

The operation of subtraction does not have the properties of addition when we focus on the numbers 0 to 20. Instead, the operation of subtraction has the following properties:

- The identity property of subtraction translates into zero subtracted from any number equals the original number and any number subtracted from itself equals zero: a-0=a and a-a=0
- The equality property states that when both sides of an equality have the same number subtracted from them, the two sides remain equal: a=b, then a-c=b-c
- Subtraction is not commutative within the set of {0, 1, 2, 3…20} a-b is not equal to b-a
- Subtraction is not associative in the way addition is associative. (a-b)-c is not equal to a-(b-c) although terms can be grouped together in the following way: (a-b)-c = a-(b+c)

Closure of the operation means that if we add two numbers in the set the answer will also be a number in the set. Addition and subtraction operations with this set are not closed operations.

There are three defining properties of the equivalence relationship: symmetry, reflexivity, and transitivity.

- Symmetry: if a=b then b=a
- reflexivity: a=a
- Transitivity: if a=b and b=c then a=c

Often children in kindergarten begin to build an understanding of the equals sign and because the sign is used in addition and subtraction equations the children often impose a meaning upon the symbol of "insert answer here" rather than equals.[10] For children to be able to understand that an equal sign is not just the symbol that comes after the numbers being added or subtracted, teachers should use terms such as "justify the relationship" or introduce the equals sign with symbols before using it in addition equations.[11]

As children are introduced to these operations and their properties, children will come to understand that addition is about combining and subtraction is about difference. We can build the understanding of the relationships between addition and subtraction so that subtraction comes to be known as reversing the actions involved in addition.

Function

In seminar we defined a mathematical function as a relationship between two sets having three components: one set, another set, and the relationship. In this unit the function is taking two numbers from the set {0, 1, 2 …20} and having an output. The input for function is any number within the set, the output could be from the same set or outside the set when conducting operations such as repeated addition, doubling a number, or ordering numbers to tell which number is greater: x+ 3= x +2+1. For the output to be within the set, I restricted the domain, such that the output stays within the set {0, 1, 2, 3…20}.

Abstract Reasoning

To be able to reason through a problem situation the children need to see and understand relationships. Providing reasons and justifications for answers can and should begin in kindergarten. Reasoning includes justification for methods of problem solving and strategy choices. My current curriculum program begins problem solving instruction with acting out practice. Acting out a situation is an effective way to help children see relationships. Author Marilyn Burns identifies the problem solving strategies most often employed in the primary grades as[10]:

look for a pattern
construct a table
make an organized list
act it out

draw a picture
guess and check
work backward
write an equation
solve a simpler (or similar) problem
make a model

Of the problem solving strategies listed above, I plan to begin my kindergarten students' problem solving journey with the acting it out strategy and move into drawing a picture. I want the children to think about and experience the situation before they begin to put pencils and crayons to paper to show their thinking. After the children are experienced with acting and drawing situations, I will then introduce making a model, constructing a table, and looking for a pattern. After we delve into our addition unit, I will introduce the writing an equation strategy. I want the children to spend time reasoning and working with problem situations before they are required to write an equation to represent their thinking.

Strategies

I try to use a variety of teaching strategies daily to keep students engaged. Some of the strategies used in this unit are:
Cooperative learning - in cooperative learning, students work together in heterogeneous teams to master the material presented. Three concepts are integral to successful cooperative learning: individual accountability, equal opportunities for success, and team rewards.
Learning Centers- these are activities set up around the classroom where small groups of students work independently on various skills using familiar instructional materials.
Work Stations - these are areas of the classroom where students work on activities to reinforce skills or concepts previously taught, either alone or in small groups, using instructional materials, puzzles, and games.
Direct Instruction - teacher-led presentation of content to students through demonstration, lecture, or viewing/listening to other media sources.

The largest strategy focus for this unit will be the formation of Mathematics Circles. I have used literature circles in my English Language Arts instruction for many years now. In Literature Circles, children form temporary student led discussion groups to develop deep understanding of a piece of literature. I began to think that forming groups to work in a similar fashion to solve word problems might be an effective strategy for mathematics instruction as well. It will allow students to spend more time talking, reasoning, and sharing their thinking.

Mathematics Circles

I will define Mathematics Circles as small, temporary discussion groups where student members take on different roles while working together to solve problems. When group work is completed the groups share their process, thinking, and solutions with the whole class. The first step to beginning a Mathematics Circle is to have the students practice doing each of the jobs that they could be assigned in the groups (see Appendix B). As you introduce a word problem to the class, begin by assigning every student the job of *Detective*. Have them "locate" the important information in the word problem. Once the children have demonstrated an understanding of that job continue to introduce the other jobs, one at a time. Allow all students to have practice with each job over a period of time. I have listed some possible jobs and titles below. This is not a comprehensive list, jobs can be deleted and added as needed depending on the needs of your students.

Student jobs and job titles:
Director - is the person responsible for organizing the work and group discussion
Newscaster - is the person responsible for sharing out and summarizing the problem solving work done by the group.
Detective - is the researcher of the group, the person responsible for locating the important information (within the word problem) necessary to understand and solve the problem.
Illustrator - is the person responsible for mapping the problem solving process and solution with pictures and/or words.
Doctor - is the person responsible for determining which strategies and operation(s) will be used to solve the problem.

Once the children have all had practice and have built an understanding of how to work in teams of small groups, you can create heterogeneous groups of four to five students. Next, assign one of the jobs to each student in the group so that each group will have a *Doctor, Director, Illustrator, Detective,* and *Newscaster*. A student can double up on jobs on days when groups are short members. Assigning jobs can be a thoughtful process where a student is assigned a job for additional practice with that skill if he or she is struggling or students could be allowed "free choice" to pick one of the jobs to do during circle time.

Word problems can be presented whole group or each group can be given a different problem to solve depending on the focus of the lesson. When each group is presented with the same problem to solve, many different strategies for solving can be discussed and all students will be engaged in the sharing and discussion. If different problems are presented discussions can be rich, as students in different groups may have suggestions for different strategy use. The quantity of word problems presented during one mathematics circles session should be limited. The idea of the circles is to allow for deep thinking and discussion, not to complete a certain number of word problems in a given period of time.

One Mathematics Circles session will last 30 to 45 minutes, including time for discussion and debriefing at the end. During Mathematics Circles the students should be allowed access to any sort of manipulatives they have used in class, however the most important part of the circle time is the discussion within the group and the reasoning before the manipulative use and drawing begin.

Activities

This unit is designed to be taught throughout the school year, not during one specified period of time. The development of Mathematics Circles will begin in the late fall with my kindergarten students, after procedures for group work have been taught, rehearsed, and closely monitored. Work with problem solving strategies will begin in the early fall and strategies will be slowly introduced throughout the whole year.

Lesson 1

Objectives: The students will decompose numbers less than or equal to 10 into pairs in more than one way, by using objects or drawings, and record each decomposition by a drawing or writing an equation.

Essential Questions:
- How do I determine the best method to solve the given situation?
- How do I know which materials to use to help me problem solve?

Activity: Allow children to work in pairs, small groups, or mathematics circles to foster discussion and reasoning. Assign the students a number and challenge them to work together to find as many different combinations as they can using two parts (part-part-whole, parts unknown).

Prompts for discussion: How many different combinations did you discover? How did you determine the best way to solve the situation? Which materials did you use to help you problem solve? Why did you choose *those* materials?

Lesson 2

Objectives: The students will solve addition and subtraction word problems, and add and subtract within 10. The students will represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.

Essential Questions:
- How do I determine the best method to solve the given situation?

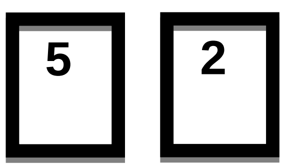
• Why do I need mathematical operations?
• How do mathematical operations relate to each other?
• How do I know which mathematical operation to use?
• How do I know which materials to use to help me problem solve?

Activity: Present a story problem to the children and have them work in their mathematics circles to problem solve. To help engage students use student names in the situation. You can present the situation in a variety of ways, below the situation is presented as join - change unknown.

Marisa and Maya collected shells on the beach and put them in a bucket. Marisa put 3 shells in the bucket, Maya put some shells into the same bucket. Now there are 5 shells in the bucket. How many shells did Maya put in the bucket?

The situation could also be presented with two number cards. Challenge the children to determine which number is the total and which number is the amount of shells Maya put into the bucket and have them justify their answers.

Marisa and Maya collected shells on the beach and put them in a bucket. Marisa put 3 shells in the bucket, Maya put _____ shells into the same bucket. Now there are ____ shells in the bucket.



Prompts for discussion: Which operation did you use to solve this situation? How did you determine the best way to solve the situation? Why did you add or subtract? How are addition and subtraction number sentences (operations) that the groups chose related to each other? Which materials did you use to help you problem solve? Why did you choose *those* materials?

Lesson 3

Objectives: The students will solve subtraction word problems, and subtract within 5. The students will represent subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations. The students will add and subtract fluently within 5.

Essential Questions
• How do I determine the best method to solve the given situation?
• Why do I need mathematical operations?

- How do I know which mathematical operation to use?
- How do I know which materials to use to help me problem solve?

Activity: Present a story problem to the children and have them work in their mathematics circles to problem solve. To help engage students use student names in the situation. The situation below is presented as separate - initial unknown.

Maya had some pencils. She gave two to Marisa. Now Maya has three pencils left. How many pencils did Maya have to begin with?

Prompts for discussion: Which operation did you use to solve this situation? How did you determine the best way to solve the situation? Why did you add or subtract? How are addition and subtraction number sentences (operations) that the groups chose related to each other? Which materials did you use to help you problem solve? Why did you choose *those* materials?

Lesson 4

Objectives: The students will solve subtraction word problems, and subtract within 10. The students will represent subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.

Essential Questions
- How do I determine the best method to solve the given situation?
- Why do I need mathematical operations?
- How do I know which mathematical operation to use?
- How do I know which materials to use to help me problem solve?

Activity: Present a story problem to the children and have them work in their mathematics circles to problem solve. To help engage students use student names in the situation. The situation below is presented as separate - change unknown.

Marisa had 10 pretzels. She gave some to Maya. Now Marisa has 7 pretzels. How many pretzels did Marisa give to Maya?

Prompts for discussion: Which operation did you use to solve this situation? How did you determine the best way to solve the situation? Why did you add or subtract? How are addition and subtraction number sentences (operations) that the groups chose related to each other? Which materials did you use to help you problem solve? Why did you choose *those* materials?

Learning Center Activities

Missing Part Game: Provide children with a set of 10 counters and a work mat (Appendix C). One child places the 10 counters on the work mat separating them into two parts while a second child looks away. Then the first child folds the flap of the work mat over one of the circles, hiding one part of the set. The second child states and writes an addition or subtraction sentence. The partners switch, taking turns until all combinations have been made.[13]

Appendix A

Addition and Subtraction Situations

	Result Unknown	Change Unknown	Initial Unknown
join	5 children were playing tag. 3 more children joined the game. How many children are playing tag altogether? a+b=?	5 children were playing tag. Some more children joined the game and now there are 8 children playing. How many children joined the game? a+?=c	Some children were playing tag. 3 more children joined the game. Now 8 children are playing tag. How many children were playing before? ?+b=c
separate	8 children were playing tag. 3 children left to play on the slide. How many children are playing tag now? c-b=?	8 children were playing tag. Some children left to play on the slide. Now 5 children are playing tag. How many children left to play on the slide? c-?=a	Some children were playing tag. 3 left to play on the slide. Then there were 5 children playing tag. How many children were playing tag before? ?-b=a
	Whole Unknown	Part Unknown	Both Parts Unknown
part-part-whole	5 boys and 3 girls are playing tag. How many children are playing tag altogether? a+b=?	8 children are playing tag. 5 are boys the rest are girls. How many girls are playing tag? a+?=c c-a=?	5 children are playing recess. How many could be boys and how many could be girls? ?+?=c
	Difference Unknown	Larger Unknown	Smaller Unknown

compare	5 children are playing tag. 3 children are playing on the slide. How many more children are playing tag than on the slide? a+?=c 5 children are playing tag. 3 children are playing on the slide. How many fewer children are playing on the slide than are playing tag? c-a=?	2 more children are playing tag than playing on the slide. 3 children are playing on the slide. How many children are playing tag? a+b=? 2 fewer children are playing on the slide than are playing tag. 3 children are playing on the slide. How many children are playing tag? b+a=?	2 more children are playing tag than playing on the slide. 5 children are playing tag. How many children are playing on the slide? c-b=? 2 fewer children are playing on the slide than are playing tag. 5 children are playing tag. How many children are playing on the slide? ?+b=c

Adapted from CCSS

Appendix B

Mathematics Circles
Student Jobs

Director
Asks: How should we begin?
Helps team talk about their ideas. Guides the group to complete their task.

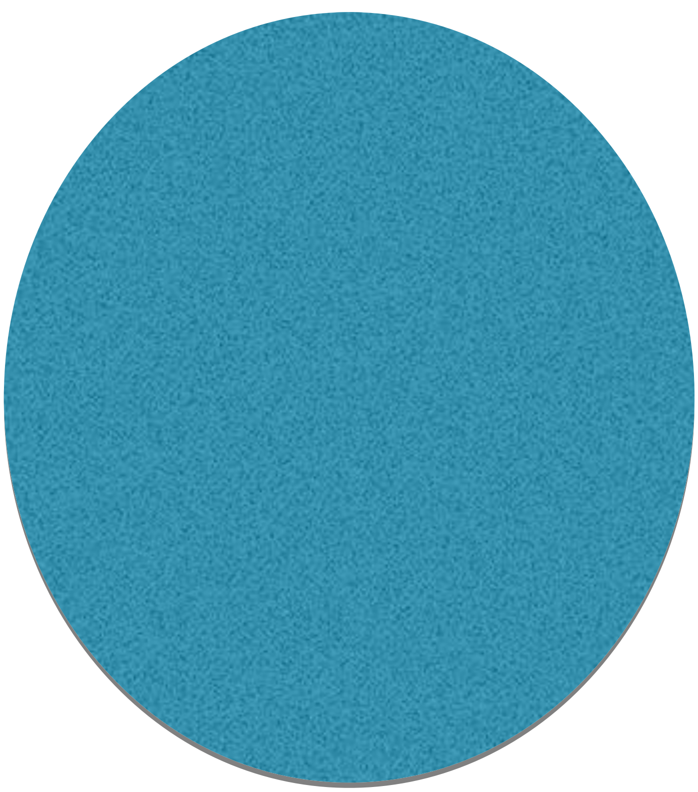
Detective
Asks: What is the important information? What are we trying to find out?
Locates the information needed to solve the problem.

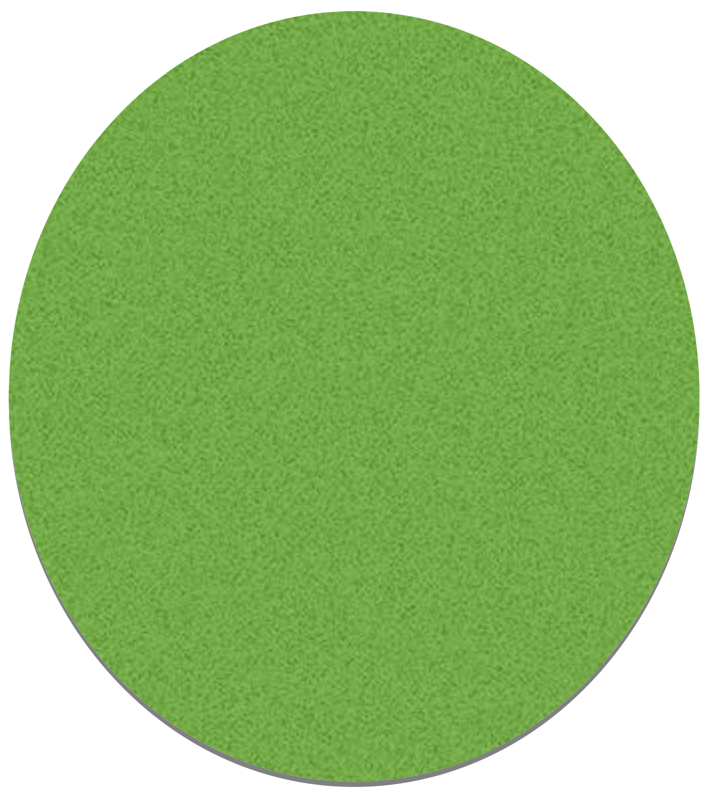
Illustrator
Asks: How will this idea look?
Draws or writes the group's ideas.

Doctor
Asks: Which operation do we need to use to solve the problem?
Looks for the most efficient way to solve the problem.

Newscaster
Asks: What steps did we take to solve the problem.
Tells what the group did to solve the problem and reports the solution to the class.

Appendix C
Work Mat





Appendix D

Common Core Standards for Mathematics
Understand addition, and understand subtraction.

Standard	Lesson
CCSS.MATH.CONTENT.K.OA.A.1	
Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	Lesson 2 The students work to solve and record their thinking for a join - change unknown word problem. Lesson 3 The students solve and record their thinking for a separate - initial unknown word problem. Lesson 4 The students solve and record their thinking for a separate - change unknown word problem.
CCSS.MATH.CONTENT.K.OA.A.2	
Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.	Lesson 2 The students solve a join - change unknown word problem. Lesson 3 The students solve a separate - initial unknown word problem. Lesson 4 The students solve a separate - change unknown word problem.
CCSS.MATH.CONTENT.K.OA.A.3	
Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).	Lesson 1 The students try to find every combination pair of a number less than or equal to 10.
CCSS.MATH.CONTENT.K.OA.A.5	
Fluently add and subtract within 5.	Lesson 3 The students subtract to solve a separate - initial unknown problem.

Endnotes

[1]Burns, Marilyn. *About teaching mathematics: a K-8 resource*, 17.
[2] Van De Walle, John A.. Elementary and middle school mathematics: teaching developmentally, 5E, 37.
[3]Ibid, 37.
[4]Kilpatrick, Jeremy, Jane Swafford, and Bradford Findell. *Adding it up: helping children learn mathematics, 10.*
[5]Baroody, Arthur J., Michael Eiland, and Bradley Thompson. "Fostering At-Risk Preschoolers' Number Sense." 84.
[6]http://www.corestandards.org/assets/CCSSI_Math Standards.pdf. 6.
[7]Howe, Roger. "Three Pillars of First Grade Mathematics, and Beyond.” 185.
[8]Ibid, 185.
[9]http://www.corestandards.org/assets/CCSSI_Math Standards.pdf. 88.
[10]Baroody, Arthur J., and Herbert P. Ginsburg. "The Effects of Instruction on Children's Understanding of the "Equals" Sign.” 198
[11]Ibid, 211.
[12]Burns, Marilyn. *About teaching mathematics: a K-8 resource*, 19.
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Van De Walle, John A.. Elementary and middle school mathematics: teaching developmentally. 5th ed. Boston: Allyn and Bacon, 2004.

This is a comprehensive text for elementary through middle school mathematics teachers. It addresses planning, teaching, assessment, and technology use in the mathematics classroom.

Curriculum Unit Title: Using Reasoning to Solve Problem Situations in Mathematics

Author: Gretchen Wolfe

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Students will work in Mathematics Circles to collaboratively solve addition and subtraction problem situations.
Mathematical problem solvers apply a variety of strategies and methods to solve problem situations.
The language of mathematics is communicated through symbols used to represent and describe relationships.

ESSENTIAL QUESTION(S) for the UNIT

How do I determine the best method to solve the given situation?
Why do I need mathematical operations?
How do mathematical operations relate to each other?
How do I know which mathematical operation to use?
How do I know which materials to use to help me problem solve?

CONCEPT A

Decomposing numbers.

ESSENTIAL QUESTIONS A

- How do I determine the best method to solve the given situation?
- How do I know which materials to use to help me problem solve?

VOCABULARY A

part
whole

CONCEPT B

Solving addition problem situations.

ESSENTIAL QUESTIONS B

- How do I determine the best method to solve the given situation?
- Why do I need mathematical operations?
- How do mathematical operations relate to each other?
- How do I know which mathematical operation to use?

VOCABULARY B

join
justify
equal
sum

CONCEPT C

Solving subtraction problem situations.

ESSENTIAL QUESTIONS C

- How do I determine the best method to solve the given situation?
- Why do I need mathematical operations?
- How do I know which mathematical operation to use?
- How do I know which materials to use to help me problem solve?

VOCABULARY C

separate
justify
equal
difference

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

www.mathwire.com

www.funbrain.com