Proof and Reasoning with Fractions

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Introduction

Teaching is my passion. Every August, I am excited to meet my new group of students. This year is no different. I have been fortunate enough to have taught at only two schools during my twenty four years of teaching, all in the same district. For nineteen of those years, I have taught at West Park Place Elementary School. During those years, I have taught both third and fourth grade, the latter being my position for the last eight years. Fractions are a major part of the math curriculum in grades 3-5. This unit is to help strengthen and build upon student understanding of fractions.

West Park Place Elementary School is part of the Christina School District. The Christina School District is the largest public school district in the state of Delaware. It is comprised of approximately 17,400 students and 2,500 employees. The district includes the City of Newark and its surrounding suburban areas as well as part of the City of Wilmington. Christina School District has 18 elementary schools, 4 middle schools, and 3 traditional high schools. It also includes the Delaware School for the Deaf, the Delaware Autism Program, Douglass Alternative School, REACH and Options Programs, Networks Career Training Program and the Sarah Pyle Academy. West Park is located in Newark, Delaware and is within walking distance of the University of Delaware. It is a small school of approximately 370 students from kindergarten to fifth grade. West Park is home to many programs that contribute to the diversity of the school: Delaware Autistic Program (DAP), Realistic Educational Alternatives for Children with Disabilities (REACH), English Language Learners (ELL), and Montessori Academies at Christina (MAC). According to the school profile for 2014-2015 school year as reported by the Delaware Department of Education, about 20% of West Park's population are English Language Learners (ELL), representing many different cultural backgrounds. West Park's population is comprised of 22% African American, 19% Asian, 10% Hispanic, 45% White and 4% Multi-Racial. In addition, special education students with the exception of REACH and DAP are integrated into the regular classroom setting. Special education students make up 12% of the West Park population.

Since we are such a small school, our fourth grade team is made up of two classrooms. I am the team leader for fourth grade and teach all subjects: reading, writing, math, science and social studies. For the past two years, I have also been involved with a Math Teacher Leader project for the Christina School District. The group is comprised of elementary, middle and high school math teachers and elementary and middle/high school math specialists. This program has spurred my interest in delving deeper into math, helping students to grasp concepts more deeply and inspire a love of math in my students.

Rationale

Through my work with Math Teacher Leader Project and West Park's Common Core Guiding Team, I have taken a closer look at my own teaching and the struggles of my students. How can I help them make the connections? How can I develop the 8 mathematical practices in my classroom to develop confident problem solvers? How can I get students to defend their thinking and question the thinking of others? I want my students to question why, not just accept math as that's just how you do it or that's just the way it is. It is when we begin to question and reason that we begin to understand. I find in fourth grade that when you ask a student to explain their thinking in math, they tell you the steps that they followed to solve a problem. They have a difficult time explaining why they solved it that way or what their reasoning behind it was.

Research supports the conclusions that memorization, drill and templates are less likely than discussion, projects and teamwork to create lasting skills or deep understanding.[1] Furthermore, students who think about what they are doing and why they are doing it are more successful that those who just follow the rules they have been taught.

In developing an idea for my unit, I decided to focus on fractions. Fractions and decimals are an integral part of the third-fifth grade math curriculum. With the CCSS shifts, fractions are introduced in grade 1 as partitioning rectangles or squares into two or four equal shares and describing the shares using vocabulary such as halves, fourths, and quarters. In grade 2, the concept of thirds is added. In grade 3-4, students work with fractions within other shapes and other quantities. They compare and order fractions, represent them on a number line, explain the equivalence of fractions or compare fractions by reasoning about their size, and express whole numbers as fractions. In grades 4-5 students begin adding, subtracting and multiplying fractions. Division of fractions is introduced in grade 5. The relationship between decimals and fractions is also introduced in grade 4. Skills are continually built upon; without a strong understanding of fractions and decimals by the end of fourth grade, students struggle to keep up in the upper grades. Through my experiences in teaching third and fourth grade, I have also noticed the role multiplication and division play in understanding of fractions. Students who have a strong grasp of their multiplication and division facts and fact families are more successful than the students who are still struggling to learn their facts. Students need to have mastered this content in order to have the number sense and multiplicative thinking that is necessary for a deep understanding of fractions. It is my goal through this unit, which is primarily to be used with fourth grade students but could be adapted to support third or fifth grade students, that students are able to reason and justify their thinking relating to fractions.

The unit will address writing standards and speaking and listening standards in addition to the math standards. Writing 4.1, Writing 4.10, Speaking and Listening 4.1, and Math 4.NF.1 through 4.NF.4. In addition, activities/lessons throughout the unit will address the standards for mathematical practice.

Content

In my experience teaching third and fourth graders about fractions, students rely on rules. The struggle occurs when students cannot remember the rules or when the rules will not work in a particular situation. Students begin experiences with fractions in earlier grades and more formal instruction begins in third grade. Many students have difficulty remembering what they learned in previous years. They are not retaining the information. In speaking with middle and high school mathematics teachers in my seminar, students still have difficulty with fraction skills/concepts in the upper grades. Students do not understand the underlying principles of fractions.

Reasoning

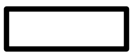
Without the deep understanding, students cannot reason in relation to fractions. The word *reasoning* indicates that we use common sense, good judgement and a thoughtful approach to problem solving, instead of pulling numbers from word problems and applying rules and operations without much thought to what the problem is really asking. Typically, reasoning is associated with mental, free-flowing thought processes in which the relationship among quantities is analyzed instead of following rules or mechanical procedures.

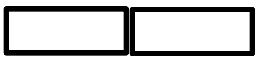
Rational Numbers

Rational numbers are represented in the form $\frac{a}{b}$. Proportionality is a key part of rational numbers. If a student cannot reason proportionally, they do not understand rational numbers. A key problem in elementary mathematics instruction relating to fractions is that students are taught the whole-part relationship and then begin operations involving fractions. This does not provide elementary students the time and opportunity to develop important ideas relating to fractions.

The Unit

One of the first understandings with fractions that students need is the concept of unit. In early elementary years, students learn to count by matching a number with an amount of objects in a set being counted. The unit, “one”, always meant a single object. With fractions, students have to develop the understanding that the unit can change. It can be more than one object or it can be a packaged amount. For example, a single granola bar can represent a unit (simple unit) or a package of four granola bars can be a unit (composite unit). A unit can look like some of the following illustrations.

 If this is one unit or one whole, then this would be $\frac{1}{2}$. 

 If this is one unit or one whole,

then this would be $\frac{1}{2}$. 

As you can see, the two representations of $\frac{1}{2}$ are very different in size. The size of the fraction is dependent on the size of the unit. This is very confusing for students and is something that students need experience in working with units or various sizes.

What Is A Fraction?

Part of the confusion students face, stems from the understanding of what is a fraction. A fraction is a form for writing numbers. It involves two integers with a bar between them. It is also a non-negative rational number. A fraction symbol can be used to represent a value that is not a rational number such as $\frac{-3}{4}$ or $\frac{\pi}{2}$. Furthermore, 0 can be in the numerator but not the denominator. Fractions and rational numbers are not the same thing but rather fractions are a subset of rational numbers. All rational numbers can be written as fractions.

Proportional reasoning

Proportional reasoning requires students to analyze and explain proportional relationships. It is the deep mathematical thinking that begins in late elementary school years and grows throughout middle and high school. It solidifies the elementary concepts and opens their minds to more complex mathematical ideas and concepts. Without the strong understanding of rational numbers and the ability to reason proportionally, students will struggle with the more complex mathematical ideas.

One of the most important types of thinking for proportional reasoning is to explain or analyze change in absolute and relative terms. Absolute terms refers to additive thinking. To explain this idea, think of 2 animals. Animal A is 6 feet tall. Animal B is 8 feet tall. After 2 years have passed, Animal A is 10 feet tall and Animal B is 12 feet tall. If we question after two more years, will both animals grow the same amount, we can respond in both absolute and relative terms. In absolute terms, we see that after two years, both animals have grown 4 feet. We are just looking at the growth without it depending on anything else. We see that 6+4=10 and 8+4=12. The have both added the same amount of height. Relative terms refers to multiplicative thinking. To respond in relative terms, we can relate the growth to its present length. Animal A has grown 4 feet or 4/6 of its original length. Animal B has grown 4 feet or 4/8 of its original length. Since $\frac{4}{6} > \frac{4}{8}$, they will not grow the same amount based on their original sizes. Proportionally, Animal A will grow more. Another example of absolute and relative terms is there are 3 boys and 6 girls. In absolute or additive terms, there are 3 more girls than boys (added 3). In relative or multiplicative terms, there are twice as many girls as boys (multiplied by 2). Neither viewpoint is right or wrong. They are just two ways to look at a situation; however, in order to develop more powerful thinking, students need to see beyond absolute terms and be able to think in relative terms.

Relative thinking is involved in the understanding of several important ideas in mathematics instruction. One idea is the relationship between the size of pieces and the number of pieces. Another is the need to compare fractions to the same unit. Furthermore, the meaning and the size of a fractional number are dependent on relative thinking. Finally, the relationship between equivalent fractions and equivalent fraction representations depend on relative thinking for a deep understanding.

Quantities That Are Not Quantified

Quantities that are not quantified are another important concept to promote understanding and reasoning with fractions. A quantity is a measurable amount. When something is not quantified, the measurement or amount is not given. For example, you can compare two trees without measuring them if they are positioned next to each other. You can visually compare to see which one is taller. Additionally, if a group of adults share some pizzas, we can reason that if more adults share less pizzas the next day, then each adult will receive less pizza. On the other hand, if fewer adults share more pizza, then each adult would receive more pizza. We can use this idea to help children compare fractional amounts. If we are to determine which of the following fractions is larger, $\frac{5}{9}$ and $\frac{6}{7}$, we can use the above situation. There are 5 pizzas being shared by 9 adults. The next day, there were 6 pizzas being shared by 7 adults. There were more pizzas being shared by fewer adults so they each received more pizza. This means $\frac{5}{9} < \frac{6}{7}$. An additional example for students to compare two fractions is yesterday I watered some plants with some water. Today I watered more plants with less water. Which day did the plants receive more water? The denominator always needs to be the number of people or things sharing the quantity (people and plants) and the numerator needs to be the amount that is being shared (pizzas or water). We can compare the fractions $\frac{5}{15}$ and $\frac{3}{17}$ using such an example. Yesterday I used 5 cups of water to water 15 plants. Today I used 3 cups of water to water 17 plants. It is important for students to reason with relationships before we introduce operations with fractions to students.

Representations of Fractions

Students need to develop the many different meanings behind a single fraction through various activities to develop the deep understanding of fractions and the ability to reason. A single fraction such as $\frac{3}{4}$ can have numerous meanings and representations. One such meaning is in relation to time. We can represent 45 minutes as $\frac{3}{4}$ of an hour. Another representation can be a quantity of a group of single units such as: John and his 2 friends ordered 3 large pizzas- one pepperoni, one sausage and one cheese. They divided each pizza into 4 equal parts and they ate one piece of each of the pizzas altogether. They ate 3 (1/4 pizzas). The unit could be a package or group that contains individual parts. For example, a package of cupcakes comes 3 to a package. You cut the package into 4 equal pieces before opening it and then ate one portion. You ate (1/4) of 3 cupcakes. The fraction could also mean a ratio. The following situation explains such a situation. For every 3 red m&m's in a package, there are 4 blue. The ratio is 3:4 or ¾ (3 red to 4 blue). There are many more meanings behind the fraction ¾. These are just a few. Developing these ideas of various meanings and representations behind a single fraction will help to develop the deeper understanding.

As we have discussed in seminar, to understand fractions deeply, students need to develop the knowledge that fractions can be represented as different ideas. A fraction can be a measurement, a quotient, an operator, a ratio, and even a part-whole relationship. Each of these ideas are explained below.

Fraction as a Measurement

If $\frac{a}{b}$ is a measure, a represents the # of units and b represents the equal units. Look at my representation of a number line below in Figure 1. If 0 to 1 represents 1 inch, the inch has been divided into 4 equal sections. The unit is $\frac{1}{4}$ inch.

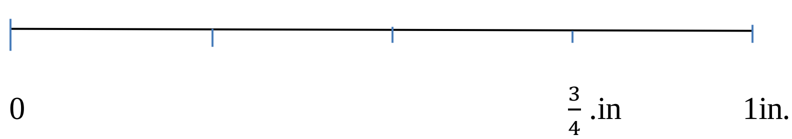


Figure 1

Fraction as a Quotient

The following problem illustrates the idea of a fraction as a quotient. 4 children equally share 3 pizzas. How much does each get?

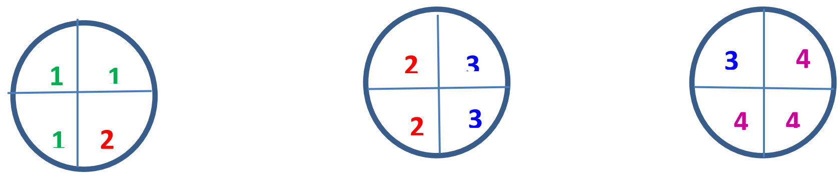


Figure 2

As Figure 2 above shows, $\frac{3}{4}$ of a pizza is the quotient of 3 pizzas divided by 4 children. ($\frac{a}{b}$ is the quotient of a divided by b). Each child receives three $\frac{1}{4}$ slices of pizza.

Fraction as an Operator

To show this concept, examine the following problem: In a party, $\frac{3}{4}$ of the party goers were females. If a total of 12 people went to the party, how many females were at the party? $\frac{a}{b}$ is like a function. You are finding $\frac{3}{4}$ of 12 or $\frac{3}{4}$ x 12. In the illustration below (Figure 3), each little circle represents one of the 12 party goers. The little circles within the large circle represent the amount that were females.

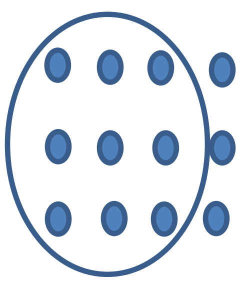


Figure 3

Fraction as a Ratio

A ratio deals with two quantities which could be parts and wholes, parts and parts or 2 wholes. The following is an example in which a ratio can be recorded as a fraction: At Appoquinimink High School, the male and female students have a 3 to 4 ratio. This can be represented by the fraction $\frac{3}{4}$. For every 3 male students, there are 4 female students.

Fraction as Part-Whole

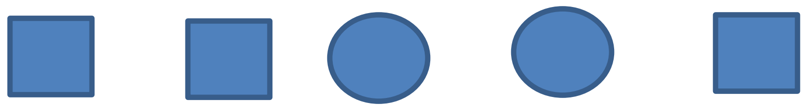


Figure 4

From the above image (Figure 4), we can see fractions such as $\frac{3}{5}$ or $\frac{2}{5}$ with 5 being the whole. Before determining the fraction, students need to determine what the whole is.

All of the above situations are essential understandings for students to develop before students are able to work with fractions and operations with fractions.

Why Are Fractions Hard to Understand?

There are many reasons why children and adults have difficulty with fractions. In order to help students develop “fraction sense”, we need to first understand some of these reasons. First, students need to be able to think about numbers in a different way than when they are working with whole numbers. For example, the number such as 23 represents a specific quantity. The number 2 and 3 represent the values 20 and 3 based on their position in the number. When we use the same digits, 2 and 3, in a fraction such as $\frac{2}{3}$, the 2 and 3 represent a relationship. Even though the relationship between the numerator 2 and the denominator 3 doesn’t change across contexts, the way the fraction is represented across those contexts does. The following illustrations, show my point.

When considering $\frac{2}{3}$ as a number the 2 represents two one-thirds, and the whole, or unit is one.

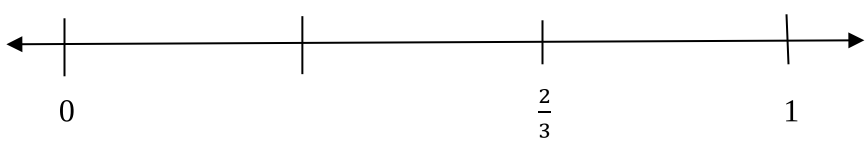
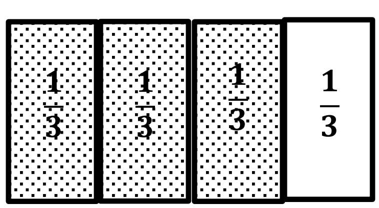


Figure 5

When considering $\frac{2}{3}$ as part of an area, the 2 represents two replications of the area that is one-third of the whole.



$\frac{2}{3}$ of the rectangle is shaded.

Figure 6

When $\frac{2}{3}$ is considered as a measure, the 2 is two repetitions of the distance that is one-third of the whole.

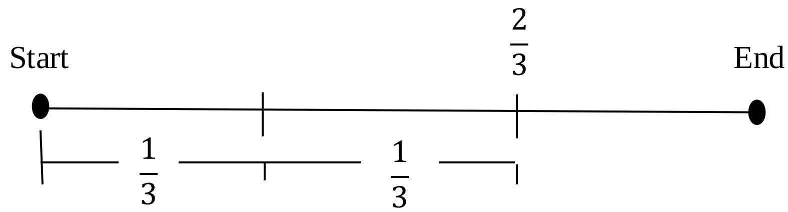


Figure 7

When $\frac{2}{3}$ is considered as part of a set, the 2 could mean two items, four items, twelve items, and so on, depending on the size of the entire set.

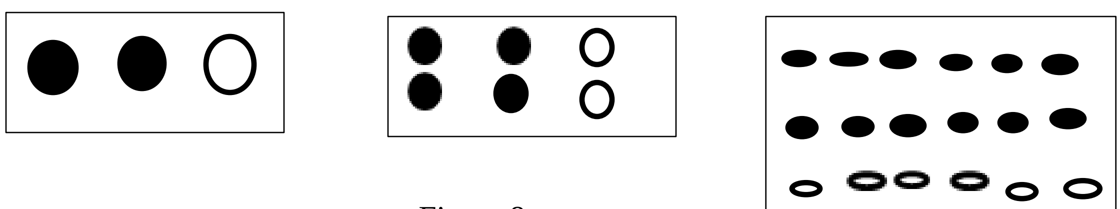


Figure 8

Research has identified several factors that likely contribute to students' difficulties with fractions, including but not limited to:

1. The way that fractions are written.
2. Classroom practices designed to help students make sense of fraction values and notation that inadvertently mask the meaning of fractions.
3. Students' overreliance on whole number knowledge.
4. The many meanings of fractions, such as measure and ratio.[2]

Essential Strategies for Supporting Fraction Sense

Through my research for this unit, I gained a great deal of understanding of the difficulties that students face in regards to fractions. I also learned how students needed to be able to think and that they needed to develop "fraction sense". Now I needed to figure out how to develop that sense in my students. What activities or strategies could I use to foster this type of thinking with my students? I came across ten essential strategies to implement in my classroom. [3]

1. Provide opportunities for students to work with irregularly partitioned, and unpartitioned, areas lengths, and number lines.
2. Provide opportunities for students to investigate, assess, and refine mathematical "rules" and generalizations.
3. Provide opportunities for students to recognize equivalent fractions as different ways to name the same quantity.
4. Provide opportunities for students to work with changing units.
5. Provide opportunities for students to develop their understanding of the importance of context in fraction comparison tasks.
6. Provide meaningful opportunities for students to translate between fraction and decimal notation.
7. Provide opportunities for students to translate between different fraction representations.
8. Provide students with multiple strategies for comparing and reasoning about fractions.
9. Provide opportunities for students to engage in mathematical discourse and share and discuss their mathematical ideas, even those that may not be fully formed or completely accurate.
10. Provide opportunities for students to build on their reasoning and sense-making skills about fractions by working with a variety of manipulatives and tools, such as Cuisenaire rods, Pattern Blocks, Fractions Kits, and ordinary items from their lives.

The last two strategies are not specific to fraction instruction and are essential instructional techniques regardless of the mathematical content you are teaching.

Classroom Activities

I used a combination of activities modified from Investigations in Number, Data and Space, Math Connects and Beyond Pizzas and Pies to create my classroom activities. I really wanted to develop the deep understanding of fractions with my students.

Lesson 1: Partitioning

Enduring Understanding:

A fraction is made up of two integers-one on top and one on the bottom. The two integers are separated by a line which can be horizontal or slanted. The top integer or numerator tells how many equal parts the fraction contains, and the bottom integer or denominator tells how many equal parts the whole has been divided into.

Essential Question:

How can I partition or divide shapes or sets of items equally to represent a fraction?

Objective:

This activity can be used as an activating activity to determine students' knowledge of fractions from third grade and build on that knowledge to develop a deeper understanding.

Procedure:

Provide students with shapes of various sizes. Have students divide the shapes into halves. As students are working, observe how students are dividing. Are they dividing the shapes into equal parts? Have students share/discuss with a partner why they divided the shapes the way they did. If you observe misconceptions such as the figure below, have students explain their thinking to clear up misunderstandings.

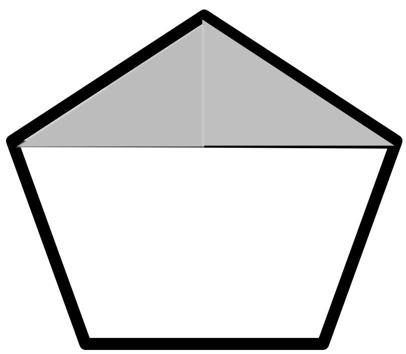
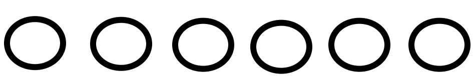


Figure 9

Repeat with dividing figures into thirds, fourths, sixths and eighths. Have students also show unit fractions such as 1/2, 1/3, 1/4, 1/6, and 1/8 on a number line and by shading items in a set such as the figure below:



Show 1/2 or 1/3 of the set.

Figure 10

Students should be given opportunities to explore the concept with concrete (countable objects) and continuous (shaded regions) examples.

Intervention:

If students are struggling with the activities, Cuisenaire Rods or pattern blocks would also work well. Given a pattern block, have students find the block that would divide the shape in half. Repeat for other fractional relationships. With Cuisenaire Rods, you can use the 10 cm rod for one whole and have students find the rod which divides the whole in half, thirds, sixths, etc. You can also change which rod would be one whole and repeat so students understand that the size of a particular fraction is not always the same. It depends on the size of the whole.

Lesson #2: Part/Whole Relationships.

Enduring Understandings:

The size of each part of a fraction must be equal. The greater the number of parts needed to make a whole, the smaller each individual part is. Different fractions can be used to represent the same amount of a whole, depending on the size of the parts being used. Any fraction with the same number as the numerator and denominator equals one whole.

Essential Question:

How do you determine the whole when given a fraction?
How do you determine the fraction of a whole when pieces are not of equal size?

Objective:

This activity is used to build students understanding of the whole-part relationships and determine students' level of understanding.

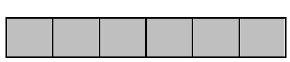
Procedure:

Pose one of the questions below to students. Have them work with a partner to determine the whole. Have them explain their reasoning to each other. Have each pair meet with another pair to explain their thinking. As a group, discuss what helped you determine the whole? What does the drawing and value of the fraction tell you about the whole?

If  = 1/4, what is the whole?

If  = 1/3, what is the whole?

If  = 2/3, what is the whole?

If  = 3/4, what is the whole?

Next give student pairs one of the following examples below. Ask the pairs to determine what fraction of the whole is shaded. Have them explain their thinking in written form after discussing their reasoning with their partner. Continue to have students work with these types of problems and share their thinking to build deeper understanding of the whole-part relationship.

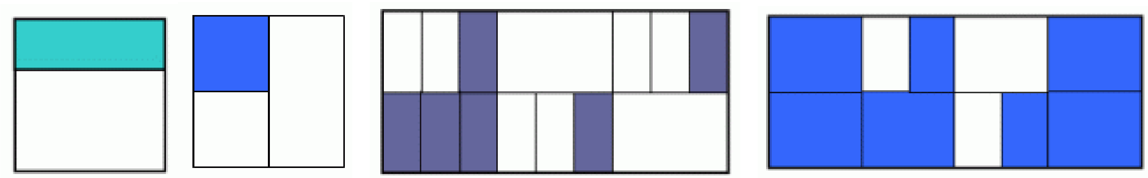


Figure 11

Lesson 3: Equivalent Fractions

Enduring Understandings:

A fraction whose numerator and denominator are the same is equal to one whole. Multiplying or dividing both the numerator and denominator by the same integer (multiplying or dividing by one) will result in an equivalent fraction. In order to deeply understand equivalency of fractions, students need to see fractions as numbers not just parts of areas or sets.

Essential Questions:

How do you determine if two fractions are equivalent?

Objective:

Students will build a deeper understanding of the concept a/b = a*n/b*n where a, b and n are all integers.

Procedure:

Provide students with an item that is 12 cm in length such as a Mr. Sketch marker and a supply of Cuisenaire rods. (Cuisenaire rods are wooden or plastic blocks that range in length from 1 to 10 cm. Each rod of a given length is the same color.) Ask students "How many brown rods long is the marker (or the item you have chosen)?" Some students will say, "One brown rod plus a purple rod"; some will say, "One and one half brown rods." Have students justify their responses. Some students may relate the one brown rod and one purple rod as one and one half brown rods. Make sure students understand that the denominator of the remainder length was two because the purple rod is one half of the brown rod which was the measuring unit. You can use Appendix A as a tool for recording their measurements. Next instruct students to measure the marker again, but this time measure the remainder length using a red rod. Students should notice that the measurement should be one and two fourths. Make sure students understand that because you need 4 red rods to make one brown rod, the denominator of the remainder is four. Record their measurement under another way. Finally have students measure the marker a third way but this time using the white rod for the remainder length. Students should notice that the measurement should be one and four eighths. Record their measurement as the third way. Repeat this process with different objects and use the brown rod as the measure unit. Students will have the opportunity to work with different equivalent fractions such as 1/4 = 2/8 and 3/4 = 6/8. Continue to have students justify their responses.

Lesson 4: Comparing Fractions to Landmarks

Enduring Understandings:

The size of the whole must be considered when comparing fractions. Benchmark fractions can help you compare and order fractions.

Essential Questions:

How do benchmark fractions help us to compare and order fractions?

Objective:

Students will use area models, number lines, verbal justification and benchmark numbers to compare fractional parts and justify their thinking.

Procedure:

Have students work in pairs. Give each pair of students a copy of Appendix B and C. Students should cut out the cards in Appendix B which will be used to label their groups. Students will also need to cut out the fraction cards in Appendix C. Students should place the 7 cards with landmark fractions on the floor or table. With their partner, they need to sort the fraction cards and place them under the appropriate card. Observe students as they are working making sure students are explaining their reasoning with their partners of why they think a particular fraction belongs under a certain card. After students have sorted their fractions, have them create a number line from 0 to 3 using a string on the floor. Have them place the following numbers on their number line. 0, 1, 2, and 3. Have students discuss and place the fractions on their number line. They should justify their reasoning of why they think a fraction should be placed in a particular spot on their number line with their partner. Observe students working listening for justification of their reasoning.

Lesson 5: Adding with Fractions

Enduring Understandings:

Numbers, including fractions can be written in a variety of ways (decomposing and composing of numbers). Wholes can be divided into different and equivalent fractional parts that will allow ease of addition and subtraction of fractions. Using visual models helps to understand and compare the relative size and equivalent parts of a whole.

Essential Question:

How does estimating fraction sums help when adding fractions?

Objective:

Students will use reasoning to estimate fractional sums.

Procedure:

Begin by writing the following expression on the board: $\frac{1}{2}+\frac{3}{4}$. Ask the class, "Is the sum more or less than 1?" Explain that they don't have to figure out the exact answer. Have them share their thinking with a partner before collecting ideas from the class. Listen for students sharing ideas of combinations that equal 1 and for comparing fractional amounts. Some students may also visualize the amounts in their head. Have them share their ideas. Continue the process with two more expressions such as: $\frac{4}{8}+\frac{2}{8}$ and $\frac{5}{6}+\frac{7}{8}$. The second expression will be more challenging for students. Students may notice that both fractions are close to 1. You may want to further the discussion of this expression by asking "Do you think the sum will be closer to 1 or 2?" Again, at this point students should not be expected to find the exact sum. Next, have students work in pairs to add other fractional amounts such as:

$\frac{1}{2}+\frac{3}{8}=$ ________________

$\frac{1}{3}+\frac{1}{2}+\frac{2}{3}=$ ______________

$\frac{5}{6}+\frac{1}{3}=$ _________________

Have them justify their reasoning by estimating sums to determine if the sum is more or less than 1. They may draw visual models or explain their reasoning of how they determined the sum. Students that struggle may benefit from using 4 x 6 rectangular arrays to shade in fractional amounts. (See figure 12)

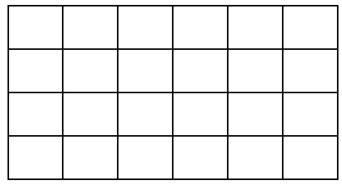


Figure 12

Standards addressed:

Writing 4.1**:** Write arguments to support claims in an analysis of substantive topic or texts, using valid reasoning and relevant and sufficient evidence.
Writing 4.10**:** Write routinely over extended time frames and shorter time frames for a range of tasks, purposes and audiences.
Speaking and Listening 4.1**:** Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
Math 4.NF.1**:** Explain why a fraction a/b is equivalent to a fraction (n x a) / (n x b) by using visual fraction models.
Math 4.NF.2**:** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record results of comparisons with symbols >, <, =, and justify the conclusions.
Math 4.NF.3**:** Understand a fraction a/b with a>1 as a sum of fractions 1/b.
Math 4.NF.4**:** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

Appendix A

Measuring with Cuisenaire Rods

Item Being Measured	First Way	Second Way	Third Way
Marker			
Pencil			
Book			

Appendix B

Landmark Fraction Labels

Equal to O	Between 0 and 1/2
Equal to 1/2	Between 1/2 and 1
Equal to 1	Between 1 and 2
Equal to 2 or greater than 2	

Appendix C

Fractions for Sorting

$1\frac{3}{4}$	$\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{3}$
$\frac{4}{5}$	$\frac{6}{8}$	$1\frac{1}{4}$	$\frac{3}{6}$
$\frac{5}{3}$	$\frac{2}{4}$	$\frac{2}{6}$	$\frac{0}{4}$
$\frac{9}{4}$	$\frac{8}{6}$	$\frac{1}{4}$	$\frac{5}{4}$
$\frac{5}{6}$	$\frac{2}{5}$	$\frac{3}{12}$	$\frac{6}{3}$
$\frac{7}{8}$	$\frac{4}{10}$	$\frac{1}{5}$	$\frac{8}{12}$
$\frac{0}{2}$	$2\frac{1}{2}$	$\frac{8}{8}$	$\frac{3}{2}$
$\frac{2}{12}$	$\frac{1}{3}$	$\frac{9}{6}$	$\frac{2}{3}$
$\frac{1}{6}$	$\frac{3}{3}$	$\frac{4}{2}$	$\frac{1}{8}$
$\frac{5}{2}$	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{4}{3}$

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Curriculum Unit Title | Proof and Reasoning with Fractions | **Author** | Linda Bacon

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

A fraction is made up of two integers-one on top and one on the bottom. The top integer or numerator tells how many equal parts the fraction contains, and the bottom integer or denominator tells how many equal parts the whole has been divided into. A fraction whose numerator and denominator are the same is equal to one whole.
Benchmark or landmark fractions can help you compare and order fractions. The size of the whole must be considered when comparing fractions. Wholes can be divided into different and equivalent fractional parts that will allow ease of addition and subtraction of fractions.

ESSENTIAL QUESTION(S) for the UNIT

How can I partition or divide shapes or sets of items equally to represent a fraction?
How do you determine if two fractions are equivalent?
How do benchmark fractions help us compare and order fractions?
How does estimating fraction sums help when adding fractions?

CONCEPT A	CONCEPT B	CONCEPT C
Partitioning	Equivalent Fractions	Comparing Fractions
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How can I partition or divide shapes or sets of items equally to represent a fraction?	How do you determine if two fractions are equivalent?	How do benchmark fractions help us compare and order fractions?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
Numerator denominator partition integer Halves thirds fourths sixths eighths	Equivalent remainder justify numerator denominator	Landmark fractions Benchmark fractions compare Order Justify

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES