Multiplicative Relationships: Fostering Students' Understanding of Ratios

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Objective

The last few years have been very exciting for us sixth grade mathematics teachers. The nation, for the most part, has become familiar with Common Core State Standards – Mathematics (CCSS-M)[1] and the shifts in not only content but the addition of the Mathematical Practices that describe how students should interact with the mathematics they are learning and using to solve problems. A major focus, according to the Common Core State Standards Initiative website; "In grade 6: Ratios and proportional relationship, and early algebraic expressions and equations"[2]. In direct response to this new focus, the Christina School District re-designed their scope and sequence for grade 6 mathematics. We previously did not address ratios, proportional relationships, and/or algebraic expression and equations as formally as CCSS-M is requiring. Our pacing guides at the time underwent major overhauling and still just this last June there was debate about the order of the units for the 2015-2016 school year.

There has been a great deal of good that has come with the adoption of the CCSS-M in my opinion. Teachers were forced to look at the CCSS-M document and begin having conversations again about the mathematics we were teaching and what topics were no longer sixth grade topics (introduction of fractions and addition, subtraction and multiplication of fractions) and reacquaint ourselves with some topics previously focused on in later grades (ratios, proportional relationships, and algebraic expressions and equations) that needed to be more formally introduced and mastered now in sixth grade. New resources were purchased to support the move to implement CCSS-M. For example, my school has used the *Connected Mathematics* as our main teaching resource since 2006. We currently have 1st, 2nd, and 3rd editions in our building, but I must say most of our teaching is supported using the 2nd and 3rd editions, in sixth grade. We purchased a teacher's edition and one student book of all the units in the 3rd edition and whole grade level sets of *Data About Us* and *Variables and Patterns: Focus on Algebra*; both to help with higher rigor demanded in CCSS-M for data and statistics and equations and expressions. In addition, the district purchased a unit from Encyclopedia Britannica *Mathematics in Context* series called *Models You Can Count On* to support students thinking with ratios and proportional reasoning. This particular unit introduces students to different models they can use to support solving problems involving proportional thinking; for example, the ratio table (specifically mentioned in CCSS-M, see Appendix A), bar model, double number line and number line. These additional resources were much improved in their development of lessons to support implementing the CCSS-M.

I was particularly excited to be working with the *Mathematics in Context* series again; although I am a huge fan of *Connected Mathematics* series and really enjoy teaching those units. Back in 1998 when the National Science Foundation was beginning to fund research into problem-based curriculum I had the opportunity to teach from some the pre-published versions of some *Mathematics in Context* units along with participating in workshops at the National Council of Teachers of Mathematics yearly conventions with some of the authors. Although Christina School District choose *Connected Mathematics* as their main teaching resource; I have participated in state-wide training that supported *Mathematics in Context* and have used *Mathematics in Context* materials as support materials for years. However, over the last couple of years we have been implementing the unit *Models You Can Count On* I have not seen the benefit of the tools with the majority of my students. Throughout the year, students would be confronted with a problem involving proportional thinking and not as often as I would have liked students did not utilize one of the tools we had studied. Some of my sixth grade teaching peers said they have just reverted to cross multiplying for solving the ratios; I really have a hard time supporting that decision with sixth graders when we are just starting to formalize their thinking around ratios.

I started to think about why students do not seem to see the power of the models as I did as a teacher when I was introduced to them. I have to say over the last couple of years I have really pushed myself to try and use ratio tables whenever possible; although I love the model and it has become a go to tool for me…breaking the cross multiply habit was very difficult for me, especially since I had practiced it for over 25 years. I have to say forcing myself to utilize ratio tables when solving problems; I believe it has made me a much strong mental mathematician too. I can easily scale up or down mentally in my head since I do moves (doubling, times by 10, multiplying) that make sense to me. Using the ratio table, bar model, double number line or number line helps me organize and keep track of my thinking while solving a problem. Here is where I think students do not see the power of the tools because I am not sure that they have had the opportunities to solve problems involving ratios without the tools to see how "messy" they can get. I wonder if in the past I have been so focused on teaching the models/tools that I lost sight of the mathematics…the reason to utilize the models/tools. In this unit I am hoping to offer students opportunities to solve some problems and see how utilizing the models/tools in our unit *Models You Can Count On* can help them organize and communicate their thinking. Another goal I have is that students gain an understanding that, "When two quantities are related proportionally, the ratio of the one quantity to the other is invariant as the numerical values of both quantities change by the same factor."[3]

Time always seems to be an issue in my classroom; there never seems to be enough time to "cover" everything. In the past couple of years I have been the teacher on our grade level team that always seems to be behind all the other teachers in pacing. I have really worked hard over the summer to map out exactly how much time I have for each unit and when mandated assessments will happen throughout the year; I am actually keeping that map on my desk and regularly refer to it. I am already a little behind, but I am now making much wiser decisions when planning lessons to keep on pace with my colleagues. I do think students need to time to play with the math; they need time to make connections and pose their own questions. Students need time to make sense of it all. I am hopeful that the extra time I allow students to "play around" will pay off later in the year when they have deeper understandings and can make connections among topics throughout the year. The two launches I am sharing with you were developed to introduce the idea of ratios to students and give them some time to solve problems where ratio tables, bar models, double number lines and number lines are useful tools for solving problems and communicating students' thinking.

School Background

Gauger-Cobbs Middle School had a population of 1210 students for the 2014-2015 school year broken down as 384 sixth grade students, 407 seventh grade students, 419 eighth grade students. The staff is numbered at 136, with 101 counted as Instructional Staff; last year teachers were 64% of the staff. Over the last two school years the racial enrollment has been identified as 40% African Americans, 35% White, 20% Hispanic/Latino and 5% American Indian, Asian, and/or Multi-Racial. The break down for staff is about 76% White, 23% African American, and 1% American Indian. Approximately 34% of the students are classified as low income and about 19% of the student population has been identified as Special Education.

I teach three blocks (85-90 minutes) of mathematics each day, along with an intervention block (45 minutes – comprised of 15 students, 7 of which are from another teacher). Two of my three sections are considered sixth grade mathematics classes and I have one section that is identified as Honors, this group of students will have a mix of sixth and seventh grade units (operating with positive and negative numbers and additional units on proportional relationships and equations and expressions) with the plan for the higher preforming students to move into a high school math class (an integrated mathematics course); while the rest of the class will continue in Honors with a mix of seventh and eighth grade units next year, followed by the high school math class their eighth grade year. This current year I have only two students with Individual Education Plans and they are for speech only. I have several students with 504 plans; however, it seems I am sent a new 504 plan weekly lately. Towards the end of September, all students were asked to take a computerized test to provide a measurement of their growth (my effectiveness) over the school year. The mathematics assessment had a total of 33 points and students were each given a percentage indicating the amount of point questions they answered correctly, see Appendix B for a table summarizing my students' performance in September.

CONTENT

According to *Developing Essential Understanding of Ratios, Proportions, and Proportional Reasoning for Teaching Mathematics in Grades 6-8*, "Proportional reasoning is a milestone in students' cognitive development. An understanding of proportionality develops slowly over a number of years" and goes on to say that how complex it is to reason proportionally and how some adults still do not reason proportionally.[4] As a sixth grade teacher I may be the first time students are being formally introduced to ratios and thinking proportionally; however, according to the CCSS–M students should be very familiar with ratios, rates and be able to apply proportional thinking to solve problems. This just being one the many reasons I feel more and more pressure each and every year when teaching mathematics. Research says students' thinking about mathematical concepts takes time; but at the end of the year I need each student to master the concepts and be able to solve problems.

Lucky I re-familiarized myself with two resources that helped me make sense of ratios and how to develop opportunities to help students to start thinking proportionally. The first resource that helped me develop my activities is *Developing Essential Understanding of Ratios, Proportions, and Proportional Reasoning for Teaching Mathematics in Grades 6-8*. Besides highlighting 10 essential understanding found in Appendix C; I found how the authors organized the essential understandings with questions and topics very helpful when developing my activities and keeping the math goal focused (Appendix D). I really focused on the questions that match up with topic ratios when developing my activities and to help me prepare. I challenged myself to make sure I was clear on the questions being asked, this was a quite a task at first for me. However, many re-readings of the chapter 1 and working through the included problems really helped me deepen my own understanding.

Another key aspect of this resource I anticipate helping me is when I analyze student work and prepare to pose the next task for them. The authors identified as, "…shifts in their [students] thinking as they become increasingly adept at forming ratios, reasoning with proportions, and creating and understanding rates."[5] The first shift is having students move from focusing on one quantity to realize the importance of two quantities. Another shift is for students to move away from just an additive comparison to multiplicative comparisons. The third shift is to have students become more efficient with their ability to find equivalent ratios; for example, instead of keeping the "composed-unit" and making the same iterations (doubling repeatedly), they begin to see that a jump of multiplying by 8 instead of 3 sets of doubles. The last shift is for students to be able to make infinite amount of equivalent ratios, not just a few "easy" ones. In my few years of experience teaching ratios tables, I worry that I pushed the "easy" steps – doubling, halving, times by ten – hoping that it was more accessible for students.

My second resource I am super glad I reacquainted myself with was *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers*[6] I particularly appreciated her take on ratio tables. Lamon highlighted that “Ratio tables (also called proportion tables) can be useful in facilitating proportional reasoning. If used properly, they can be tools that facilitate powerful ways of thinking. However, when they are presented as convenient devices for keeping work organized, they fail to facilitate any kind of reasoning at all.”[7] Lamon highlighted that students who persistently “fill-out” the ratio table additively for the most part, in interviews with student, do not realize that the columns have the same relationships. She would go on to comment, “Success with additive strategies do not necessarily encourage the exploration and adoption of more efficient strategies and should not be interpreted as proportional reasoning”[8] Yikes that is the whole reason I was using the model of ratio tables was to help develop proportional reasoning.

A resource that I found quite inspiring was a lesson posted on the website Share My Lessons called Introduction to Ratios[9]. In the lesson, a teacher Molly Sims shares a lesson using the situation of beads on a necklace to introduce ratios to her class. She poses the problem and allows students to work in small groups/partners. After a few different groups share their thinking, the teachers gives them the vocabulary to talk mathematically about ratios. She ends the lesson by posing some questions for students to think about before coming back into class tomorrow.[10] I designed two activates to be utilized before jumping into the unit to help introduce (informally at first) ratios to students – one modeled after Sim’s lesson and the second activity is another opportunity for students to show how they are thinking about situations involving ratios. The third activity is a project that can be used as a summative assessment for students to complete at the end of the unit of study.

Activities

Beads on a Necklace

I modeled this lesson right from the lesson *Introduction to Ratios* from Share My Lesson website[11]. The lesson starts by the teacher, Stephanie Mills, posing the problem “A necklace has two red beads for every three yellow beads. How many yellow beads are there is the necklace has six red beads.”[12] I changed up the problem to use blue and yellow beads, our school colors, and I used a larger given number. I posed the following problem for my students: This year Gauger-Cobbs Middle School is going to sell bracelets made of blue and yellow beads for students to show their school spirit. Each necklace will have two blue beads for every three yellow beads. How many yellow beads are there if the necklace has twelve blue beads? I will give students some private think time (4-5 minutes) and then ask students to move into groups of 3-4 classmates to come to consensus as a group on their answer. Once students are in groups, each group will be given a blank piece of 8.5 by 11inch paper to illustrate their answer and thinking to support their answer.

Just like in the video, I would share each group's response to look for similarities and differences. One of the strengths I felt from this lesson was the share out, at that time the teacher capitalized on connections between ratios and vocabulary students already knew; for example, pattern and multiples. It is during this time the teacher stressed the two quantities (blue and yellow beads in this example) and their relationship to each other. Finally, she introduced the term ratios and its definition. She also introduced the three different forms in which ratios can be expressed using the word to, a semicolon, and with a fraction bar. My plan is to follow that same progression.

If there is time and I think students are okay to think some more about ratios, I would pose the following questions? We just found that if there were 12 blue beads, there would be 18 yellow beads; could there be 12 yellow beads? If so, how many blue beads would be on a bracelet with only 12 yellow beads? Could a bracelet have 20 blue beads? If so, how many yellow beads would be on the bracelet? Could a bracelet have 21 blue beads? If so, how many yellow beads would be on the bracelet? Could I have a bracelet with a total of 75 beads? How many beads would be yellow and how many beads would be blue? As I type this, I wonder if students would pose similar questions to what I have prepared.

Mini-Posters with partners

For this activity I wanted to give students different scenarios involving ratios and have them work with a partner to solve the problems and complete a mini-poster (11.5 by 17 inches or larger) showing their solution and how they thought about the problem. I want to have students partner up and then have them pick a card (or maybe two), each card having a different ratio problem on it. See below for sample cards:

Jamie works at a shoe store, during his shift he sold 20 pairs of sneakers. If the ratio of pairs of sneakers to pairs of boots was 4:3, how many pairs of boots did he sell?	Kelly works at a hotdog stand and she sold 64 hotdogs with cheese. If the ratio of hotdogs sold with cheese compared to hotdogs with no cheese was 8:1, how many hotdogs did Kelly sell total?
Yesterday at the library, the librarian checked out 66 books. If 18 of the books were fiction, what is the ratio of non-fiction books to fiction books check out?	On the first day of the Book Fair 28 books were sold. If the ratio of books to bookmarks sold was 7:3, how many bookmarks did the store sell?

Mrs. Burlew won 8 games of 14 Chips. If the ratio of wins to losses was 2 to 1, how many games did Mrs. Burlew play total?	At the basketball game last night, the concession stand sold 54 items total. If 9 of the items sold were nachos and rest were pretzels, what is the ratio of nachos to pretzels?
At Evans' Wayout Wings you can either order blue cheese or ranch with your wings. The ratio of blue cheese to ranch is 7 to 1. If 56 people choose blue cheese, how many selected ranch?	The school store sold Gauger-Cobbs Middle School 50 blue t-shirts. If the ratio of blue t-shirts to gold t-shirts is 10 to 3, what is the combined amount of blue and gold t-shirts sold?
Tre is reading a book that has 26 chapters. He has already finished reading 10 chapters, what is the ratio of chapters needed to read to chapters already read?	A recipe calls for a ratio of sugar to flour to be 3:2. If you use 24 ounces of sugar, how many ounces of flour would you need to use?

I would like to have each problem solved by at least two of the groups. As students finish their mini-posters I would like to have them hang them up so that we could have a gallery walk and students could see how different partners solved their same problems. I am sure groups will finish at different times, so I would have a worksheet of all the problems available for students to continue to work on; the gallery walk would also give students a time to see if their answers are complete.

Ratio Recipe Project

This activity is a take-home project that students would complete at the end of completing the unit *Models You Can Count On*[13]. In this activity students find a recipe and then show, using an extended ratio table, the amounts of ingredients needed to scale the recipe up to serve 36 people and scale it down to just have one serving. I would definitely mock up some projects so that I could share with the class and show how to use the scoring rubric to "score" their own work before submitting the projects.

RATIO RECIPE PROJECT

For this project you will need to:

1. Choose one recipe from the internet, cookbook or home. The recipe must have ***6-8 ingredients*** and make ***10-20 servings***.
2. List the ingredients and their amounts to show what you would need to make 36 servings and again to make a single serving. You must show all of your work in a ratio table!
3. Create a display (construction paper or poster board are acceptable) that includes the following:
   - Original Recipe (Including the ingredients, directions for making, and amount of servings)
   - Scaled Recipe– Ingredients and new amounts needed for a single (one) serving.
   - Scaled Recipe– Ingredients and new amounts needed for 36 servings.
   - Show ALL of your work. *You can use one combined ratio table or individual ratio tables.*
4. Check your final work using the attached rubric.
5. **Extra credit: Make the new recipe for the class!**

RATIO RECIPE PROJECT SCORING RUBRIC

<table>
<tr><th></th><th colspan="4">Point Values</th></tr>
<tr><th rowspan="2">Original Recipe</th><th>3</th><th>2</th><th>1</th><th>0</th></tr>
<tr><td>• Original recipe (ingredients, number of servings, and directions) is displayed.
• 6-8 ingredients
• 10-20 servings</td><td>Two requirements</td><td>Only one requirement</td><td>NO requirement</td></tr>
</table>

<table>
<tr><th></th><th></th><th>10</th><th>8</th><th>6</th><th>4</th><th>2</th><th>0</th></tr>
<tr><th rowspan="4">Ratio Table – Whole Numbers and Fractions only</th><th>First Colum</th><td></td><td></td><td></td><td></td><td>All ingredients listed with units from original recipe.</td><td>Not listed</td></tr>
<tr><th>Second Column</th><td></td><td></td><td></td><td></td><td>Amount of original ingredient.</td><td>Not included</td></tr>
<tr><th>36 Servings</th><td>Student correctly calculates the new amounts for each ingredient for 36 servings and has supporting work on a ratio table.</td><td>Student has a minor error or two that is quickly found in supporting work on the ratio table. The strategy should have produced a correct answer, but due to a minor calculation error incorrect amount found.</td><td>Student correctly calculates MOST the new amounts for each ingredient and has supporting work on a ratio table.</td><td>Student correctly calculates the new amounts for some of the ingredients and has supporting work on a ratio table.</td><td>Student correctly calculates the new amounts for 1 or 2 of the ingredients and has supporting work on the ratio table.
OR
Student has all the correct new amounts for each ingredient; however, there is no supporting work on a ratio table.</td><td>No new ingredient s were calculated
OR
Student has some correct amounts but no supporting work on a ratio table.</td></tr>
<tr><th>Individual Serving</th><td>Student correctly calculates the new amounts for each ingredient for 1 serving and has supporting work on a ratio table.</td><td>Student has a minor error or two that is quickly found in supporting work on the ratio table. The strategy should have produced a correct answer, but due to a minor calculation error incorrect amount found.</td><td>Student correctly calculates MOST the new amounts for each ingredient and has supporting work on a ratio table.</td><td>Student correctly calculates the new amounts for some of the ingredients and has supporting work on a ratio table.</td><td>Student correctly calculates the new amounts for 1 or 2 of the ingredients and has supporting work on the ratio table.
OR
Student has all the correct new amounts for each ingredient; however, there is no supporting work on a ratio table.</td><td>No new ingredient s were calculated
OR
Student has some correct amounts but no supporting work on a ratio table.</td></tr>
</table>

	3	2	1	0
Display	Display is neat and free of errors (grammatical and visual)	Display may have errors, but the errors do not interfere with understanding the information presented.	Display has errors that interfere with understanding information being conveyed.	Display is very difficult to read due to errors.
		Poster was turned in on time.	Poster is fewer than 5 days late.	Poster was 5 or more days late.
	• Scoring Rubric attached • Project scored by student (each row has a check mark) • Parent signed project score	Two requirements	Only one requirement	NO requirement
Extra Credit	• Food presented on due date • Was recipe from project • Prepared appropriately to serve (forks, spoons, or bowls if necessary) • Shared food with class			

35 total points

A – more than 32 points earned
B – 29, 30, 31 points earned
C – 25, 26, 27, 28 points earned
D – 21, 22, 23, 24 points earned
F- 20 or fewer points earned

Conclusion

This unit is an exciting unit for me in that it provides an introduction to ratios, before I ask students to use ratio tables to solve problems involving ratios. I think the two introduction activities will give students an opportunity to experience ratios (for some students it may be their first time formally). I am really looking forward to sharing the Ratio Recipe Project with the students. I think I will not only gain performance task opportunity, but one that might also give students an opportunity to share a little about their culture and family with the class. I am hoping that students will take this activity as a way to show their talents in the kitchen too. I am looking forward the the tasting day and celebrating with the students.

Appendix A: Common Core School Standards and Mathematical Practices[14]

Common Core School Standards

Understand ratio concepts and use ratio reasoning to solve problems.
CCSS.MATH.CONTENT.6.RP.A.1
Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
CCSS.MATH.CONTENT.6.RP.A.2
Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]
CCSS.MATH.CONTENT.6.RP.A.3
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
CCSS.MATH.CONTENT.6.RP.A.3.A
Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
CCSS.MATH.CONTENT.6.RP.A.3.B
Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
CCSS.MATH.CONTENT.6.RP.A.3.C
Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
CCSS.MATH.CONTENT.6.RP.A.3.D
Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Mathematical Practices

CCSS.MATH.PRACTICE.MP1 Make sense of problems and persevere in solving them.
CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively.
CCSS.MATH.PRACTICE.MP3 Construct viable arguments and critique the reasoning of others.
CCSS.MATH.PRACTICE.MP4 Model with mathematics.
CCSS.MATH.PRACTICE.MP5 Use appropriate tools strategically.
CCSS.MATH.PRACTICE.MP6 Attend to precision.

CCSS.MATH.PRACTICE.MP7 Look for and make use of structure.
CCSS.MATH.PRACTICE.MP8 Look for and express regularity in repeated reasoning.

Appendix B: Student performance

Class sections	Range of scores	Average score	Percent correct on specific standards 6RP.A.1	6RP.A.2	6RP.A.3
Regular 1	9 – 33%	20%	15%	15%	8%
Regular 2	6 – 27%	19%	33%	7%	9%
Honors	18 – 48%	32%	17%	42%	31%

Appendix C: Essential Understandings[15]

Essential Understanding 1
Reasoning with ratios involves attending to and coordinating two quantities.

Essential Understanding 2
A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.

Essential Understanding 3
Forming a ratio as a measure of a real-world attribute involves isolation the attribute from other attributes and understand the effect of changing each quantity on the attribute of interest.

Essential Understanding 4
A number of mathematical connections link ratios to fractions:

- Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
- Ratios are often used to make “part-part” comparisons, but fractions are not.
- Ratios and fractions can be thought of as overlapping sets.
- Ratios can often be meaningfully reinterpreted as fractions.

Essential Understanding 5
Ratios can be meaningful reinterpreted as quotients.

Essential Understanding 6
A proportions is a relationship of equality between two ratios. In a proportion, the ratio of two quantities remains constant as the corresponding values of the quantities change.

Essential Understanding 7

Proportional reasoning is complex and involves understanding that-

- Equivalent ratios can be created by iterating and/or portioning a composed unit;
- If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship; and
- The two types of ratios – composed unites and multiplicative comparisons – are related.

Essential Understanding 8
A rate is a set of infinitely many equivalent ratios.

Essential Understanding 9
Several ways of reasoning, all grounded in sense making, can be generalized into algorithms for solving proportion problems.

Essential Understanding 10
Superficial cues present in the context of a problem do not provide sufficient evidence of proportional relationships between quantities.

Appendix D: Essential Understandings, Questions, Topics[16]

Essential Understanding	Question	Topic
1	How does ratio reasoning differ from other types of reasoning?	Ratios
2	What is a ratio?	
3	What is a ratio as a measure of an attribute in a real-world situation?	
4	How are ratios related to fractions?	
5	How are ratios related to division?	
6	What is a proportion?	Proportions
7	What are the key aspects of proportional reasoning?	Proportional Reasoning
8	What is a rate and how is it related to proportional reasoning?	
9	What is the relationship between the cross-multiplication algorithm and proportional reasoning?	
10	When is it appropriate to reason proportionally?	

Resources

America Achieves. 2013 *Introduction to Ratios.* Prods. TES connect and American Federation of Teachers. Accessed September 2015. http://www.sharemylesson.com/teaching-resource/introduction-to-ratio-50017837/.

Bright, George W., and Bonnie Litwiller, eds. *Classroom Activities for Making Sense of Fractions, Ratios, and Proportions: 2002 Yearbook*. Reston, VA: National Council of Teachers of Mathematics, 2002.
Activities to accompany 2002 Yearbook

Encyclopædia Britannica, Inc. *Britannica Mathematics in Context*: *Models You Can Count On* Chicago, Ill.: Encyclopædia Britannica, 2010.

"Key Shifts in Mathematics." *Common Core State Standards Initiative.* Accessed October 8, 2015. http://www.corestandards.org.
This page highlights the shifts in the CCSS-M.

Kilpatrick, Jeremy, W. Gary Martin, and Deborah Schifter, eds. *A Research Companion to Principles and Standards for School Mathematics*. Reston, Va.: National Council of Teachers of Mathematics, 2003.
Chapter 7, pages 95-113: “Fractions and Multiplicative Reasoning” by Patrick W. Thompson and Luis A Saldanba was the chapter I specifically read to research this unit.

Lamon, Susan J. *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers*. Mahwah, N.J.: Erlbaum, 1999.
Resource for teachers to deepen their understanding of the content.

Lappan, Glenda, Elizabeth Difanis Phillips, James T. Fey, and Susan N. Friel. 2014. Connected Mathematics. Boston: Pearson Education, Inc.
This is the main teaching resource I use in my classroom.

Litwiller, Bonnie H., and George W. Bright. *Making Sense of Fractions, Ratios, and Proportions: 2002 Yearbook*. Reston, Va.: National Council of Teachers of Mathematics, 2002.
This resource has a collections of articles addressing fractions, ratios, and proportions throughout elementary school to middle school, contains both activities and research.

Lobato, Joanne, and Amy B. Ellis. *Developing Essential Understanding of Ratios, Proportions, and Proportional Reasoning for Teaching Mathematics in Grades 6-8*. Reston, VA: National Council of Teachers of Mathematics, 2010.
A resource to use to deepen understanding of ratios and proportions.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Retrieved from http://www.corestandards.org/math.
This page listes and explains the Common Core Mathematics standards and Mathematical practices.

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[1] National Governors Association Center for Best Practices & Council of Chief State School Officers 2010

[2] Key Shifts in Mathematics

[3] Lobato, et al. 2010, 11

[4] Lobato, et al. 2010, 48

[5] Lobato, et al. 2010, 61

[6] Lamon 1999

[7] Lamon 1999, 176

[8] Lamon 1999, 177

[9] America Achieves 2013

[10] Sims 2013

[11] America Achieves 2013

[12] America Achieves 2013

[13] Encyclopædia Britannica, Inc. 2010

[14] National Governors Association Center for Best Practices & Council of Chief State School Officers 2010

[15] Lobato, et al. 2010, 12-3

[16] Lobato, et al. 2010, 14

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KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Ratios are a relationship between two quantities that can be represented using words, semicolon, and fractional notation. Ratios are a multiplicative comparison between two quantities.

ESSENTIAL QUESTION(S) for the UNIT

How can I use ratios to represent relationships between two quantities?

How can I use multiplicative reasoning to find equivalent ratios?

CONCEPT A	CONCEPT B	CONCEPT C
Ratios are a relationship between two quantities.	Multiplicative thinking can be used to find equivalent ratios.	Ratio tables are one way to show equivalent ratios.
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
What is a ratio? How can I communicate the relationship within a ratio?	If I know a ratio, how I can I find an equivalent ratio given one of the quantities?	How can I use a ratio table to show equivalent ratios?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
Ratio	Equivalent	Extended ratio table Scale up Scale down

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Poster paper (per group) and Mini-poster - 11.5 inches by 17 or 24 inches (per partner group).