Reasoning with Factors: Developing Algebraic Thinking In Middle School

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Introduction

School and District Demographics

I teach at Gauger-Cobbs Middle School - one of the four middle schools in the Christina School District. We are one of the largest middle schools in the State of Delaware. In the 2013-2014 school year, we had a fall enrollment of 1,137. That jumped to 1,210 for the fall enrollment in the 2014-2015 school year. Our sister schools had enrollments of 467 students at Bayard, 823 students at Kirk and 995 students at Shue. Being so much larger in population, we have been closed to Choice for the past several years. The district Choice Program allows families to apply for attendance at a school which is not the feeder school based on residence. Each year a Choice Open House event offers parents an opportunity to view the schools in the district and ask questions to try to find a best-fit school for their child. This is done at all levels K-12. Applications open in the fall and are due in early winter. Students are placed and if they are accepted to a Choice school, that building then becomes the "feeder" school for that child for the remainder of their time at that level. For example, I choice my son into an elementary school for Kindergarten and that building because his new "feeder" school through fifth grade with no additional paperwork required. This process provides stability for the family, but also means that there are few spots in subsequent years available for Choice students until those students age out of that building. Students who attend a child care in the school's feeder neighborhoods are automatically granted Choice. Students with older siblings already in attendance at a school are given priority. Any student accepted to Choice must then have their own transportation to and from school. We also have a larger student body than two of the district's three high schools (Christiana – 843, Glasgow – 992, Newark -1,555).

Gauger-Cobbs is a 6^{th} through 8^{th} grade building. This past school year, we began with an enrollment of 384 sixth grade students, 407 seventh grade students, and 419 eighth grade students. Our school was comprised of 39.8% African American students, 34% white students, 20.6% Hispanic/Latino students, 2.6% Multi-Racial students, 2.5% Asian students, and 0.4% American Indian students. Our staff was comprised of 76.2% Whites, 22.8% African Americans, and 1% American Indians. We had 33.9% of our students identified as low income, 19.1% identified as receiving special education services, and 2.7% as English Language Learners. We had promotion rates of 100% in both sixth and eighth grades from the 2013-2014 school year, and 99.5% for seventh grade.

In the 2013-2014 school year, our building had 768 suspensions and 3 expulsions. The number of students suspended and/or expelled was 247, which represents 22% our of student population, and is greater than both the district (15%) and state (13%) averages.

This school year, I will be teaching three classes of co-taught classes of eighth grade mathematics, which is our building Setting A for special education services. Last year I taught in that same program in both sixth and eighth grade. The year prior, I was the building mathematics intervention teacher, having six classes comprised of up to sixteen students who were identified as needing additional support in the area of math. That class served as their Response to Intervention (RTI) support during the day, and served both students with and without Individual Education Plans (IEPs). Both last year and this year, I also have one shortened block each day of "Academic Support". This class provides additional time in a smaller group setting for students with IEPs to meet and receive additional time and instruction for mathematics or English Language Arts. My responsibilities are to provide the additional mathematics instruction based on the unique needs of each learner. I provide students with additional time as indicated in their IEPs as well as other accommodations and modifications, work on individual IEP goals, work to build prior knowledge, preview and review material, and complete paperwork necessary for the IEP (gathering data for quarterly updates, working on transition activities). I have a co-teacher who specializes in providing the additional English Language Arts instruction for that course.

Classroom Demographics

The classroom I will focus my work on this year is my Academic Support classroom. This class is currently comprised of 19 students; 12 male and 7 female. There are 6 Caucasian students, 9 African American students, and 4 Hispanic students. There are two students with specific behavior plans focused on positive interactions with peers and staff to limit physical outbursts of violence. There is one student who is currently undergoing cancer treatment and is therefore frequently is absent and missing key concepts in understanding of math. Last school year this student was in a hospital setting and is therefore also adjusting to the routines and demands of traditional school. There is one student who struggles with fine motor control and uses a typing device with word predictive software which presents a challenge when completing math work. Another student is deaf in one ear and needs additional visual prompting to keep pace with the general curriculum. I also have a student who has needs specific to her Cerebral Palsy. She comes with a one-on-one paraprofessional to assist with certain physical needs and attend to health concerns, as well as to scribe for her and assist her in managing materials (taking things from her backpack, helping her to grasp her pencil, to operate her wheelchair). She also has unique needs related to toileting and eating. She tires easily when writing and either needs someone to scribe for her or the use of her laptop. She is proficient in using the laptop but it also requires physical effort and often takes so much time that it puts her behind. To accommodate this we provide her with copies of our class

notes so that she can spend more time thinking about and engaging in the mathematics than putting her energy into physically coping down those notes.

On the most recent state test, three students scored a level 3: Meets the Standard, four students scored a 2: Below Standard, and eleven students scored a 1: Well Below Standard. Based on STAR Math testing data, two students scored above the seventieth percentile, three students scored between twenty-fifth and thirtieth percentile, four students scored between sixth and twelfth percentile, and ten students scored at or below the fifth percentile. Two students are performing currently at grade level (8^{th} grade), one performing two years below grade level (6^{th} grade), two are three years below grade level (5^{th} grade), four are four years below grade level (4^{th} grade), seven are five years below grade level (3^{rd} grade), one is performing six years below grade level (2^{nd} grade), and two are currently on a first grade level for math. They have extreme gaps in their math knowledge being so far below grade level. They have difficulty solving multi-step math problems even with a calculator and concrete examples because they do not understand they underlying properties of the formulas they are trying to use. They also do not have a sense of when the answer they have on the calculator is incorrect due to user error because of their poor number sense.

Objectives

For this curriculum unit, I will focus on eighth grade mathematics. My goal is to create a series of warm-up activities and mini-lessons to review and reinforce the concepts related to square numbers in a way that helps students to engage with the concept and prepare them to be able to factor expressions with square numbers or square variables in a way that makes sense to them, not just following a prescribed series of steps. Students who continue to struggle with the actual number values have more difficulty when they later work with variable representations. Square numbers are unique and therefore having a familiarity with them and being able to immediately recognize a number as being square allows students to more quickly identify possible strategies for solving problems.

Familiarity with factor pairs will aide students in identify patterns and thereby find an entry point for solving more complex algebraic problems. I hope to help my students to feel more comfortable with this concept and to be more successful when we work with it in the curriculum.

Rationale

Helping Parents Help Their Children

We see it every day on social media - another attack on the common core and how math "these days" is ruining our children. Parents are intimidated by strategies and assume the math has changed. They feel helpless to assist/support their children and voice/vocalize

their frustrations over and over. This creates a learned helplessness and an internalized negative response towards math and learning. Students take on their fears and frustrations as their own. Parents are crying out for help - always saying this is too difficult for them to understand and therefore impossible for their children. The most common complaint heard in parent conferences is that the math is too hard and that the book comes with no help/explanation. This “new math” is over their heads and they can’t or won’t help. In reality, the math is the same - it hasn’t changed. We offer additional pathways to understanding and show students multiple modalities to solve problems and to be more well-rounded problem solvers, but the math remains the same. What parents need and are really asking for is a way to help their children find success (who doesn’t want their child to be a great student). Parents frequently share in conferences that things were easier in elementary when they knew the math. The concepts in secondary math are still based on those facts that parents do know and that educators often find students are lacking. By continually reinforcing those facts and getting parent buy-in that they can help I hope to improve student performance with math.

I have found in the past that students become frustrated and unsure of how to work through math problems related to square numbers due to unfamiliarity with factors, squares, and basic number sense. We discuss squares in relationship to finding area in sixth grade, then in eighth they can recall to square the unit in an area problem, but do not understand that squares and square roots are like opposite sides of the same coin; one operation will undo the other. When presented with problems requiring them to factor or distribute, they don’t see that relationship and become stuck in their thinking, needed an extensive amount of review and practice that I hope to lesson based on the practice done in these warm-ups and mini-lessons. Their parents’ negative attitudes toward math and frustrations when trying to help also lead to student’s thinking that this math is beyond their grasp and their becoming quickly frustrated and choosing to give up rather than persist with the content. For some students, these concepts haven’t been made a natural part of student thinking. In school, concepts are often taught in isolation both within the math classroom and between subjects. Students do not see the connection or correlation between topics and therefore do not make attempts to solve problems beyond the boundaries of what they have learned in that particular lesson. I liken it to the adage that to the man with a hammer, everything is a nail. Students believe they can only use the hammer to drive a nail, rather than using any other tool even one that would work just as well. When solving equations, they believe they must always draw a line through the equal sign and walk through step by step (cancelling b, then cancelling m) just as they have been shown in class. In reality, there is not requirement for drawing a line through the equal sign – this is a tool given to help students remember to do the same thing to both parts of the equation. It is also equally as valid to begin by cancelling m, but this requires a better understanding of what you are doing than when you cancel b first. Students without the basic understanding of what they are truly doing and why those particular steps work, have a harder time working with more complex mathematical concepts.

Progression of the Curriculum

Beginning in elementary school, students are building the tools they will need to apply to be successful with the middle school mathematics curriculum. In second grade they are introduced to area using physical representations of the square tiles that are put together to make a rectangle. In third grade they look at multiplication and division. They also find area of rectangles by tiling and using multiplication. In fourth grade students continue to work with rectangles and are introduced to the concept of factor pairs for values one to one hundred. In fifth grade students continue to work with rectangles and begin to manipulate factors to see how changing one factor will affect the product. In sixth grade, students move from just knowing factor pairs to determining the greatest common factor of two numbers. In seventh grade we introduce linear expressions – their first real taste of working with formal equations. In eighth grade, students work with exponents and square roots. All of this building to high school where students will solve quadratic equations, interpret expressions and rewrite them by factoring and distributing. *See Appendix A for grade specific standards.

Classroom Activities

In Christina School District, we follow the Connected Mathematics 2 texts to provide instruction related to the common core standards. The book titled, Shapes of Algebra, asks students to write expressions in both factored and expanded form and the factor and distribute expressions. These are concepts that students frequently find difficult to grasp and struggle to even follow a model to complete. My goal is to provide mini-lessons to activate the prior knowledge students already have related to these concepts, create a series of warm-up questions that will create a bridge between what students do know and how to complete these problems, and enable them to find success with the math with or without using the area model.

First I want to ensure that students can accurately determine the factors of numbers and understand the uniqueness of square numbers. We see some students become more fluent with finding factor pairs as they practice throughout the year. As students become more reliant on technology, calculators in particular, we hope that they do not lose the basic number sense. If they can memorize the first 20 square numbers (1,4,9,16…) then when they confront a problem that contains a square number, they will more often recognize the square and consider the square root as the most likely solution.

Activity 1

At the beginning of the school year, I will begin in those first two weeks when we are in the getting-to-know-you phase and still waiting on assemblies to end and supplies to come in, to work on a 100 chart of factor pairs. This is a review lesson that I complete to

help students learn how my classroom works (expectations, procedures, discussions) and to organize the information we know about the first 100 numbers (primes, composites, squares). We work through this lesson a few factors at a time and discuss the patterns we see for each (two is a factor for all of the even numbers, if four is a factor of a number two must also be a factor of that same number). This serves as the first touch point for square numbers and provides students with a reference guide to use if they should need it during future activities. We then have a developed together a reference chart of all factor pairs for numbers 1 – 100. Students keep a paper copy with their math journal and I have a posted copy in the classroom. We refer to this chart throughout the year and students use it as a tool to find possible solutions at home where they may not have access to a calculator. As we continue to use the chart and discuss factor pairs it becomes a familiar part of the classroom dialogue and eventually students anticipate that they will be asked for factor pairs so it becomes an automatic response to consider that when working with questions prior to the teacher having to prompt. See Appendix B for a copy of the completed chart.

Along with this activity, I teach students a verse from a song I have written about concepts in the sixth grade common core math standards. This verse refers to primes, composites, and squares. I sing it often and encourage students to sing along. See Appendix C for the lyrics to this verse of the song.

Activity 2

After completing the chart, each student will build a visual to represent square numbers 1-100 taken from *Building Squares and Discovering Patterns* by David J. Whitin and Phyllis Whitin. Students will mark out the outline on a grid of each square number, showing the factor pairs on the outside and the square number on the inside. Similar to a multiplication chart, but with a focus on square numbers. This activity is meant to help students see the connecting that square numbers are so called because given those dimensions they form a square. This will later lead to the understanding that a cube number builds a cube. It will reinforce the concept of squares and provide an additional reference sheet for students who prefer the more visual representation to the list form. See Appendix B for the completed chart. You may choose to have students color code the chart.

Along with this lesson we challenge students to memorize the first 15 square numbers. Students can earn extra credit if they can recite the first 15 squares (“One squared is one. Two squared is four.” And so on) in under one minute. To help students memorize the squares and practice, I frequently sing a song I have written about square numbers. See Appendix C for the lyrics to this song.

Activity 3

In those first two weeks, after the chart is complete, I will complete a graphing activity with students in which they work first individually to graph the factor pairs of a number as coordinate points on a piece of graph paper. Each student will graph the factor pairs for several numbers. Then students go up to the graphing board and physically mark out these lines using different colored yarn for each set of factor pairs. After the students mark their lines and have checked each other's work for accuracy, the teacher, using a black string, will mark the line y = x on the graphing board. As a group we discuss which lines have points on the y = x line and why it is only true for some sets, those that represent square numbers with the square roots of that number being the intersecting point on the y=x line. We will post this visual graphing board in the room to remain up as a classroom decoration and talking piece for, as well taping in the individual paper graphs in student journals to provide an additional visual of what makes a square number that students can always have with them. I am adapting this activity from *Exploring Factor Sets With a Graphing Calculator* by E.P. Eisen. See Appendix B for a photo of the completed display board.

Warm-Up Activities

We then complete a series of warm-up activities over the coming weeks. Incorporating one or two questions each day along with a question related to our current curricular topic. Doing this will keep the concept of factors fresh and provide students with an access point for the warm-up activity even if they are still struggling with the other curriculum – giving them those small victories at the beginning of the block. We will scale up the difficulty of these warms ups as the days progress (finding area from two side lengths with a visual then without a visual, finding area with one numeric side length and one variable side length with a visual then without a visual, finding possible side lengths if you know the given area). In Appendix B I have provided a sampling of these warm ups. I will include 2 or 3 each day, but you may choose to focus only on these concepts or to include just one or two depending on the amount of time you have at the beginning of each class or the difficulty level of other questions you are asking at that time. I typically use only one type of question the first few weeks, then as the year progresses I will mix and match as I see students making progress or having difficulty with the questions to meet their needs and review as they need.

Area Model Activities

Beginning with a review of area for a rectangle, we will build practice for finding area using an area model. As students become proficient, we will find the combined area of two rectangles with a common side. Then we will progress to two sets of two rectangles (two paired on top, two paired below). We will practice finding the combined area during our warm-up activities for several consecutive days, then continue to work with these problems once or twice a week to help students maintain this skill as we work towards our overall classroom unit where students must factor and distribute.

Bagging Snacks Lesson Adaptation

Modified from the Connected Mathematics Project 2 book Prime Time lesson 3.3 titled Bagging Snacks. I worked with students on the modified version in which students think about ways they can sort snacks to share. I brought to class several items for a PBS reward day activity. The snacks were purchased at a local wholesale distribution center and came individually wrapped. We worked together first to list all of the factors for each snack. Then they found all of the common factors and wrote a list of what combinations would work (If they had 12 bags, each bag would contain 3 M&M packs, 7 Starburst candies, and 2 mini-muffin snacks.) They then were asked to consider how many students are in the class and which combination would allow each student to have a snack pack for the event. As the culmination of the lesson, we share the snack bags and enjoy our treats and our PBS reward activity they have chosen.

Review at Home

To help parents encourage their children to continue to practice with factors and multiples at home, and to provide them with an entry point to help their children in a way they feel comfortable, I suggest they play the game "Bang" with their children. This game can be made with some pop sticks and a jar tall enough to fit the sticks. On each stick you write a number (1-20). I make three or four of each number. You also write "Bang" on a few sticks (more for older children, fewer if you have younger children also playing). To play the game, you mix the sticks in your jar. The first player removes two sticks and uses them to state a mathematical fact. This part can be modified to be appropriate for different players. If the person who pulled the sticks states a correct fact, they keep the sticks. Play goes back and forth between the players. If a player pulls out a stick marked "Bang" they must return all of their sticks to the jar. You can play for a certain amount of time or until a player reaches the desired number of sticks. You could modify the game to pull a single stick and have to recite the square of that number.

With my own family, I have children at very different points in their academics so each on plays with a different goal in mind. My daughter in seventh grade will list the multiple the two numbers she pulls or can pull a single stick and list the square number. My son in fourth grade is currently working on multiples, but sometimes chooses to state addition or subtraction facts for review. My daughter who is in preschool pulls a single stick and names the number on the stick to practice number recognition. This game can be easily modified to meet different stages and ability levels, and continues to be fun to play as the kids have gotten older. We frequently play during longer car rides since we use a jar that has a lid and can easily be passed around from child to child without making a mess. In the end, no matter who "wins" the game, we have all practiced math facts and had fun together in the car.

Because the game can be adjusted to different ability levels, requires few materials that are all available for purchase at the local discount store for a total of $2, and parents don't require special knowledge of math concepts, it is a great game for families to engage in and encourage math practice in any home. Parents can play together with their children without "feeling dumb" which they often say is the case when trying to help with grade level homework. It promotes a positive feeling toward math and children are more likely to engage in it because it is in the format of a game rather than just going through flash cards or writing down facts. Some more competitive children like to use timers, while others are content to play at a more leisurely pace. Regardless, they are practicing and internalizing math facts that they need to know.

Appendix A

Common Core State Standards – Grade-Level Focus Standards

Math Practice 1: Make sense of problems and persevere in solving them.
Math Practice 5: Use appropriate tools strategically.
Math Practice 7: Look for and make use of structure.
8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.
8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2=p$ and $x^3=p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
8.EE.C.7.B Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Progression of Standards

Standards Related to Factors

*3.OA.B.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
*4.MD.A.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
*4.OA.B.4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

*5.NF.B.5.A Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
*6.EE.A.2.B Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*
*6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*
*7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
*HSA.REI.B.4.B Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
*HSA.SSE.A.1.A Interpret parts of an expression, such as terms, factors, and coefficients.
*HSA.SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Standards Related to Area/Area Model

*3.MD.C.7.A Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
*3.MD.C.7.B Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
*3.MD.C.7.C Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
*3.MD.C.7.D Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
*4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
*4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the

properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
*5.MD.C.5.A Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
*5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
*5.NF.B.4.B Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Standards Related to Squares/Square Numbers

*2.G.A.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
*3.MD.C.5.B A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
*3.MD.C.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
*HAS.REI.B.4.A Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
*HAS.REI.B.4.B Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Appendix B

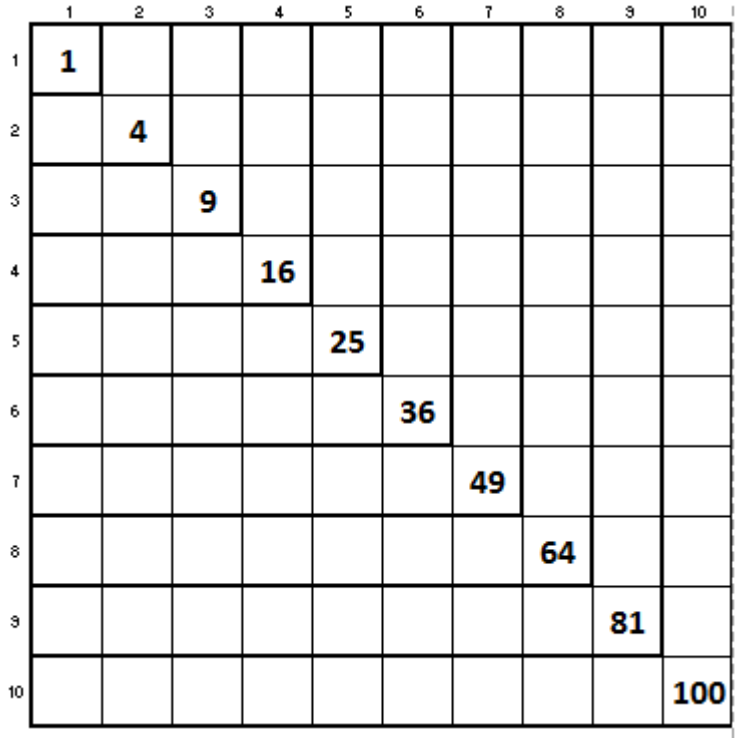
Resources and Problem Sets

Activity 1: 1 – 100 Factor Chart

1 1*1	2 1*2 Prime	3 1*3 Prime	4 1*4 2*2	5 1*5 Prime	6 1*6 2*3	7 1*7 Prime	8 1*8 2*4	9 1*9 3*3	10 1*10 2*5
11 1*11 Prime	12 1*12 2*6 3*4	13 1*13 Prime	14 1*14 2*7	15 1*15 3*5	16 1*16 2*8 4*4	17 1*17 Prime	18 1*18 2*9 3*6	19 1*19 Prime	20 1*20 2*10 4*5
21 1*21 3*7	22 1*22 2*11	23 1*23 Prime	24 1*24 2*12 3*8 4*6	25 1*25 5*5	26 1*26 2*13	27 1*27 3*9	28 1*28 2*14 4*7	29 1*29 Prime	30 1*30 2*15 3*10 5*6
31 1*31 Prime	32 1*32 2*16 4*8	33 1*33 3*11	34 1*34 2*17	35 1*35 5*7	36 1*36 2*18 3*12 4*9 6*6	37 1*37 Prime	38 1*38 2*19	39 1*39 3*13	40 1*40 2*20 4*10 5*8
41 1*41 Prime	42 1*42 2*21 3*14 6*7	43 1*43 Prime	44 1*44 2*22 4*11	45 1*45 3*15 5*9	46 1*46 2*23	47 1*47 Prime	48 1*48 2*24 3*16 4*12 6*8	49 1*49 7*7	50 1*50 2*25 5*10
51 1 * 51 3 * 17	52 1 * 52 2 * 26 4 * 13	53 1 * 53 Prime	54 1 * 54 2 * 27 3 * 18 6 * 19	55 1 * 55 5 * 11	56 1 * 56 2 * 28 4 * 14 7 * 8	57 1 * 57 3 * 19	58 1 * 58 2 * 29	59 1 * 59 Prime	60 1 * 60 2 * 30 3 * 20 4 * 15 5 * 12 6 * 10

61	62	63	64	65	66	67	68	69	70
1 * 61 Prime	1 * 62 2 * 31	1 * 63 3 * 21 7 * 9	1 * 64 2 * 32 4 * 16 8 * 8	1 * 65 5 * 13	1 * 66 2 * 33 3 * 22 6 * 11	1 * 67 Prime	1 * 68 2 * 34 4 * 17	1 * 69 3 * 23	1 * 70 2 * 35 5 * 14 7 * 10
71	72	73	74	75	76	77	78	79	80
1 * 71 Prime	1 * 72 2 * 36 3 * 24 4 * 18 6 * 12 8 * 9	1 * 73 Prime	1 * 74 2 * 37	1 * 75 3 * 25 5 * 15	1 * 76 2 * 38 4 * 19	1 * 77 7 * 11	1 * 78 2 * 39 3 * 26 6 * 13	1 * 79 Prime	1 * 80 2 * 40 4 * 20 5 * 16 8 * 10
81	82	83	84	85	86	87	88	89	90
1 * 81 3 * 27 9 * 9	1 * 82 2 * 41	1 * 83 Prime	1 * 84 2 * 42 3 * 28 4 * 21 6 * 14 7 * 12	1 * 85 5 * 17	1 * 86 2 * 43	1 * 87 3 * 29	1 * 88 2 * 44 4 * 22 8 * 11	1 * 89 Prime	1 * 90 2 * 45 3 * 30 5 * 18 6 * 15 9 * 10
91	92	93	94	95	96	97	98	99	100
1 * 91 7 * 13	1 * 92 2 * 46 4 * 23	1 * 93 3 * 31	1 * 94 2 * 47	1 * 95 5 * 19	1 * 96 2 * 48 3 * 32 4 * 24 6 * 16 8 * 12	1 * 97 Prime	1 * 98 2 * 49 7 * 14	1 * 99 3 * 33 9 * 11	1 * 100 2 * 50 4 * 25 5 * 20

Activity 2: Square Number Visuals



Activity 3: Graphing Squares

The graphing board was constructed with a piece of scrap plywood.. Screws were inserted at 1 in spacing to create a 26 x 26 grid (this creates a graph that goes from 0-25 on each axis). Teachers could alternately use peg board to make a large scale model, or use geoboards with pegs for individual student work. In retrospect, I would use a different color nail to indicate the y=x line, and use a thicker yarn to accentuate the lines better.

Image of actual board.

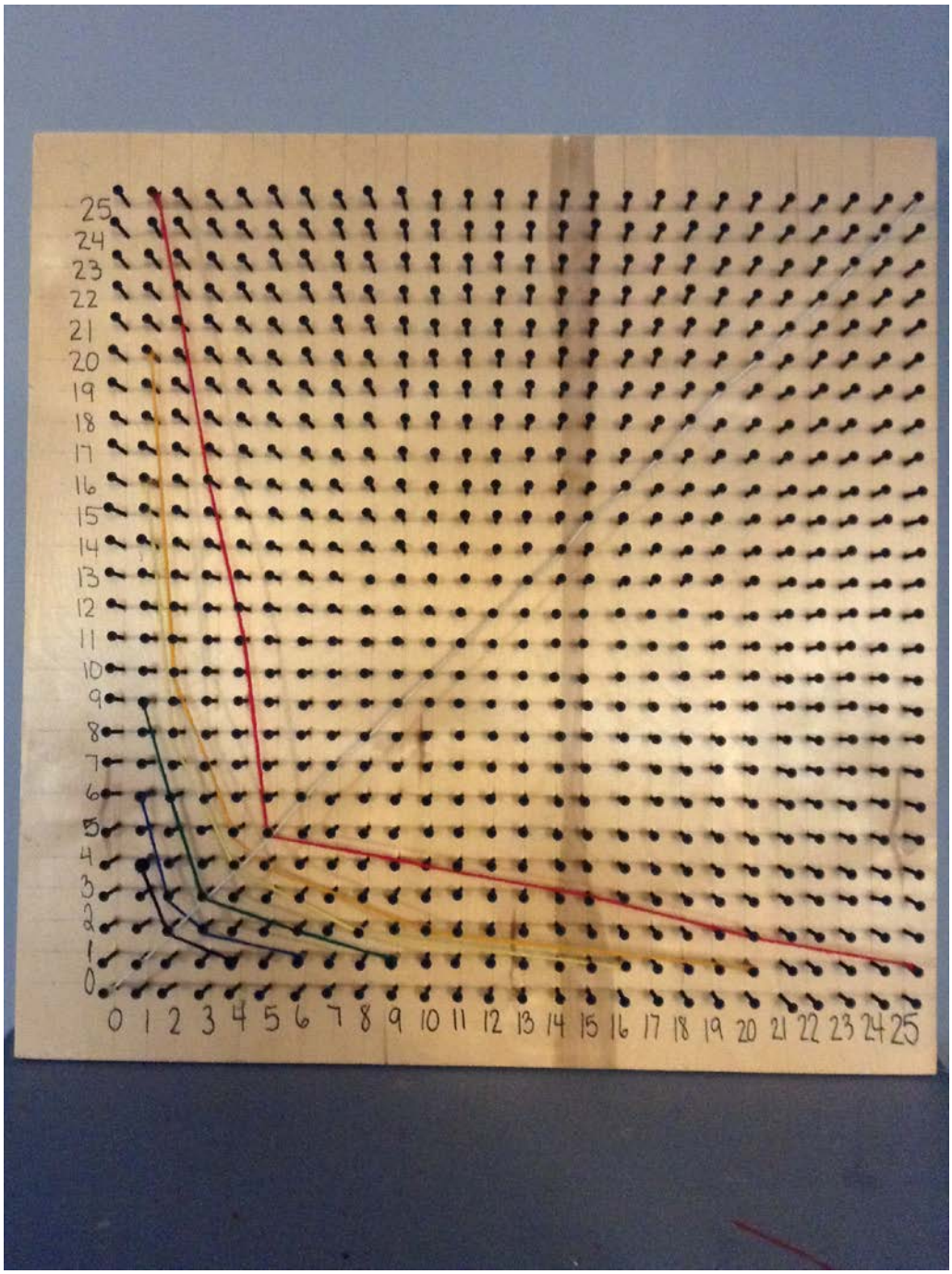
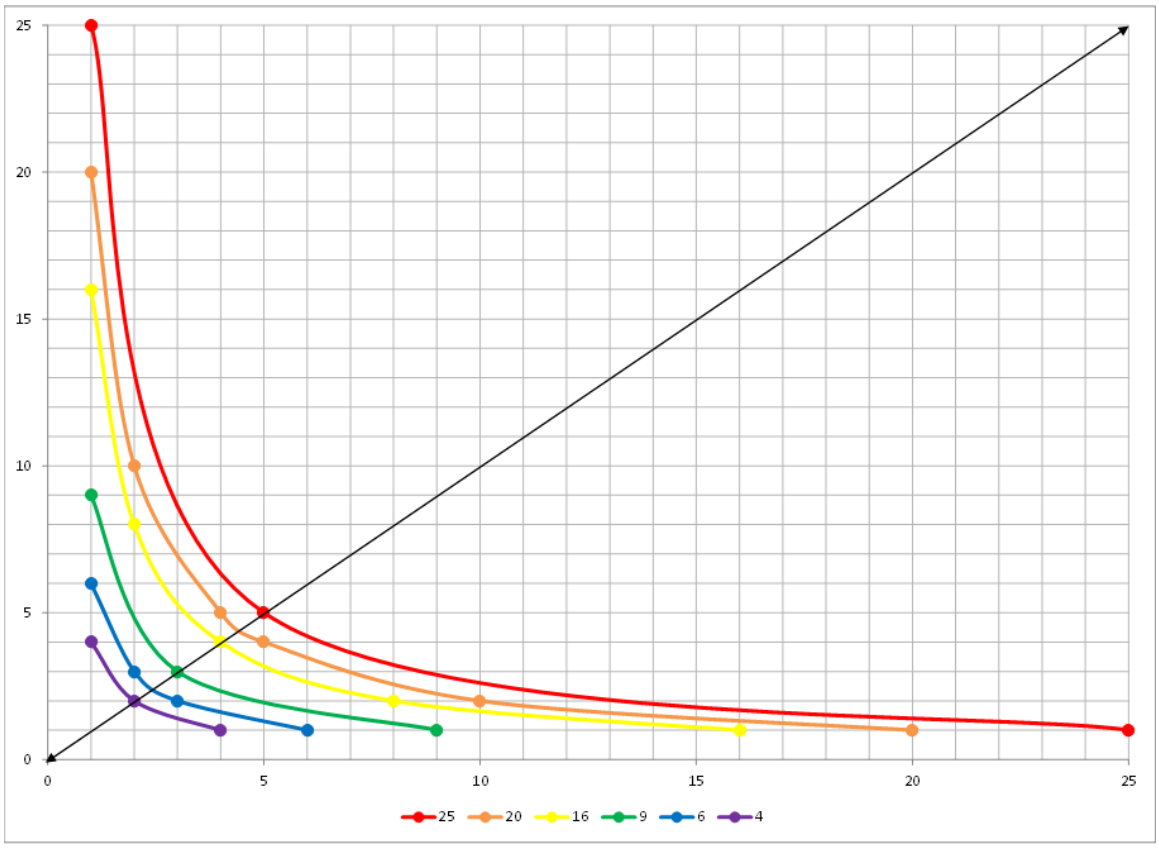


Image from Excel for students who are absent the day of the activity.



Warm-Up Activities

The series of warm-up questions can be combined to include as few or as many as time will allow within the classroom warm-up time and still allow for an activating strategy related to the current content being taught.

Fact Families			
Write the complete fact family.			
5 x 8 – 40	3 x 7 = 21	24 ÷ 6 = 4	27 ÷ 9 =3
8 x 2 = 16	6 x 3 = 18	70 ÷ 10 = 7	60 ÷ 12 = 5

Greatest Common Factor			
List the factors for each number. What is the greatest common factor?			
20 and 48	24 and 52	14 and 30	30, 42 and 96
90 and 72	85 and 68	56 and 28	24, 32, and 72

Squares and Roots			
Evaluate each expression.			
16^2	4^2	$\sqrt{49}$	$\sqrt{121}$
8^2	5^2	$\sqrt{225}$	$\sqrt{196}$

Factoring Expressions			
Factor each expression.			
3x + 15	2b – 16	6z + 18	-4f + 24
7x – 56	3g + 12	5k – 45	-9h + 54

Distributing Expressions			
Use the distributive property to simplify each expression.			
2(4+10x)	4(12x – 16)	5(2x +3)	-8(x + 4)
3(7x – 3)	7(1x + 9)	X(3x + 2)	X(1x – 4)

Area Model Activities:

Finding Area			
Find the area of each rectangle using the given side lengths. You can draw a picture to help you visualize the rectangle. Remember to include units in your answer.			
6 in by 4 in	2 in by 8 in	11 yd by 12 yd	15 in by 2 in
7 ft by 2 ft	5 km by 9 km	4 ft by 8 ft	6 km by 7 km

Finding Side Lengths			
Find the missing side length for each rectangle using the given side length and area. You can draw a picture to help you visualize the rectangle. Remember to include units in your answer.			
Side Length: 5 in Area: 30 in^2	Side Length: 12 yd Area: 60 yd^2	Side Length: 7 ft Area: 56 ft^2	Side Length: 6 mi Area: 42 mi^2
Side Length: 4 in Area: 32 in^2	Side Length: 2 cm Area: 30 cm^2	Side Length: 11 m Area: 77 m^2	Side Length: 5 km Area: 75 km^2

Using Area Model with Variables
Use the area model to distribute with variables.

	10	8
5		

	10	2
4		

	20	4
7		

	20	5
3		

	10	8
10		
2		

	10	9
10		
5		

	30	3
20		
1		

	10	5
20		
7		

	2	X
3		

	4	X
7		

	8	X
10		

	20	X
10		

	3	X
7		
X		

	9	X
5		
X		

	6	X
2		
X		

	12	X
10		
X		

Appendix C

Prime/Composite/Square Verse of 6th Grade Common Core Math Song

Sung to the tune of Froggy Went A-Courtin. “Prime numbers have exactly two factors. Mmm Hmm. Composites have more than two factors. Mmm. Hmm. Squares have an odd number of factors. Times a number by itself to get a square answer. Mmm Hmm. Mmm Hmm. Mmm Hmm.”

Square Number Song

Sung to the tune from Mozart’s Serenade No. 13 for Strings in G major, K. 525 “Eine knleine Nachtmusic”. “One. Four. Nine. Sixteen. Twenty five. Thirty six. Fourty nine. Sixty four and Eighty one. Our song is almost done. One hundred. One twenty one. One forty four. One sixty nine. One ninety six. Two twenty five. These are the squares, number pairs. All you have to do is multiply a number by itself. Like three by three. It’s easy. Now you know the squares.”

Appendix D

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KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Students will be able to use a factor pair reference sheet for numbers 1 to 100 that they have created to determine the factors of a given number and to classify numbers. Students will be able to represent perfect squares using a pictoral model based on area. They will be able to use that model to determine the square of a number and the square root of a perfect square from 1-100. Students will be able to create a graphical representation of a number based on its factor pairs and use that representation to justify if a number is or is not a square number.

ESSENTIAL QUESTION(S) for the UNIT

How does knowing the factor pairs of a number help us to classify that number? How can we represent the relationship between the factor pairs of a given number and its classification using different displays?

CONCEPT A	CONCEPT B	CONCEPT C
Classifying Numbers	Modeling Perfect Squares	Graphical Representation of Square Numbers
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How do you use the factor pairs of a number to classify it as a prime, composite, and/or square number? What logical argument can you can you use to make a broader hypothesis for classifying any number as being a prime, composite, and/or square number?	How can visually represent the square numbers between 1 and 100? How can we use the model to determine the square of a number? How can we use the model to determine the square root of a perfect square?	How can we use a graphical representation of the factor pairs for a given number to determine if the number is a square number? How can we use the graphical representation to justify the classification of a number as being a square number?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
*Prime Number *Composite Number *Square Number *Factor *Factor Pair *Multiple	*Square number *Perfect Square *Square root *Area	*Factor pairs *Square number

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

During Concept A students will work to create a chart of the factor pairs for numbers 1-100. They will learn to classify numbers as prime, composite, and/or square based on the factor pairs for a given number.

During Concept B students will create a visual representation for the perfect squares between 1 and 100. They will apply what they have learned about area to create their visual.

During Concept C students will list the factor pairs of given numbers, then create a graph using the factor pairs as their coordinates. They will graph several sets on the same graph, color coding them, then use the line y=x to determine which of the data sets represents a square number.