Providing Opportunities for Students to Reason, Justify and Prove in the High School Calculus Class

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Introduction

In 1989, the National Council of Teachers of Mathematics published its new standards, sending the mathematical and educational communities ablaze with debate.[1] As a young teacher that had recently left a doctoral program in Pure Mathematics, I found the discussions around me interesting. Being new to the teaching community, I sat quietly listening intently to the chatter that the NCTM just released standards suggesting that the two-column proof in geometry be de-emphasized. What? As I sat listening, I felt saddened that something I enjoyed immensely was being cut from the curriculum. And therein lies the problem, first, I personally had not read the new NCTM standards and judging by the high school mathematics teachers sitting around me I could later deduce that they had not read them either. Instead, they were interpreting what someone else had said or heard. From that point forward, high school mathematics teachers took large liberties in their approach to the two-column proof. Over the past twenty or so years, I have had the opportunity to meet many high school geometry teachers and found that most of them have communicated to me that they do in fact skip or de-emphasize proofs in geometry. With the removal of the proof, came the removal of the chapter on logic. It was not the intention of NCTM to de-emphasize any mathematical reasoning or proof, in fact it was their intention to offer more opportunities for students to reason and justify their thinking in geometry and in the earlier grades without the structure of the two–column proof, but however, the damage was done. Most mathematics teachers today believe that reasoning and justification is a part of the mathematics curriculum. In fact, the Common Core Standards have a set of Mathematical Practices that include the following [2]

CCSS Math Practice #3 Construct viable arguments and critique the reasoning of others
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using

concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Thus, clearly indicating that within the newly released Common Core State Standards in Mathematics Curriculum, a student should learn the basis of proof. It might be important to note that the Common Core does not use the word proof, but hints at the idea of proof through words like constructing arguments, and building a logical progression of steps to explore the truth value of a statement. That is, the proof can be included beyond high school geometry. In particular, the goal of this unit is to explore opportunities for students to reason, justify and prove in the high school calculus class.

Purpose

As I begin to think about the upcoming school year, I wondered why do I expect my students to write out each and every step when problem solving? What am I actually looking for? I can honestly say that I am frequently frustrated with the written work or lack thereof that I receive from students. I find it very difficult to follow their work or even monitor their train of thought. Am I trying to force the idea of logical progressive steps without ever engaging them in the idea of a mathematical proof? If so, how successful have I been over the years? As I examine one definition of the term proof: "A sequence of statements in which each subsequent statement is derivable from one of the previous statements", [3] I realize that I have been asking my students to provide a written account of their complete thought process, to reason, to justify and to explain without ever modeling my expectations beyond basic problem solving. By introducing the concept of proof to my students, I hope to create a classroom that will permit my students to explore ideas, make conjectures and write viable arguments. I do not want my classroom to become void of reasoning ability in such a way that mathematics simply becomes a matter of following a set of rules or procedures and mimicking examples without any thought as to why they make sense. [4]

The concept of proof may be something quite new to my students. Some of my students will have studied the concept of proof and logic in our Honors Geometry class, and some may not have seen a proof in their general Geometry class. This unit will begin with the definition or the idea of proof and an introduction to different methods of proof. After explaining the idea of proof, I will introduce students to constructing their own arguments or proofs with simpler context not requiring any calculus. The idea is that I am looking for my students to begin examining mathematical arguments with a critical eye. Do they believe what they are being told or what they read? Do they require more convincing? To get the ball rolling, I plan to introduce some easier proofs. One such example might be

If m is an odd integer, then $3m^2+4m+6$ *is also an odd integer.*
or
If m is a real number, and m, m+1, m+3, are lengths of a right triangle, then m = 3

By introducing proof in a simpler context, students will have entry points into writing a viable argument. I would be willing to bet that they will simply test an odd number in the first example, and then proudly proclaim that it works. Such as

If m is an odd integer, then $3m^2+4m+6$ *is also an odd integer.*
An expected response from a student would be if m= 3 then

$$3(3^2)+4(3)+6$$
$$3(9)+4(3)+6$$
$$27+12+6$$
$$45 \quad \textit{which is odd}$$

Checking to see if something is true does not constitute a viable argument that it will be true for every odd integer. For the second problem, I am expecting that my students will actually use what they are trying to prove and then say it is true. Such as

If m is a real number, and m, m+1, m+3, are lengths of a right triangle, then m = 3
The expected response from a student would be
If m = 3, then the sides of the right triangle will be 3, 4, and 5 and

$$3^2+4^2=5^2$$
$$9+16=25$$
$$25==25$$

Approaching this problem by assuming the end result is true first, is actually directly opposite from what the problem is asking them to validate. Obviously, the problem is trying to get students to realize that three in fact would be the only number that m could be to make the statement true. Hopefully, by designing their own argument and critiquing the work of others, my students will develop a better sense of what written work really means in the context of problem solving in mathematics. This portion of the unit will take place during the first two weeks of school. I will have my students think about what constitutes a good argument. Are they able to convince their classmates that something is true? During this phase, I do realize that some students will not speak up and question others. This might be due to a number of reasons including a fear of not wanting to be singled out as not knowing a particular idea or concept. These issues must be confronted as each student must feel comfortable within the classroom. Since my goal is to approach these easier proofs during the first two weeks of school, I hope to create a classroom environment where questioning is the norm. After the initial introduction, I plan to systematically pose “tough” problems as a way for students to reason and justify their thinking about the calculus concepts they are learning. These problems will require some productive struggle and thought to develop a chain of logical reasoning in order to convince others if they are valid or not.

My unit will be taught to three different sections of Honors Calculus at Conrad Schools of Science during the 2015-2016 school year. Most of the students in these classes have been relatively successful in their previous mathematics courses, but may have not necessarily been in an honors mathematics class before. At Conrad, a student who has successfully completed the general education track leading to Pre-Calculus or the Honors educational track leading to Honors Pre-Calculus may register for the Honors Calculus class. Note: this is not the Advanced Placement Calculus class. A student registered for the class most likely would be a senior looking to be successful on their college placement exams, or a junior enrolling in the honors calculus class before taking the advanced placement calculus course their senior year at Conrad.

An Overview of the High School and Students

My unit will be communicated to my three sections of Honors Calculus classes for the 2015 - 2016 school year. As mentioned previously, these students are a mixture of students that have successfully completed an honors mathematics pathway or a non-honors mathematics pathway. At this time, my roster indicates 80 students comprised of 61 seniors and 19 juniors. Of these 80 students it might be useful to note that 10 students have 504 plans related to extra time on assignments and tests, chunking and the use of graphic organizers and 2 students have Individualized Education Plans requiring chunking, extra time, graphic organizers, the use of calculators and preferential seating.

Although Conrad is a sixth through twelfth grade building, a large number of our incoming students consist of students from other public middle schools, private schools and parochial schools. Some of these students may have been introduced to proofs or writing mathematical justifications through previous exposure to an integrated curriculum or new common core standards, while other students will not have been exposed to justifying their reasoning at all. This unit will help develop a better understanding of the structure of writing in mathematics and what constitutes a viable argument.

Conrad Schools of Science is a life sciences magnet school under the umbrella of the Red Clay Consolidated School District. It is located in the Wilmington area and draws students from New Castle to Newark. Students apply to Conrad by application, essay, interview, and science project. Each student receives a score on each admission measurement and all students meeting a certain percentage of a given total score have their name placed into a lottery system. As a result of the lottery system, we have a somewhat diverse group of academically performing students, meaning, that students with “A” averages have an equally likely chance in the lottery as students with a “C” average. Our school is operated on a daily rotating block schedule, which means I will have approximately 85 minutes with each class every other day.

Each grade level houses approximately 160 students and I will have three sections of Honors Calculus with approximately 26 students in each section. It is interesting to note that our current senior class has 147 students and 60 of these students are registered for my class, and 38 are registered for the Advanced Placement Calculus class, which means that 66.6% of the seniors at Conrad are enrolled in a calculus class during their senior year. This statistic could and should bring to the forefront many other discussions regarding calculus at the high school level, but that is for another paper.

Conrad has a minority population of approximately 35% with 14.6% of our students qualifying for free or reduced lunch services. In 2014, 95% of our tenth graders (our current seniors) passed the state assessment in mathematics. I have no data regarding the number of juniors that successfully passed the state assessment last year (our current seniors) because the assessment was in a trial year and many of our current seniors opted out of this test due to its timing. The test was administered within one week of the Advanced Placement testing window. Many students could not process taking one more assessment during that month. As such, I thought it might be useful to know that our current senior class, with a 96% participation rate, had an average SAT score in mathematics of 499. The average rate nationally on the SAT mathematics portion was 511. This data was collected from School Profiles and I have provided a snapshot of the data in figure 1, 2 and 3.[5]

Fall Enrollment

	2012-13	2013-14
Grade 6	163	173
Grade 7	167	169
Grade 8	166	173
Grade 9	176	167
Grade 10	155	169
Grade 11	139	159
Grade 12	116	143
Total	1,082	1,153

Enrollment by Race/Ethnicity

	2012-13	2013-14
African American	10.4%	11.2%
American Indian	0.1%	0.1%
Asian	2.7%	3.9%
Hispanic or Latino	20.3%	18.9%
White	66.0%	65.1%
Multi-Racial	0.6%	0.8%

Other Student Characteristics

	2012-13	2013-14
English Language Learner	7.9%	7.7%
Low Income	36.7%	14.6%
Special Education	3.0%	2.9%
Enrolled Full Year	98.9%	N/A

Figure 1

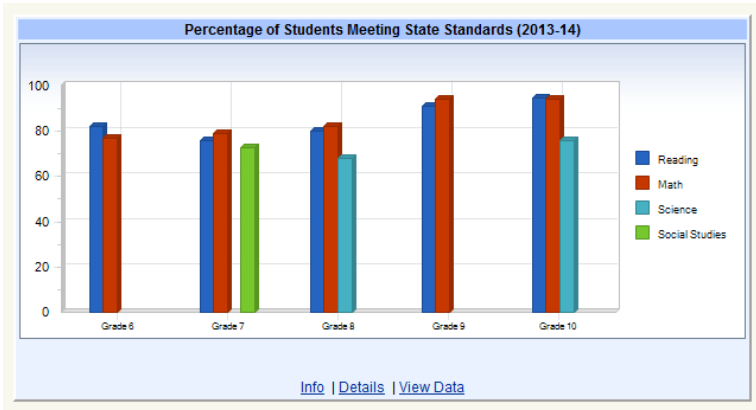


Figure 2

11th Grade School Day SAT Reasoning Test (2013-14)

	School	District	State
Math	499	461	443
Critical Reading	495	461	441
Writing	493	447	419
Total	1,487	1,369	1,303
Participation	96%	89%	93%

Info | Details

Figure 3

Unit Objectives

During this unit I would like to concentrate on the Common Core State Standards for Mathematical Practices

CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.
CCSS.Math.Practice.MP2 Reason abstractly and quantitatively
CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others.
CCSS.Math.Practice.MP6 Attend to precision

To meet this objective, I will develop the unit goals as, follows. At the conclusion of this unit students should be able to

- Define the concept of valid proof
- Identify the methods of proof
- Question the arguments of fellow students
- Present mathematics clearly and precisely to an audience of peers
- Proficiently construct logical arguments
- Understand the differences between proofs and other less formal arguments
- Distinguish a coherent argument from a fallacious one
- Make vague ideas precise by formulating them in mathematical language

History of Proof

According to Krantz, in the earliest days, mathematical concepts and thoughts were driven by the needs of the time. For instance, when the Egyptians and the Greeks were interested in surveying their land, the ideas of rectangles and triangles were explored. Then, as thoughts to building water tanks and arenas came to mind the thought of circles emerged and so ancient geometry was born. But when did the idea of a proof develop exactly? I could not find a point in history that signifies the birth of the proof. However, at some point the idea was conceived by mathematicians and philosophers that any mathematical assertions required proof or some type of justification. Once the idea was established that justification was needed, it was another leap for our ancient mathematicians to determine that they needed a set of rules for constructing this justification. It is difficult to determine when the first mathematical "proof" was introduced but according to my readings it seems the Babylonians had diagrams and tablets that showed what we now know as the Pythagorean Theorem as being true. As a result of this, ancient mathematicians had to decide what would be considered as a valid mathematical argument. And finally, it was Euclid of Alexandria who first formalized the way that we think about mathematical proofs. Euclid had developed definitions and axioms and then theorems. Thus, Euclid really defined the way that we study and define mathematics today.

What Defines a Proof?

In earlier times, if one could draw a picture, or give a compelling description, then that was all the justification that was needed for a mathematical statement to be accepted as fact. Sometimes one argued by analogy. At this time there was no standard for proof. The logical structure, or the "rules of the game", had not yet been created. Thus we are led to ask: What constitutes is a proof? A proof is a vehicle that is used in mathematics to convince someone else that a statement is true.[6] And how might one do this? We do this by creating a "tightly knit chain of reasoning, following strict logical rules that leads to a particular conclusion". Like a good story, a proof has a beginning, a middle and an end. Typically the beginning is the statement that we are assuming to be true, the middle is the tightly knit logical chain of events, and the end is the conclusion or what we are trying to prove. Thus, the difficulty lies in the middle section of the proof. "It's like putting in stepping stones to cross a river. If we put them too far apart, we're in danger of falling in when we try to cross. It might be okay, but it might not, and it's probably better to be safe than sorry". [7] For my classes this year, a proof will consist of any logical argument that can be easily followed through a series of connected steps, to justify the desired conclusion. My students may use any medium for their justifications such as pictures, diagrams, and less formal writing. The idea is that students will develop a sense of thinking in logical steps building upon the given, determining the middle and arriving at the end.

The Activities of the Unit

Activity 1: The first section will introduce the ideas of writing viable arguments through non-calculus related problems. These problems can be used earlier in the year to allow students to get a general feel of writing in mathematics with content that is accessible to them. Since students typically find it less stressful when working in a group, the opening of this unit will begin with a list of proofs for the students to attempt. Each class has seven groups of 4, so for each day I will have 7 different prove statements. Each prove statement will be written on a 3 x 5 index card and each group will randomly select a card. Students will be given enough time to think through the problem and design a strategy for their proof. Students will then be asked to share their argument with the rest of the class. As a class we will ask questions and make suggestions about what constitutes a good argument, and what does not constitute a good argument. On day three, I will introduce the principal of mathematical induction and students will be asked to apply mathematical induction to prove statements.

Day 1: Introduction to Writing Arguments
Problem Set

Problem 1 If m is an odd integer, then $3m^2+4m+6$ is also an odd integer.
Problem 2 If m is a real number, and m, m+1, m+3, are lengths of a right triangle, then m = 3
Problem 3 Let n be an integer. If n^2 is odd, then n is odd.
Problem 4 Let n be an integer. If n^2 is even, then n is even.
Problem 5 Let n be an integer. Then n^2 is odd if and only if n is odd.
Problem 6 Prove that the difference of two odd integers is always even.
Problem 7 Prove that the product of two even integers is always even

Day 2: Writing More Arguments
Problem Set

Problem 1 For every integer x, if x is odd then x^3 is odd.

Problem 2 Prove that for every integer x, x + 4 is odd if and only if x + 7 is even.

Problem 3 Prove that for every integer x, if x is odd then there exists an integer y such that $x^2 = 8y + 1$.

Problem 4 For every integer x, the integer x(x + 1) is even.

Problem 5 Prove that for any integer x, $x^2 + x + 1$ is odd.

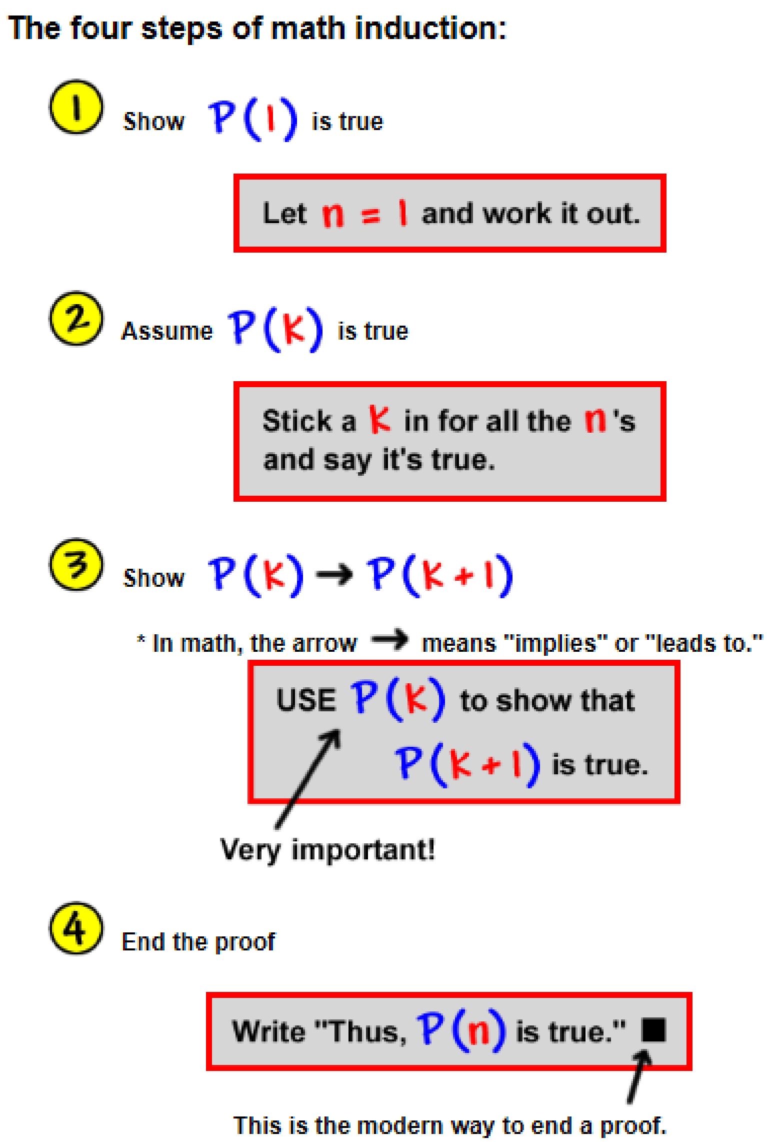
Problem 6 Prove that there is an integer solution to the equation $x^2 + y^2 = 13$.

Problem 7 Prove the Quadratic Formula, using $ax^2 + bx + c = 0$, show that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Day 3: Using Mathematical Induction

Introduction to the Principle of Mathematical Induction



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Mathematical Induction
Problem Set

Problem 1 Prove that $3^{k+1}-1$ is a multiple of 2

Problem 2 Prove that 3^n-1 is a multiple of 2

Problem 3 Prove that the sum of the odd integers would be $1 + 3 + 5 + ... + (2n-1) = n^2$

Problem 4 Prove that the sum of the natural numbers would be
$1 + 2 + 3 + ... + n = n (n + 1) / 2$

Problem 5 Prove that the sum of the squares of the natural numbers would be
$1^2 + 2^2 + 3^2 + ... + n^2 = n (n + 1) (2n + 1)/ 6$

Problem 6 Prove that for any positive integer number n , $n^3 + 2n$ is divisible by 3
Problem 7 Prove that $3^{2n+1} + 2^{n-1}$ is a multiple of 7

After enough time has elapsed in each class period. I will walk around the room, I am expecting to see students checking to see if a statement is true, but not actually proving the statement is true. We will discuss how rigorous does their argument need to be? Can you convince me that this will work for all such numbers? After prompting the students to dive deeper into the mathematics of the proof, I will have them present their arguments. As they present, I want my other students to ask questions and challenge them. I am hoping by using this approach, the conversation will begin. This lesson will be continued for three class periods. At the end of these class periods, the students will have begun to develop a sense of what I mean when I say a logical sequence of thoughts.

Activity II: This section will include problems that will be posed to the students to prove using items either reviewed or learned in the calculus classroom. Each statement will contain directions as to which definition or theorem to apply.

Problem Set

Problem 1: Using your knowledge concerning parallel and perpendicular lines, find the value of k in $2x + ky = 3$ that would make this line parallel to the line $4x - 9y = -12$. What value of k would make the lines perpendicular?

Problem 2: Apply the Intermediate Value Theorem to justify that for $f(x) = 2x^3 - 5x + 5$ there is at least one value for c such that $f(c) = \pi$

Problem 3: If a function is continuous on the closed interval [4,6] and is known to be positive at x = 4 and negative at x = 6. Apply the Intermediate Value Theorem to identify what this means about $f(x) = 0$

Problem 4: Use the Sandwich Theorem to prove that $\lim_{\theta \to 0^+} \frac{\sin \theta}{\theta} = 1$

Problem 5: Find the value of k that would make the function $f(x) = \begin{cases} \frac{x^2 + 2x - 15}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$

continuous, justify your response by applying the definition of continuity.

Problem 6: Using the limit definition of the derivative prove that the derivative of $f(x) + g(x)$

is $f'(x)+g'(x)$

Problem 7: Using the limit definition of the derivative prove that the derivative of $f(x)g(x)$ is $f(x)g'(x)+g(x)f'(x)$

Activity III: During activity three students will be given competed “proofs” or completed problems and asked to either justify that they are correct or to identify and correct the mistake that has been made. Since academic standards call for increased rigor, students need to be able to examine their own thinking and the thinking of others. As my students begin to question the logic of others, they become more adept at examining errors, identifying flawed logic and deepening their understanding of the concept.

Problem Set

Proof 1: Find the fallacy in the proof that 1 + 1 =0

Let a = 1 and b =1 then

$$a=b$$
$$a^2=b^2$$
$$a^2-b^2=0$$
$$(a-b)(a+b)=0$$
$$\frac{(a-b)(a+b)}{(a-b)}=\frac{0}{(a-b)}$$
$$(a+b)=0$$
$$a+b=0$$
$$thus\quad 1+1=0$$

Proof 2: Find the fallacy or mistake in the proof that $\cos^4 x-\sin^4 x=1-2\sin^2 x$

$$\cos^4 x-\sin^4 x=1-2\sin^2 x$$
$$(\cos^2 x+\sin^2 x)(\cos^2 x-\sin^2 x)$$
$$(\cos^2 x+\sin^2 x)(1)$$
$$(1+\sin^2 x+\sin^2 x)(1)$$
$$1-2\sin^2 x$$

Proof 3: Find the fallacy or mistake in the proof that the solution to $\sqrt{5x+2} = -4$ is $x = \frac{14}{5}$

$$\sqrt{5x+2} = -4$$
$$5x + 2 = 16$$
$$5x = 14$$
$$x = \frac{14}{5}$$

Proof 4: The equation of the tangent line to the curve $f(x) = x^3$ is $y = 3x^3 - 6x^2 + 8$ at x = 2

$$f(x) = x^3$$
$$f'(x) = 3x^2$$
$$f(2) = 8$$
$$y - 8 = 3x^2(x - 2)$$
$$y - 8 = 3x^3 - 6x^2$$
$$y = 3x^3 - 6x^2 + 8$$

Proof 5: Find the fallacy or mistake in the proof of the derivative of $f(x) = \sqrt{1 - x^2}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\sqrt{1-(x+h)^2} - \sqrt{1-x^2}}{h}$$

$$\lim_{h \to 0} \frac{\sqrt{1-x^2+2xh+h^2} - \sqrt{1-x^2}}{h} \bullet \frac{\sqrt{1-x^2+2xh+h^2} + \sqrt{1-x^2}}{\sqrt{1-x^2+2xh+h^2} + \sqrt{1-x^2}}$$

$$\lim_{h \to 0} \frac{1-x^2+2xh+h^2-(1-x^2)}{h(\sqrt{1-x^2+2xh+h^2} + \sqrt{1-x^2}}$$

$$\lim_{h \to 0} \frac{1-x^2+2xh+h^2-1+x^2}{h(\sqrt{1-x^2+2xh+h^2} + \sqrt{1-x^2})}$$

$$\lim_{h \to 0} \frac{2xh+h^2}{h(\sqrt{1-x^2+2xh+h^2} + \sqrt{1-x^2})}$$

$$\lim_{h \to 0} \frac{h(2x+h)}{h(\sqrt{1-x^2+2xh+h^2} + \sqrt{1-x^2}}$$

$$\lim_{h \to 0} \frac{2x+h}{\sqrt{1-x^2+2xh+h^2} + \sqrt{1-x^2}}$$

$$\frac{2x}{\sqrt{1-x^2} + \sqrt{1-x^2}}$$

$$\frac{2x}{2\sqrt{1-x^2}}$$

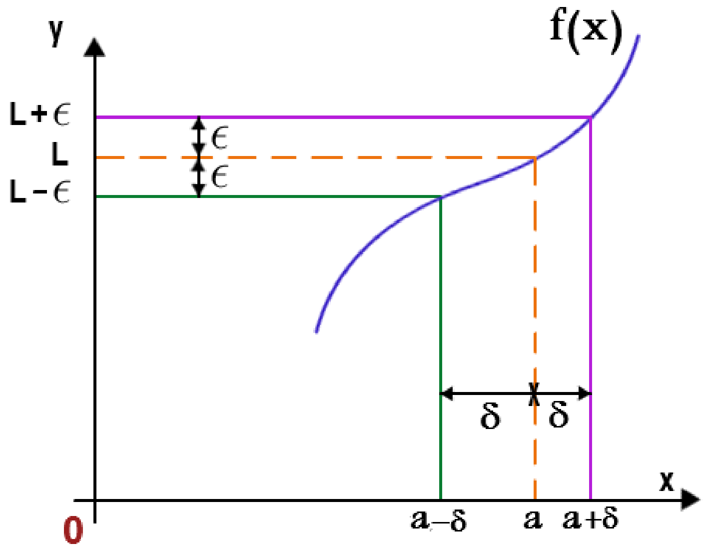
$$\frac{x}{\sqrt{1-x^2}}$$

Activity IV: In this activity I plan to develop the epsilon delta definition for students

The formal definition of a limit using epsilon and delta

Let *f* be a function defined on some open interval that contains the number *a*, except possibly at
x = *a* itself. Then the statement $\lim_{x \to a} f(x) = L$ means that for every number ε > 0 there exists a
δ > 0 such that if 0 < |*x* - *a*| < δ, then |*f(x)* - *L*| < ε.

This is illustrated in the diagram below, if we know that $\lim_{x \to a} f(x) = L$, then given any epsilon value greater than zero, I can find a neighborhood around x = a such that its limit will be within epsilon units of L.



As an example I can look at the above function as $f(x) = (x-5)^3 + 4$ and we can say that $\lim_{x \to 5} f(x) = 4$

Thus by using the epsilon delta form of the limit I can show that given any epsilon value greater than zero, I can find a delta

$0 < |x-5| < \delta$, then $|f(x) - 4| < \varepsilon$

$|(x-5)^3 + 4 - 4| < \varepsilon$

$|(x-5)^3| < \varepsilon$

$|(x-5)|^3 < \varepsilon$

$\sqrt[3]{|(x-5)|^3} < \sqrt[3]{\varepsilon}$

$|x-5| < \sqrt[3]{\varepsilon}$

Which means given any epsilon such as $\varepsilon = 0.027$, then $\delta = \sqrt[3]{0.027} = 0.3$

Using the Epsilon Delta to Practice Formally Defining Limits
Problem Set

1. True or False: The epsilon-delta definition of a limit starts with a $\delta > 0$ for which an $\varepsilon > 0$ can be found.

2. Given $\lim_{x\to 1}(-3x^3 + x + 4) = 2$, use the epsilon-delta definition of a limit to find values of δ that correspond to $\varepsilon = 0.1$

3. Prove using epsilon delta form that $\lim_{x\to 3}\frac{2}{x+3} = \frac{1}{3}$

4. Prove using epsilon delta form that $\lim_{x\to 9}(2+\sqrt{x}) = 5$

5. Using the definition of limits, find the delta which corresponds to the $\varepsilon = .02$

$$\lim_{x\to\frac{\pi}{4}}\sin x = \frac{\sqrt{2}}{2}$$

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Curriculum Unit Title: Providing Opportunities for Students to Reason, Justify and Prove in the High School Calculus

Author: Michelle Northshield

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

To explore mathematical ideas, make conjectures and construct viable arguments using the concept of proofs
This unit focuses on the Common Core Mathematical Practices *(MP1 – Make sense of problems and preserve in solving them. MP2 – Reason abstractly and quantitatively. MP-3 Construct viable arguments and critique the reasoning of others. MP – 6 Attend to precision*)

ESSENTIAL QUESTION(S) for the UNIT

How can students be encouraged to justify, reason and prove in a high school calculus course?
What constitutes a viable argument or proof?

CONCEPT A	CONCEPT B	CONCEPT C
Introduction to writing viable arguments using non calculus problems.	Introduction to proving concepts from algebra through calculus	Formal Epsilon Delta Proof
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
What constitutes a viable argument? What do we mean when we say prove?	What is mathematical induction? Is this justification viable? If not, explain how to construct a viable argument from the given submitted proof.	How can we prove the limit of a particular function?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
Proof even, odd, multiple,	Direct proof, indirect proof and Mathematical induction parallel and perpendicular Limits Product Rule, Quotient Rule	Epsilon –Delta limits

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Cheng, Eugenia. "How to Write Proofs: A Quick Guide." 1 Oct. 2004. Web. 13 Dec. 2015. <http://cheng.staff.shef.ac.uk/proofguide/proofguide.pdf>.

Fitzgerald, Franklin. "Proof in Mathematics Education." *Journal of Education*: 35-45. Print.