Helping Students Justify and Reason Division as the Inverse of Multiplication

Moira Snider

Introduction:

I recently attended the International Literacy Association's National Convention in St. Louis, Missouri. While reviewing my notes, I came across a sentence I had written down during a workshop: **Whoever explains learns.** It occurred to me that I could base my Delaware Teacher Institute unit for Mathematical Proof and Reasoning around this simple sentence. **Whoever explains learns,** compliments many of the 8 Common Core Standards of Mathematical Principles. Making sense of problems and persevering in solving them, reasoning abstractly and quantitatively, construct viable arguments and critique the reasoning of others, and modeling with mathematics are a few principles that could be touched on with that seemingly simple statement. Thinking back throughout my teaching years, it is evident that students struggle with problem solving and persevering through them. I hope to have developed a unit that incorporates that principle, and deepens their understanding that division is the inverse of multiplication. The guiding Common Core State Standard (CCSS) for this unit is Operations & Algebraic Thinking.

Demographics:

The Christina School District is one of Delaware's largest public school systems, serving about 17,400 students. The district serves both suburban and urban areas of Northern New Castle County. Christina School District has 18 elementary schools, 4 middle schools, and 3 high schools. I teach at Thurgood Marshall Elementary School. It is a suburban K-5 school located in the Newark/ Bear area. Enrollment for 2015-2016 school year is 733 students. Our population includes 33% African American, 19 % Asian, 8% Hispanic/Latino, 34% White, and 6% Multi-Race. Of that population 9% are English Language Learners from diverse backgrounds, 20 % are Low Income, and 7% have Individualized Education Plans.

Class Structure:

My 2015/16 third grade classroom consists of 25 students with a wide range of academic abilities. It is a co-teaching classroom. I am the 'regular education' teacher and my partner, Kelly Schneider is the 'special education' teacher. This year we have 7 students with I.E.Ps. The range of disabilities runs from learning disabled to behavior to blindness. There is also a paraprofessional in our room assigned to work one to one with our blind student. Additionally, we have 5 students with 504 plans. My daily Math block occurs at

the end of the day. We begin our math block using the Smart Board for our math calendar. This consists of math routines such as Today's Number, Number Talks, Patterns and Algebra, Geometry, and Problem Solving. It is a fast paced review of topics which changes daily. It normally takes about 10 minutes. Then we present to the whole class the math lesson for that day. This usually takes about 20- minutes. The rest of the time, approximately 45 minutes, is spent in flexible groups. My partner will take the neediest students and work with them for the entire 45 minutes. The other students are then placed into groups and they will rotate through three 15 minute placements. One is 'teacher time', the next is 'independent work', and finally they will work at a center with a partner. They will rotate through four centers over the course of a week, one center per day. The four centers consist of problem solving, computers, fast fact fluency, and math games. However, Fridays are slightly different. Instead of breaking into flex groups, a new math games is introduced and practiced as a whole class. This is also a time when the special education teacher will pull and work more intensely with students needing extra instruction. The main curriculum used is *Math Connects* from McGraw Hill (2011). It is supplemented with other resources including Investigations in Numbers, Data, and Space by TERC.

Rationale:

I hope to develop a unit that deepens my students' understanding of both multiplication and division and their relationship to each other, while focusing on persevering and making sense of problems. Multiplication and division understanding is essential for students in grade three. Having a solid background with these two operations lays the foundation for a deeper understanding of more complex mathematics later in life. I feel *Math Connects* does not emphasize nor does it develop a strong foundational understanding of multiplication and division for my students. It relies heavily on rote memorization of facts. "Understanding of these operations is limited if it does not also include an understanding of how and why these algorithms work."[1] For students with a learning disability, research has shown that exposure to concrete and semi-concrete representations are helpful in allowing them to succeed with multistep problems[2]. This is what is missing in the *Math Connects* curriculum for multiplication and division. My unit will expose students to concrete, hands on learning. There will be a strong emphasis of using models to explain multiplication and division problems. Another area I want my unit to focus on is the use of both partitive and measurement type of division problems. The partitive model of division lets the total number of groups known, but the size of each group is unknown. On the other hand, the measurement model of division shows the size of each group but the number of the group is unknown. Exposing students to many different division scenarios, both partitive and measurement will help increase their conceptual understanding. Having a firm grasp on division and the part/whole connection lays the foundation for understanding fractions as well.

Content:

When I was told I was going to be in Mathematical Proof and Reasoning seminar I was a little intimidated. It definitely wasn't my forte in school and how would I ever teach this to third graders? Well, after many seminars and listening and learning from my professor and colleagues, I came to the conclusion that I could share my new learning with my students. Mathematical Proofs and Reasoning boils down to explaining and justifying work. This is something we are always asking our third graders to do, justify or prove their answers. When we ask students to 'show' their work, we are asking them to share their reasoning. Mathematical proof and reasoning is essential for doing all mathematics, even the mathematics for elementary school students. As we all know, learning is a social process. Students are able to learn mathematical concepts through interactions with their peers, through repeated negotiations and struggle, and finally through shared understanding. This process of working through problems, explaining, proving, and sharing their thinking allows students to become 'owners' of new learning[3]. **Whoever explains learns!**

Problem Solving:

One of the eight Common Core mathematical practices states that proficient students make sense of problems and persevere in solving them. Unfortunately, this is a struggle for some children. Mathematical problem solving can be difficult. As teachers we must encourage productive struggle and model how to persevere while problem solving. In order for this process to flourish we must create a classroom atmosphere where risk taking is promoted, valued and honored. Students should feel respected and know that their mathematical thinking is heard. It has been suggested that productive struggle can be fostered using four strategies. The first one is teacher **questioning**. Teachers ask probing questions to move along a student's thinking. This allows a student to clarify and perhaps revise their original thinking to a problem. Next, teachers **encourage** students to reflect on their work and the fact that they have struggled. We need to remind students that it is what persevering looks like. The third strategy is to allow for **time**. Time to struggle, time to reflect, and time to work! Finally, **acknowledge** this struggle as a part of the learning process[4]. Productive struggle helps students make meaning and sense of mathematical concepts.

Collaborative Protocols:

Learning is a social process. When our students share or justify their explanations to problems they are actually cementing their own thinking. As their peers question them this allows them to clarify their thinking if needed. Cooperative learning can encourage mathematical concepts to be fully understood. One way of supporting cooperative learning is through the use of the strategy **Think/Pair/Square.** First, a problem is posed and each student individually works on it for a set amount of time. Five minutes would probably suffice for elementary students. Then, students pair up, and work on the

problem together. Both will have to explain their thinking. After enough time has been given for discussion and for finding a solution, those partners will then partner up with another set of partners—the square! Together all four will discuss and share their thinking. Finally, each group can then share their solution and strategies with the class. This strategy of Think/Pair/Square can easily be differentiated by level of problems and partners. One idea would be to color code problems on sheets of paper and students would then have to find a partner who has the same color as they do. This strategy actively engages students and allows for more communication and time for them to persevere when problem solving.

Another collaborative protocol is the use of **Gallery Walks**. This activity allows students to display, discuss, and provide feedback to their peers' work. Once students have 'squared up' and collaboratively worked on and solved a mathematical problem, it is hung around the classroom. Students are then instructed to walk around the room and analyze and evaluate the work. I explain that in a real art gallery, people often walk around looking at the artist's pictures and compliment and critique their work. It is helpful to review expectations or guideline prior to beginning a gallery walk. A helpful document titled; *Peer Critique Protocol*[5] was discovered from The Engage NY website. It describes four non-negotiables: **Be Kind, Be Specific, Be Helpful,** and **Participate.** It also give guidelines about how to critiques a peer's work. It suggests ways for students to respond such as, "I notice…" or "I'm confused by this part….." Time spent modeling expectations prior to a Gallery Walk will lead to a more harmonious and productive experience where students feel comfortable explaining, justifying and discussing work. The Peer Critique Protocol can be found in Appendix B.

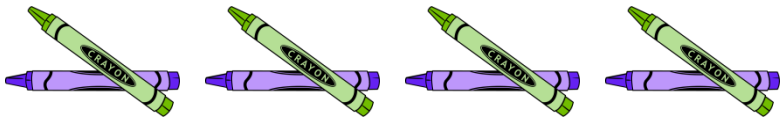
Math Knowledge:

There are nine Common Core standards in the domain of Operations and Algebraic Thinking that deal with both multiplication and division. By the end of third grade our students are expected to be able to represent and solve problems involving multiplication and division. Understand properties of multiplication and the relationship between multiplication and division. Multiply and divide within 100. Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Having a firm grasp of the concepts behind both multiplication and division will help our students as they encounter more advanced mathematic later in their school years.

Multiplication:

Student often use their addition knowledge to begin to make sense of multiplication. This is why multiplication is usually introduced as **repeated addition**. This is when a certain number of items are grouped together and repeatedly displayed. They are equivalent groups. $4 \times 2 = 8$ is modeled using 4 groups of 2 crayons.



2 + 2 + 2 + 2 = 8

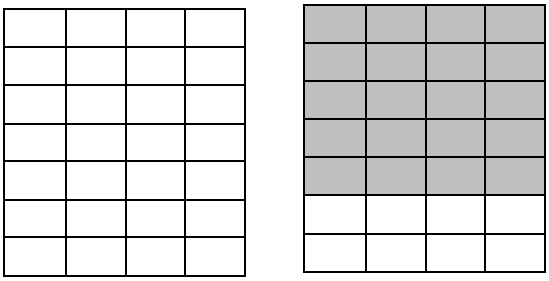
A **number line** can also model multiplication when equal sized jumps are made.

0 2 4 6 8 (4x2=8)

Skip counting by multiples is a strategy used to represent multiplication. When skip counting, students count by the second factor the number of times the first factor tells them to. For example, 4 x 2 = 8 would be modeled with skip counting by saying 2,4,6,8.

Multiplication can also be modeled using **arrays**. This is when objects are arranged into rows and columns. 4 x 2 could be represented as:

Area models also reinforce the concept of multiplication and are similar to arrays. The product is the number of square units, or its length times its width. Arrays and area models are wonderful tools that can be used to introduce the Distributive Property of Multiplication and reinforce the idea of decomposing numbers. For example, to decompose 7 x 4, we could split the array into two pieces: (5 x 4) + (2 x 4).



The **Cartesian product** is another way to visualize multiplication. This occurs when two independent sets are known. Each item in one set is paired up with items in the other set. The product is the total number of possible joined items. For example: There are two types of ice cream cones, sugar and waffle. There are four choices of ice cream, vanilla, chocolate, strawberry, and cookie dough. How many different choices do you have? This could be modeled with ordered pairs.

(sugar, vanilla) (sugar, chocolate) (sugar, strawberry) (sugar, cookie dough)
(waffle, vanilla) (waffle, chocolate) (waffle, strawberry) (waffle, cookie dough)

It is important to represent multiplication in a variety of ways. If students only view multiplication as repeated addition we are limiting their conceptual understanding. They will develop misconceptions when they begin working with quantities other than whole numbers. One way to prevent this from happening is to think of multiplication as a **scalar operation**. This is where the first factor 'operates' or act upon the other factor. In the number sentence 3 x 4 = 12, the factor 3 will 'scale' or resize the second factor, 4. For example, if a recipe calls for 3 eggs per serving, how many eggs will be needed for 4 servings? The first factor, **3** eggs, will resize the second factor of 4 to 12 because we need 3 times as many eggs for 4 servings.

Thinking back to arrays, in the problem: If there are two rows of marbles and each row contains three marbles, how many marbles altogether? A student could draw an array depicting two rows of three.

Viewing multiplication as a scalar operation, we should notice that since there are three marbles in the first row and there are a total of two rows, there is two times as many marbles as the first row. In this case two is the scalar factor (**2** x 3 = 6). There are two times as many as three. The first factor describes what is to happen to the second factor. The first factor 'scales' or changes the second factor. This change is multiplicative. When our students have this understanding of the scalar as the multiplication powerhouse and are given many opportunities to represent and discuss an assortment of problems, they will have a richer understanding of multiplication.

Division:

Division can be defined by its inverse relationship with multiplication. Division word problems can be classified as either partitive or quotitive. The chart below explains the difference between the two types.

	Partitive Model of Division (Equal/ Fair Shares) $15 \div 3$	**Quotitive Model of Division** (Measurement/ "*Goes into*") $3\overline{)15}$
Example:	Mrs. Snider bought 25 apples to be shared equally among her 5 students. How many apples will each student get?	Mrs. Snider has 25 inches of ribbon. If it takes 5 inches to make one bow, how many bows can she make?
The Known:	The number of items or the size of the group. (25 apples) Total # of groups. (5 students)	The number of items or the size of the group. (25 apples) The # of items per group. (5 inches per bow)
The Unknown:	The # of items per group. (? apples per student)	The # of groups. (? # of bows)
Common Strategy Used to Solve:	Splitting a set/ partitioning a collection	Repeated subtraction

Students need to be given many opportunities to work through real world division problems so they can develop a thorough understanding about division. Since the partitive model of division deals with fair shares, it is often used to introduce division. Students are able to easily connect to the idea of sharing items or quantities evenly. However, we must also expose students to problems that deal with the quotitive or measurement model of division to promote a deeper understanding. This will be helpful when our students begin working with fractions, decimals, and algebraic expressions. Refer to Appendix A for more division problems that deal with both the partitive and measurement model of division. In addition to having students work through real life problems, they should also be given the opportunity to create their own division scenarios. It will help develop greater conceptual understanding.

Vocabulary:

Only after giving ample practice with working through problems and having students model their thinking using concrete manipulatives, pictures, or language should the standard algorithm be introduced. Students will then be able to make a conceptual connection to the vocabulary of multiplication and division.

Multiplication:

Multiple - The multiple of a number is the product of the number and any other whole number. (2, 4, 6, 8 are multiples of 2.)
Factor - A number that will divide evenly into another number. (The factors of 8 are 1, 2 and 4).
Product - The answer when any two or more numbers are multiplied together.

2 x 4 = 8 (factor x factor = product)

Division:

Dividend- size of the group or quantity to be divided into
Divisor- number of groups or the quantity you are dividing by
Quotient- number obtained by dividing one quantity by another or the answer

24 ÷ 6 = 4 (**dividend** ÷ **divisor** = **quotient**)

4 (quotient)
(divisor) 6$\sqrt{24}$ (dividend)

Division can be explained by its inverse relationship to multiplication. It is this relationship between multiplication and division that lays the foundation for fact families.

2 x 4 = 8 4 x 2 = 8 8 ÷ 4 = 2 8 ÷ 2 = 4

Having a solid grasp on division begins with having a clear understanding of related vocabulary. We can help our students with this new terminology by making connections to vocabulary they already know about multiplication; factor, missing factor, and product. In the problem: You have 30 cents to spend on stickers. If one pack cost 10 cents, how many packs can you buy? The standard algorithm would be, $10\sqrt{30}$. By relating division terms to multiplication we could say the total number of items (30) is the dividend or the product, the number of each group (10) is the divisor or factor, so the quotient would be the number of groups or the missing factor.

missing factor
(factor)10$\sqrt{30}$ (product)

However, when problem solving students need to be able to distinguish which fact to use based on the situation. Students need to be able to justify or prove why they may choose division over multiplication. This is where spending time developing a true conceptual understanding of both multiplication and division comes into play. The time spent working through real life problems will offer an advantage to our students and improve their conceptual understanding.

Learning Activities:

The following learning activities are suggested to develop student understanding of multiplication, division, and their relationship to each other.

Activity One:

Essential Questions: What is multiplication and how is it used in the real world? What are the different ways of representing multiplication?

Procedure: Introduce, read, and discuss the book, *Each Orange Had 8 Slices: A Counting Book* by Paul Giganti, Jr[6]. Display the numbers 3, 15, and 5 and ask students to create their own multiplication stories similar to the ones read about in the book using those three numbers. Review and discuss correctly written multiplication stories. Display any incorrect stories. Ask students to pair up with their 'elbow' partner and try to figure out what operation they saw shown in the incorrect examples. Discuss. Allow students the opportunity to revise or confirm their multiplication stories.

Next, show how multiplication can be modeled and represented in different ways using a graphic organizer. Introduce the four square organizer. See Appendix C for the organizer or it could be simply done by folding plain paper. The four square organizer shows multiplication as Equal Groups, Repeated Addition, Array, and Multiples. Discuss and support students as they build their own organizer. Use 3 x 5 = 15 as the example. Discuss how this organizer could help to create multiplication stories. Allow students to orally come up with new stories.

Activity Two:

Essential Question: What makes a good multiplication story?

Procedure: Students will eventually work in groups of four. Gather colored index cards. Since I have 25 students in my class there were 6 sets of 4 colored index cards. One set had five in them. Write three numbers on each card. Each set will have different numbers. For example the four yellow index cards had 3, 12, and 4 written on them. The 4 blue index cards had 32, 8, and 4 written on them. Hand out index cards to individual students.

Instruct them to work independently to create a multiplication story that would go with their numbers, solve the problem, and represent their solution using the four square organizer. Give them 5 minutes to work and **think** independently. After five minutes, students will find a partner with the same colored index card. The **pair** will then collaborate together, evaluate each other's story, revise if needed or develop a new story together. Give about eight minutes for this work. After time is up, the pair will find another pair with the same color index cards. This group of four or the '**square'**, will have the challenge of choosing the best story-or developing a new one- and displaying it onto a large piece of chart paper. They also need to include the four square organizer related to their multiplication story. Give about fifteen minutes to complete the task.

Then display the chart papers around the classroom for a gallery walk. Students will rotate around the room observing, analyzing, and evaluating the other pieces of work. Instruct the students to use a post-it note to record down their observations, and then leave it on the chart paper. Remind students to **Be Kind, Be Specific, Be Helpful,** and **Participate.** Then the groups return to their poster/ chart paper and make any needed revisions based from their peers' observations/ feedback. Debrief with a class discussion.

The following learning activities will be used to develop student understanding of division and how it relates to multiplication. You will see that the activities are similar to the ones already introduced.

Activity Three:

Essential Questions: What is division and how is it used in the real world? What are the different ways of representing division?

Procedure: Introduce, read, and discuss the book, *The Doorbell Rang* [7] by Pat Hutchins. Have the students use manipulatives to model the sharing of the cookies as the book is read. Record the related division sentence for each scenario throughout the book. Discuss. Write down the numbers 5, 15, and 3. Have students develop division stories using those numbers. Share and discuss. Introduce a new four square graphic organizer for division. This represents division as Equal Groups, Repeated Subtraction, Arrays, and Number Line. Model it and support students as they complete it for their division story of $15 \div 3 = 5$. See Appendix D for organizer.

Activity Four:

Essential Question: What makes a good division story?

Procedure: Using the 'think, pair, square' and gallery walk format, students will create division stories. Each set will have a different set of numbers. The same index cards could be used again or present them with new ones. This time they will be instructed to

work independently to create a division story that would go with their numbers, solve the problem, and represent their solution using the four square organizer. They will then pair up, then 'square' up and finally display their completed story and organizer. Then students will rotate around the room observing, analyzing, and evaluating work. They will leave feedback on the chart papers using post-it notes. Students will finally return to their original chart and make any needed revisions based on feedback from their peers.

Activity Five

Essential Question: How is division related to multiplication?

Procedure: Explain how arrays can help represent division as the inverse of multiplication. Show completed multiplication/ division organizer. (Appendix E) Discuss, guide, and support students as they complete their own organizer. (Appendix F) Finally, using the 'think, pair, square' and gallery walk format hand out the index cards and have them create **both** multiplication and division stories from the 3 numbers on the index card. Follow the same format as previous activities.

Conclusion:

These learning activities will help to develop students' conceptual understanding of both multiplication and division and how they relate to each other. As stated earlier, learning is a social process. The goal is that through collaboration, negotiations with their peers, justification of their work, and perseverance throughout the activities, students will build upon their learning. **Whoever explains learns!**

Notes:

[1] Otto, Albert Dean. *Developing Essential Understanding of Multiplication and Division for Teaching Mathematics in Grades 3-5*. Reston, VA: National Council of Teachers of Mathematics, 2011.

[2] Hord, Casey, and Jill Newton. "Investigating Elementary Mathematics Curricula: Focus on Students With Learning Disabilities." *School Science and Mathematics*, April 1, 2014, 191-201.

[3] Cai, Jinfa, and Cliff Sloyer. "Making Indirect Reasoning Accessible: A Problem-Based Learning Approach." *EduMath 17, 2-15.*

[4]Warshauer, Hiroko. "Strategies to Support Productive Struggle." *Mathematics Teaching in the Middle School*, March 2015, 390-393.

[5] https://www.engageny.org/resource/grades-3-5-ela-curriculum-appendix-1-teaching-practices-and-protocols (Gallery Walk/Peer Critique Protocol)

[6] https://www.youtube.com/watch?v=hZbSeWnVHRQ (video of Each Orange Has 8 Slices)

[7] https://www.youtube.com/watch?v=ESHLF92_rBw (video of The Doorbell Rang)

Bibliography:

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Magrude, Robin, and Cindy Jong. "Beyond Cookies: Understanding Various Division Models." *Teaching Children Mathematics*, February 1, 2014, 367-72.

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Warshauer, Hiroko. “Strategies to Support Productive Struggle.” *Mathematics Teaching in the Middle School*, March 2015, 390-393.

Appendix A

Partitive/ Fair Sharing

Two people share eight cookies. How many cookies does each person get if they share fairly?

Ms. Smith has 21 students in her class. She needs to separate them into three groups. How many students will be in each group?

You have 30 cents to spend. You buy 3 packs of stickers. How much did each pack cost?

Quotitive/Measurement:

Four cookies are handed out to each person. There are eight cookies. How many cookies will each person get?

Ms. Smith has 21 students. She wants to divide them into groups of seven. How many groups will she have?

You have 30 cents to spend on stickers. If one pack cost 10 cents, how many packs can you buy?

Appendix B

Peer Critique Protocol

Non-Negotiables

1. Be Kind: Always treat others with dignity and respect. This means we never use words that are hurtful, including sarcasm.

2. Be Specific: Focus on particular strengths and weaknesses, rather than making general comments like "It's good" or "I like it." Provide insight into *why* it is good or what, specifically, you like about it.

3. Be Helpful: The goal is to positively contribute to the individual or the group, not to simply be heard. Echoing the thoughts of others or cleverly pointing out details that are irrelevant wastes time.

4. Participate: Peer critique is a process to support each other, and your feedback is valued!

Guidelines

1. Have the author/designer explain his or her work and explain exactly what type of critique would be helpful (in other words, what questions does he or she have or what is s/he confused about that s/he would appreciate help with).

2. The critique audience should begin comments by focusing on something positive about the work ("warm" feedback), then move on to constructive sharing of issues or suggestions ("cool" feedback).

3. When critiquing a peer's work, use "I" statements. For example, "I'm confused by this part," rather than "This part makes no sense." Remember the three important phrases:
   - "I notice…."
   - "I wonder…."
   - "If this were my work, I would…."

4. Use questions whenever possible. For example, "I'm curious why you chose to begin with…?", or "Did you consider adding…?"

Appendix C

Equal Groups:	**Repeated Addition:**
Array:	**Multiples:**

Equation:

Appendix D

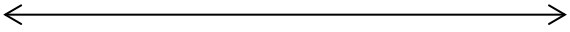
Equal Groups

Repeated Subtraction:

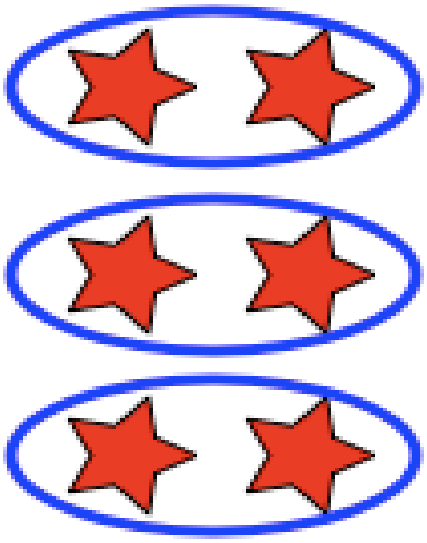
Equation:

Array:

Number Line:



Appendix E

Multiplication	**Division**
There were 3 friends. Each friend got 2 cookies. How many cookies altogether?	There were 6 cookies to be shared among 3 friends. How many cookies did each friend get?
 	

Appendix F

Multiplication	Division

Appendix G

Common Core State Mathematical Standards:

Operations & Algebraic Thinking:

CCSS.Math.Content.3.OA.A.3
Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.

CCSS.Math.Content.3.OA.A.4
Determine the unknown whole number in a multiplication or division equation relating three whole numbers

CCSS.Math.Content.3.OA.B.5
Apply properties of operations as strategies to multiply and divide

CCSS.Math.Content.3.OA.C.7
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division

Common Core State English Language Art Standard:

CCSS.ELA-Literacy.SL.3.1
Engage effectively in a range of collaborative discussions

CCSS.ELA-Literacy.SL.3.1.c
Ask questions to check understanding of information presented, stay on topic, and link their comments to the remarks of others.

CCSS.ELA-Literacy.SL.3.6
Speak in complete sentences when appropriate to task and situation in order to provide requested detail or clarification.

Curriculum Unit Title	Helping Students Justify and Reason Division as the Inverse of Multiplication	**Author**	Moira Snider

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

The operation of both division and multiplication is used to solve every day real world problems. There are different ways of reasoning about a problem which could lead to different ways of modeling and representing the problems with different equations.

ESSENTIAL QUESTION(S) for the UNIT

How is multiplications and division used in the real world?
What is the relationship between multiplication and division?

CONCEPT A	**CONCEPT B**	**CONCEPT C**
Multiplication	Division	Inverse Relationship
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How is multiplication used in the real world?	How is division used in the real world?	What is the relationship between multiplication and division?
VOCABULARY A	**VOCABULARY B**	**VOCABULARY C**
multiple factor product	dividend divisor quotient	inverse operation fact family

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

Colored index cards

Chart paper and markers

Copy of Each Orange Has 8 Slices and The Doorbell Rang