First Graders Have Something To Prove

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Introduction

I hated ninth grade Geometry! Especially the third marking period when we were taught proofs. I failed that marking period. Up until that point I was an A math student. I was great at memorizing. But I just could not understand how to tell someone why adjacent angles on a line would have the same measurement, or how a circle could help me find the measurement to the angles of a triangle. That did not deter me from taking Algebra II/Trig, even though this information could have been helpful when learning about sine, cosine and tangent. I was back to memorizing. I still cannot tell you what these terms mean. Apparently, much of my mathematics education was memorizing. No one ever really expected me to know why math works the way it does, they just expected me to be able to plug values into different algorithms and come up with the correct answer. I don't remember if we discussed why the algorithms worked, or if we worked cooperatively to find solutions. I do remember my mother laughing when I had to use my "air pencil" to figure out how much I would save if the store was giving 15% off.

The following might explain this problem. Thomas Carpenter and colleagues researched how young children learn addition and subtraction and they speculated that the transfer from informal modeling and counting to the use of algorithms is a "critical stage". It appeared to them that when children learn formal procedures they stop analyzing the problems they are solving. This says to me that we teachers are not doing a good enough job connecting the procedure with the process. It also may be that using an algorithm is so much easier for a child. Consider this outline of the process of solving a simple addition problem:

> "To solve a typical problem one must first understand its implied semantic meaning. Quantifying the elements of the problem comes next (e.g. choosing a unit and counting how many). Then, the implied semantics of the problem must be expressed in the syntax of addition and subtraction. Next the child must be able to carry out the procedural (algorithmic) steps of adding and subtracting. Finally, the results of these operations must be expressed.[1]

Not only do they need to set up an equation for solving, they need to understand what the numbers represent and what is to be done with them. Once they are taught the formula, it's a matter of memorizing rather than processing. It is in their best interest that we expect students to know and be able to explain the numbers being manipulated in that formula! In fact, this has been made quite apparent later in our school year when the students start learning two-digit addition. There is one specific example on our unit test where the students are given superfluous information in a word problem to test whether they are making sense of the numbers given. It is something like this: Jordan has 20 pieces of candy that cost 5 cents and 9 pieces of candy that cost 3 cents. How many pieces of candy does Jordan have? If the students have just memorized the algorithm for addition, how will they know which of the four numbers to use in their answer? Should they add all four numbers?

Researchers Ball and Bass wrote about this need for reasoning and they stated that just knowing the mathematical procedures is not enough because then a person is unable to use those procedures flexibly in diverse cases. Additionally, as the acquisition of mathematical knowledge is cumulative, if a student does not understand the reason behind the algorithms and knowledge has been forgotten, it is difficult for that person to reconstruct that lost knowledge.[2] To illustrate this, imagine a farmer who learned about perimeter in elementary school. He remembers the formula $2l$ X $2w$=the perimeter. But he now owns a piece of property that is not perfectly rectangular. What good is it to him to know that formula if he does not know how to reconstruct the meaning of it?

Rationale

You may argue that mathematical proof has no place in the first grade classroom. If I were telling you that first graders would have to develop a formal, written proof for their work, then I would whole-heartily agree. But why not start them down the path of thinking in terms of proof? How can we get them to understand that subtraction is the inverse of addition, so that they do not try to take twelve objects from a set of seven (assuming we are working with only the set of natural numbers and zero)? That is, students may be able to understand subtraction by "proving with additions." If they are taught to think of what subtraction is and the meaning behind the algorithm, then I believe they will see when they have made such an error. Additionally, I think developing skills of discussion and argument at this young age, an understanding of modifying ideas, and collaborating to gain a solution are worthy of our time as these skills are not only useful in mathematical reasoning but across curricula.

I have found over the years that six year olds want to know "why" and, when there is hard work to be done, they are motivated to make full effort if the reason for it is meaningful to them. A researcher working on how students come to understand math indicated that what makes sense to the child has dominance over what is imposed as an extrinsic rule.[3] I have also learned that they like to talk - a lot. It is necessary to help them learn the importance of mathematical reasoning and what it means to prove a theory, because mathematics is not just each individual making sense of the world. The ideas and concepts that collectively make up Mathematics, are ideas and concepts that were discovered, explored, and evaluated by many people over centuries of time and now are understandings that are shared as a discipline with "practices and norms that are collective, not individual or idiosyncratic”.[4] Additionally, reasoning needs encouragement. Take the idea suggested by Cara Crosby in her curriculum unit that when children disagree during a game (like hide-and-seek) they may work together to come up with a new rule that changes or clarifies what they had in place originally. This is very similar to how mathematicians hash out theorems or scientists work to find conclusions to hypotheses.[5]

Overview

I am hopeful that this curriculum unit will be used to help young students realize that “why” is just as important as “how” when learning algorithms for addition and subtraction. I want this unit to instill this idea past first grade so that when these same students are working with multiplication/division, fractions, exponents, polygons, and angles they continue to look to make sense of the steps to concluding a result. All too often, we teachers are pushing our students to memorize facts and procedures and our students cannot transfer what they have learned to more complex problems.

This curriculum unit will be designed for teachers of first grade, however it may benefit struggling second graders. I will be teaching first grade students in a middle class, suburban community. The 2015 demographics of my class of twenty-four are: 58% male, 70% non-white, 30% children classified as learning English as a second language, and 87% benchmark in DIBELs (a measure of literacy). The 2014 demographics of my school with an enrollment over 700 are 33% African American, 33% White, 34% Other, 20% Low Income, and 80% or more meet expectations in both math and reading. Students at this school are expected to enter first grade being able to solve simple addition problems with sums to 10, being fluent with sums to five. By the end of the year, these same students are expected to master addition and subtraction of equations with sums to twenty. We use the Houghton Mifflin Harcourt Math Connects curriculum which is aligned with the Common Core State Standards. Some teachers are using Number Talks strategies to develop number sense. The children have math workbooks which are quite colorful, use pictures of manipulatives, but the program writers also have the students working rather quickly with abstract symbolism versus hands-on problem solving. Teachers at my school are expected to provide whole group instruction to introduce concepts, small-group instruction focused on individual needs, one-on-one check-ins (for RTI), and leveled, independent work within a 70 minute period.

My team and I have often discussed how our students struggle to learn subtraction facts when they seem to easily master addition facts. That is why I have chosen to focus this unit on three standards[6]: CCSS.MATH.CONTENT.1.OA.B.3, CCSS.MATH.CONTENT.1.OA.B.4, and CCSS.MATH.CONTENT.1.OA.D.7.

This curriculum unit is focused on the mathematical practice set forth by the Common Core curriculum that students in all grades levels should be able to construct viable arguments and critique the reasoning of others. In order to do that, students must be taught how to construct an argument, what constitutes a “viable” argument, and how to critique the reasoning of others in a positive and constructive way. They also need to be given time to discuss mathematical problems and understanding.

The activities found in this unit have been developed with the young child in mind. Therefore, they will be developmentally appropriate with an element of play (concrete and hands-on). The activities will be standards based, however there will be opportunities for divergent thinking because that is how discussion will be fostered. The students will also write about what they are learning, but they are not expected to record a “formal proof.” Again, with the unit, I am hopeful to develop in my students a way of thinking that will transfer to many different lessons (and content) throughout the year.

Content

An enduring understanding for anyone that teaches math is that the development of mathematical reasoning is a process rather than a skill that can be directly taught. This is not to say that teaching discrete skills is useless or less important than developing reasoning. Rather, what is one without the other? Since reasoning is a process, shouldn’t that process begin concurrently with teaching discrete skills? This can be simplified by simply asking young children “How do you know?” when they state an answer to a problem. Mathematicians spend lifetimes trying to answer this question: “How do you know a conjecture to be true?” or “not true?”

The methods used in school mathematics common proof types are direct proof, proof by exhaustion, proof by contradiction, existence proof, and proof by mathematical induction.[7] Direct proof tends to be more difficult as your arguments have to be so specific and logically based so as to ensure that there is no question that a statement is in fact true or false. Indirect proof is a bit easier to use because you simply have to prove one contradiction of a conjecture (or supposition) to be true and the original conjecture is deemed false. I have chosen to focus on indirect proof for the purposes of this curriculum unit as it may prove easier to help first graders understand the reasoning behind subtraction. You may have heard of the game of Elimination or finding one of a series of statements that doesn’t fit or belong to a set. In the study of vocabulary, you may have used Frayer models to help students understand vocabulary words. Two of the four “squares” would be “What it is” and “What it isn’t.” The “What it isn’t” part is called a counter example and we will use this throughout this unit.

Before we begin with concepts of indirect proof however, I will outline some concepts with respect to subtraction on which this unit will primarily focus. I will also share some key features of developing reasoning with young students and I will share the type of environment within which mathematical proof takes root.

It is important to know with which set of numbers our students work. In first grade we work with the set of natural numbers, also called the counting numbers and can be thought of the positive whole numbers. If we think of a number line each number on the number line is one unit from the number before it. For example, 1 is exactly one unit from the origin of the number line (also known as zero). 2 is exactly two units from zero and one unit from one, and so on to infinity. Unit is a very important concept for our students to understand, and using the number line can help to alleviate misconceptions later when these same students begin to learn about fractions.

With the natural numbers ($N = \{1, 2, 3, 4, \ldots\}$) we also work with 0. Zero is known as the identity element for addition. Our students need to understand that this is the set of numbers with which we are working. That is because there are certain properties that control what we can and cannot do with regards to addition and subtraction. Those properties are the Commutative Property, the Associative Property, the Additive Identity property, and the Distributive Property. When adding we have what is known as closure of the operation of addition. In other words, the answers that we get when using numbers within the confines of these properties are also a part of the set within which we are working (N and 0). However, there is not closure when we are working with subtraction. Take the commutative property for example. Stated simply, when two numbers are added, the sum is the same regardless of the order of the addends or, $a + b = b + a$. We cannot say the same for subtraction. Note if we use the values 2 and 4: $2 - 4 \neq 4 - 2$. The additive identity property states the sum of any number and zero is the original number. When subtracting this only works with when 0 is the subtrahend (the number that is begin subtracted from the minuend). For example, $4 - 0 = 4$. Our students need to understand these concepts so that they limit their errors when subtracting. There may be a way we can get our students to this understanding if we develop a “disciplined means of reasoning” before we begin using the algorithm for subtraction.

Researchers have found certain features of the mathematics classroom to be useful to developing reasoning skills. One way of thinking about reasoning is “productive struggle.” These features include using open-ended questioning, scaffolding reasoning, promoting predictions and encouraging multiple conjectures. Additionally, the teacher needs to help students to become skillful listeners and to teach students how to ask questions of their peers in a non-threatening, inquiry-based manner[8]. This last feature is most important. According to the authors, simply allowing students to discuss their thinking about a problem with one another fails to promote a strong mathematical community unless the students and teachers are carefully listening. The teacher’s role as facilitator should be to guide students to understand misconceptions without condemning or criticizing. Teachers need to learn how to restrain telling students what to do. Students also need to acknowledge that struggle is natural. Indeed, teachers need to ask leading questions that help students to develop their own ideas for solving problems or predicting outcomes. Students need to be able to formulate generalizations by struggling with a problem over sessions and to be allowed to justify their thinking and to consider how ideas can be expanded or shifted[9]. By providing opportunities for students to share multiple representations and communicate their ideas with peers, the teacher can encourage this discourse as well as gather data about her/his students' understanding[10].

In addition to the features of productive struggle, there are features of the setup of a mathematical classroom that will help a teacher be more successful in developing these critical thinking skills. Teachers need to allow time. Time should be used to pose problems or questions that may not be in our curriculum; problems that are open-ended and do not require one answer responses. These problems should require students to collaborate on their responses. This requires allowing students the necessary time. They need time to modify or strengthen original arguments. They need time to think about a problem, sometimes deferring collaborative discussion for days.[11] They need time to construct and share representations. Our school district has developed both problems of the day and problems of the week which could be used as the vehicle for developing this type of math class. The students should have some experience with collaborative strategies like think-pair-share and collaborative teams where each participant has a specific role. They need to be taught appropriate methods of sharing, namely that one person speaks while the other listens, then the listener restates what they have heard and either asks a clarifying question or elaborates on the original statement. Teachers can use interactive modeling in the beginning of the year to establish these norms of collaborative work.

Once the means of discussion are established we are then able to teach the process of reasoning. There is one thing to keep in mind when teaching students about proof. It is different than simply providing data and drawing a conclusion. "A mathematical proof comprises a logical argument with carefully stated assumptions, statements using precise language and definitions, and reasoning used to reach a valid conclusion."[12] There are also several different types of proof. A direct proof has a chain of statements, each of which follows logically from the previous one. A proof by exhaustion relies on checking all cases (usually with the aid of a computer). Proof by contradiction (or indirect proof) is simply proving that the negation of your original statement is true, therefore making the original statement false (or visa versa). An existence proof determines that a particular object actually exists, although the proof may not produce the object in question. There is also proof by mathematical induction, which is useful if there is a continuous pattern that you wish to prove. You simply have to prove the first two or three to be true and then state that the rest are true based on these first two or three proofs.

When one constructs a proof, it is important to understand that once something is proved, there are no counterexamples that contradict the proof. A proof is true in all circumstances under the conditions by which it was constructed. Thus, not only does understanding a proof constitute being able to recognize what is and what is not a proof, it also must include recognizing that a proof means that there are no exceptions from the proof.[13] When one constructs a proof, there is mathematical notation specific to proof so that anyone can read and try to understand the proof. P is used to represent one statement; Q is used to represent another. There is little English language used in the terms: if, then, suppose, true and false. These are terms even our youngest students can come to understand with practice. The notation will be unnecessary to use. However, it is important for the teacher to understand the mathematics behind the notation. We can make up new statements from old ones. If P and Q are statements, then we have:

Statement	Notation
P and Q	$P \wedge Q$
P or Q	$P \vee Q$
If P then Q (or P implies Q)	$P \Longrightarrow Q$
P if and only if Q (or P and Q are equivalent)	$P \Longleftrightarrow Q$
not P (or not Q)	$\sim P\ (or \sim Q)$

This is the distinct language that mathematicians use which differentiates mathematical proof from everyday language. Take as an example $y = x^2$: when doing algebra we know that depending on the values of x and y, this can be a true or false statement. But when doing mathematical proof, this statement can only be true or false because of the Law of the Excluded Middle. There are principles of proof which state that i) every statement is either true or false (the Law of the Excluded Middle) and, ii) no statement can be both true and false (the Law of Non-contraction). So, when constructing a proof, it will be necessary to make known the reasoning being used. For young mathematicians this can be done by modeling with manipulatives or drawings. Proving a conjecture (a statement to be proved) can be done by experimenting. As the experimenting moves forward it may become apparent that a counter-argument is proven to be true, and the original conjecture can then be proven false.

The process follows: you first make a statement or declarative sentence. Some examples: It is raining. I am a woman. $For\ x\ in\ N, x > 0\ is\ an\ even\ number$. These statements will either be true or false depending on the weather, who is making the second statement, or the value of $x\ in\ N$. Once stated, a conjecture is made as to the truthfulness of the statement. Take the statement,
$For\ x\ in\ N, x > 0\ is\ an\ even\ number$. I will suppose that this is a false statement. I then have several techniques I could employ to prove that my conjecture is accurate and that the original statement is false: guesswork, examples/counterexamples, use prior knowledge and brainstorm in a cooperative group. This is the experimenting noted above. As more information is gathered about the original statement, the students will better understand the process.

Classroom Activities

Following are five activities which may prove helpful to developing the kind of thinking we want our students to carry throughout their mathematical development. As stated above, teachers need to teach how to have a discussion, and more importantly how to listen to others.

Lesson #1 – Using Graphs as a Model of Subtraction

Enduring Understanding

When asked to compare to quantities on a graph (bar graph or pictograph) to figure out how many more or less one quantity has than the other, the student understands that subtraction is the most efficient method to finding the difference in the two quantities.

Essential Question

How can I use a graph to represent a compare subtraction problem?

Objective

Students will use graphs to compare quantities and determine which of two quantities is greater than or less than the other, and determine how many more or less of one than the other.

Materials and Setup

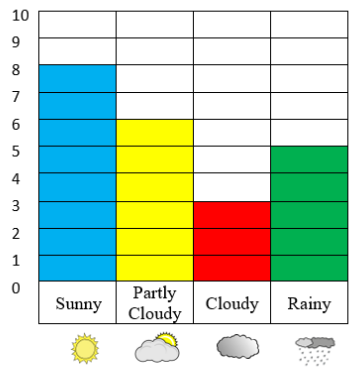
You will need images of completed graphs. It would be beneficial to use graphs that the students themselves have created. We use graphs throughout our school year, especially for weather. The students will need two different colored sets of interlocking cubes as counters. They will also need paper and pencil (or dry erase boards).

Procedure

Students have worked with graphs in kindergarten and first grade and they are most adept at figuring out which of the choices has the most. Often, they need much more practice with the question of which has the fewest (or fewer) but after many examples they pick this up pretty quickly. However, in my experience, when asked how many *more/less* of one *than* an another, most first graders will respond with the answer of which one of two has more or less, and not *how many* more or less.

For example:

How many more sunny days than partly cloudy days are there?



In order for the students to grasp the concept of how many more/than less than, the students will recreate the two bars using the two different sets of cubes. In the above example, one set of cubes would be an eight cube tower and the second set would be a six cube tower. Once the two towers are made, have them stand them up side by side. Ask which tower has more, which tower has less? Do this to confirm their understanding of more/less. Next, have the students talk about how the towers are alike. At first this is somewhat confusing because they will automatically think color. But give them a minute to decide that they both have six cubes. Then ask how they are different. Again, they will automatically think color. Once that is addressed, focus their attention on the number of cubes in each tower. They will conclude (eventually) that the taller tower has two extras or two more than the shorter tower. Make the connection with the language eight is two more than six. Explain that when asked "how many more of one than another" this is what we are asking. You will need to repeat this process many, many times before the idea becomes solidified with most first graders. You could use different questions using the same graph, or use different graphs.

Once you get a sense that your students are understanding the concept of "how many more/less than?" do the same procedure using only the graphs. How many more sunny days than partly cloudy days are there? The students should look at the graph (without the manipulatives) and talk about how the two bars are alike (they both reach the six line) and how they are different (sunny reaches the eight line and partly cloudy does not). Once they are comfortable with using the graph to compare, then show them how to figure out how many more. Show them that the number of spaces that sunny is above the partly cloudy is how many more. This may seem like it should be an easy understanding, but again, in my experience it requires many, many examples before they are independent with this skill.

Finally, once you have worked with the manipulatives and the graphs, and your students seem to be able to understand how to find how many more of one quantity than another, you can introduce subtraction equations into their understanding. Using the same graphs that you have worked with, like the example of sunny and partly cloudy, show them how to create the equation. In fact, I went back to the cubes to help them visualize the process. Show how the cubes are alike at six, and remove those six from the tower of eight. Ask the students to explain what you just did and they should respond that you took away cubes. Remind them that this is subtracting. So you subtracted six. Ask what did you subtract six from (eight). Then show them the equation $8 - 6 = 2$. Once again, do lots of examples. Revisit this process throughout the year and with ever increasing quantities.

Lesson #2 – Understanding “Truth”

Enduring Understanding

Students will understand the meaning of the words true and false.

Essential Question

How do I know a statement to be true or false?

Objective

This lesson will build the students mathematical vocabulary needed to understand mathematical proof.

Materials and Setup

You will need a large space for a large group of students to choose sides. You will also need to think of facts that students can determine if they are true or false. See Appendix D for ideas.

Procedure

Prior to this activity we worked on sorting activities using a Venn diagram. We talked about how things can be sorted into separate groups and sometimes an item might fit into two groups at once (hence the middle section of a Venn diagram). For this activity it is important to let your students know that they will only fit into one of two categories and that no one will be standing in the middle.

Tell the students that they will play a game in which they will learn that a statement can be either true or false, but not both at the same time. Explain the rules of the game: everyone will stand in a big group in the middle of the space. The teacher will read a statement and the students decide if the statement is true or false. If the statement is true, the students will move to one side of the space, and if the statement is false to the opposite side of the space. So if you were working at your meeting space, left could be true and right could be false (or front and back). Remind them they can only be on one or the other side of the space for each statement. It may be beneficial to have the students return to the middle after each statement is discussed. Also, to keep everyone on track for mathematical thinking, you may want to use a system for random selection of speakers (I use craft sticks with the students' names). This way it is not always the same children talking. Also, misconceptions may be more apparent and generate richer discussion.

Begin with non-mathematical statements. For example: "I am a girl". The girls would move to the true side of the space and the boys to the false. Talk about why this is. Throughout this activity the students should be noticing that the statement is true under certain conditions (in this case gender). Students return to the middle and state "I am 6". Again, the students will move to either side and discuss why they chose each side. The statements are pretty simplistic to begin. You want the students to get a clear understanding of how a mathematical statement must be decisive. After a few of these types of statements, you might try an ambiguous one, like I have short hair. There should be some confusion because this is not a decisive statement. The shortness of one's hair has to be quantified in order to make a decision and you should talk about this with your students. Return to clear statements starting to bring in some math. Hold up a picture with a number of items and state the quantity, state that "I have ten fingers", or "two ears", or a number of buttons on a shirt, etc.

Once you get a sense that the students understand the concept of truth, begin to work with mathematical equations (2+4=6; 10=3+7; 4+4=5+3; 7-4=3; as well as some false statements). If you have an interactive whiteboard or projector you could have these posted, but if not, you will have to write these large enough for all to see. After each, remember to discuss how the students know that these are true. It is necessary for the students to tell how they know each statement to be true throughout the game as it is building an understanding of truth (with regards to math), as well as helping the students to begin "to construct viable arguments and critique the reasoning of others" (one of the eight mathematical practices put forth by Common Core). Additionally, it is a good method for the teacher to start helping young students become better listeners of mathematical argument.

This game could take an entire math lesson, depending on the length of the discussions. You could use this game over a period of days, beginning with the non-mathematical on the first day, then the simple recognizing number patterns the next day and then finally equations on another day. This game could also be used throughout the school year as a transition to math instruction. You could choose to focus on addition only during your addition unit and then subtraction only during that unit. Finally, you can refer to this game when you are problem solving throughout the year. "Remember when we played the True/False sorting game, how we had to tell how we know something to be true?"

Lesson #3 – Representing Addition with Story Problems

Enduring Understanding

There are real life reasons for using addition.

Essential Question

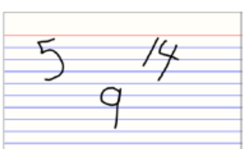
How do I use a story to represent addition?

Objective

Students will be able to show that they understand addition to be the process of joining sets that result in a larger set, using all the types of addition problems (see Appendix C).

Materials and Setup

You should group your students by heterogeneous ability into triads. They will need one pencil and paper per group. You should pre-program index cards with three numbers (two addends and the sum, e.g. 5 9 14). Use numbers that your children are working with at the time.



Procedure

Prior to this activity, students should have had some experience with reading and solving word problems (aka: story problems). According to Bloom's Taxonomy, creating is the highest order thinking, so students who have difficulty analyzing problems may struggle with this activity. Hence, the heterogeneous grouping. Before beginning this activity, I read Math Fables by Greg Tang to let the children hear the type of language that can be used with setting up story problems. After reading I told the students that they would think of real life situations where we would have to add. I asked for some ideas while we were in whole group and a few children shared these ideas: items that they eat, books in their chair bags, friends on the playground and pencils in the caddy.

Explain that the students will be given a card with numbers on them. The numbers are in no particular order, but two of them will be the addends and the third will be the sum (you will need to teach this vocabulary prior to this activity). The triads will work together to come up with a story problem that represents addition to share with the rest of the class. Assign these roles to each team member: the project manager will give a number from the card to each student in the triad, that is their part of the problem; the recorder is responsible for writing the story problem once the students come up with their story; and the reporter will be responsible for sharing the problem with the class once the project is complete. Remind the students that they must listen to one another and restate what they heard before making any argument for or against what they have heard. Give out the index cards to triads and give them five minutes to work on their story problems.

After five minutes regroup at the carpet and the reporters will take turns sharing out. With each story read, the class should be given a chance to discuss what they have heard. Facilitate the discussion by asking leading questions. “How did the three of you determine the order of the numbers?” should be a discussion throughout. If there is time, the students can be given more opportunities to work on other sets of numbers with their groups.

This activity can also be used in small group intervention. I took the stories and typed them into word problems representing the different types of join and part-part-whole problems. I place these in sheet protectors and placed them in the math activity center for early finishers. The students use wipe-off markers to solve the problems. We use the Math Connects organizer of UPSC (Appendix E) to solve word problems, so the students are able to check their answer by drawing a picture or using manipulatives.

Lesson #4 – Representing Subtraction with Story Problems

Enduring Understanding

There are real life reasons for using subtraction.

Essential Question

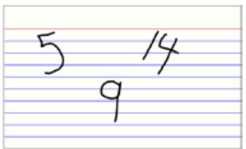
How do I use a story to represent subtraction?

Objective

Students will be able to show that they understand subtraction to be the process of separating a larger set into smaller sets, using all types of subtraction problems (see Appendix C).

Materials and Setup

This activity is set up exactly as the previous activity with a few differences. In fact, you can use the same index cards from the previous activity.



Procedure

I read Birdsong by Betsy Franco before beginning, without stating that we would be subtracting. After a few pages the students notice that the number of animals in the story are decreasing. Once they pointed this out, I reminded them that subtraction means that you will end up with a smaller number. This is key to helping them create subtraction stories correctly.

Once again, the students are grouped into triads, heterogeneously. These do not have to be the same groups as before. I like to mix my groups up frequently as the results are more dynamic if you are not always working with the same people. The roles are the same as before: project manager doles out each number, recorder writes the story, reporter shares out. The students work for five minutes to come up with real world subtraction stories. DO NOT CORRECT ERRORS!!! I cannot stress this enough. If a group creates a story with the largest number as one of the “addends” let them read it as is. Let the other students argue with them about this. Remember the research says when students create the understanding it sticks! [Notice I called the numbers addends. While this is not the correct terminology for these numbers, our first grade students will have an easier time knowing the smaller numbers as addends because of their work with addition. We do this at the risk of creating a misconception in later grades. The correct terms are subtrahend, minuend and difference. You might consider introducing the idea that what is considered the sum in addition becomes the subtrahend in subtraction. And the “other” number is the minuend. I can assure you that maybe one of your students might remember these terms. You should absolutely teach that the result of subtracting is the difference, as you can use this language with compare problems!]

After five minutes, call the class back to whole group and the reporters take turns sharing the story problems. Before the discussion begins it will be important to remind the class of how to make a mathematical argument. First, we must listen carefully so that we can understand the problem. If we have an argument against a point, we restate what we heard the person say and then tell why we disagree with the statement. This is very important to go over because there will be errors and the students will want to yell out what they think the second they hear an error. Once again, teacher plays facilitator asking those leading questions. This time, it would be prudent to allow the students to return to their triads to try a second story problem. This can be used as a formative assessment, as the discussion generated would bring to light the common errors that students make when creating subtraction equations. Having the students try again, would ensure that they had an immediate opportunity to adjust their thinking.

You may wish to take the students stories and show them the equations that can be written to represent their stories. Then in subsequent practice the students can create their own equations. This is an important step, as this process of transferring understanding of concept to representation with equations is where a lot of the mistakes happen. Also, this activity can be used for small group intervention and once again as independent work if typed up (see Lesson #3).

Lesson #5 – Guess My Number

Enduring Understanding

Deductive reasoning can be used to isolate an unknown number.

Essential Question

How do I use what I know to figure out what I don’t know?

Objective

Students use number sense and operations along with mathematical reasoning to find an unknown number. They will make conjectures which will be proven correct or incorrect.

Materials and Setup

There is a mass-produced game called “Guess Who?” where the players have to ask a series of questions to reduce the number of choices in order to figure out the other player’s identity. There is also a mass-produced game called “What’s My Number?” which is the same game with numbers. This game is easily reproduced for the classroom in several ways. Here are two:

Guess My Number (using the calendar).
You need a calendar displayed for all to see.

Procedure

Choose a number which can be found in the current month. Do not tell the students what the number is. Give them clues that will help them isolate the number. For example, “I am thinking of a number which is the sum of a double and between 10 and 20.” The numbers 12, 14, 16 and 18 are possibilities, so the students have to ask a question about one of those numbers in order to isolate which number it is. Questions they can ask are about its addends, about more/less than, if I add or subtract ____ will I get ____, if I subtract 10 do I get ___, etc. These must be questions that can be answered with yes or no (like 20 questions). Once a student thinks they have the correct answer they can say I think your number is _____. I have a SMART board so it is easy for me to cross out numbers once questions are asked or answered. If you do not have this luxury, you could use Post-its to cover over numbers on a calendar. Or if you have a projector, you could use a copy of the calendar in a sheet protector and use a wipe-off marker to cross out numbers. If you play this game frequently enough the students can begin to choose a number and have others ask questions of them.

Guess My Number (board game)

You need a chart with the numbers 1-50 or you can use a 100's chart once your students get comfortable with the game.
Cards numbered 1-50 (or 100) (for keeping the game honest)
You need a set of questions for the student guessing to use (See Appendix F)

Procedure

Two students can play this game as an independent (early finisher) activity. They can take turns being the guesser. I introduced this game to the whole class, after we used the calendar version for a few months.

The students place the chart in between them. One partner chooses a card from the stack of mixed up number cards. The second partner chooses a question card from the stack of questions. S/he asks the first partner the question. The first partner can refer to the chart in order to answer the question correctly. The second partner uses a dry erase marker to cross off any numbers that do not meet the criteria set forth by the answer. Play continues with the second partner choosing a question card and crossing off numbers until the first partner's number is revealed. Partner One confirms the guess by showing the number card. If time, partners can switch roles.

Appendices

Appendix A

Common Core State Standards addressed throughout this unit:

Mathematics:

CCSS.MATH.CONTENT.1.OA.B.3
Apply properties of operations as strategies to add and subtract.[2] Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)

CCSS.MATH.CONTENT.1.OA.B.4
Understand subtraction as an unknown-addend problem. For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.

CCSS.MATH.CONTENT.1.OA.D.7
Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

Appendix B

Properties of Addition

There are four mathematical properties of addition:

Commutative property states when two numbers are added, the sum is the same regardless of the order of the addends. Ex.: $5 + 3 = 3 + 5$

Associative property states when three or more numbers are added, the sum is the same regardless of the grouping of the addends. Ex.: $(1 + 2) + 3 = 1 + (2 + 3)$

Additive Identity property states the sum of any number and zero is the original number. Ex.: $12 - 0 = 12$

Distributive property states the sum of two numbers times a third number is equal to the sum of each addend times the third number. Ex.: $4 * (2 + 4) = 4 * 2 + 4 * 4$

Appendix C

Types of Addition & Subtraction Problems

PROBLEM TYPE

Join	*Result Unknown* L had four candies. She bought two more. How many candies does she have now? $4 + 2 = ?$	*Change Unknown* L had four candies. She bought some more candies. Now she has six candies. How many candies did L buy? $4 + ? = 6$	*Initial Quantity Unknown* L had some candies. She bought two more candies. Now she has six candies. How many candies did L have before buying more? $? + 2 = 6$
Separate	*Result Unknown* N had ten cars. He lost three cars. How many cars does N have now? $10 - 3 = ?$	*Change Unknown* N had ten cars. He lost some cars. Now he has 7 cars. How many cars did N lose? $10 - ? = 7$	*Initial Quantity Unknown* N had some cars. He lost 3 cars. Now he has 7 cars. How many cars did N have before he lost them? $? - 3 = 7$
Part-Part-Whole	*Whole Unknown* Six cats and five dogs are in our neighborhood. How many cats and dogs are in our neighborhood? $6 + 5 = ?$	*Part Unknown* 11 cats and dogs are in our neighborhood. Six are cats and the rest are dogs. How many dogs are in our neighborhood? $6 + ? = 11$	
Compare	*Difference Unknown* Our class has 10 girls. Mrs. M.’s class has 7 girls. How many more girls does our class have than Mrs. M.’s? $10 - 7 = ?$	*Larger Quantity Unknown* Mrs. M.’s class has 7 girls. Our class has three more girls than Mrs. M.’s. How many girls are in our class?	*Smaller Quantity Unknown* Our class has three more girls than Mrs. M.’s. Our class has 10 girls. How many girls are in Mrs. M.’s class?

	or $7+?=10$	$7+3=?$	$?+3=10$ or $10-?=7$

Appendix D
A List of Possible Questions for the True/False Game

I am a boy.
I am seven.
I have a pet.
I have a fish.
I am wearing pants.
I am in a school.
I live in an apartment.
I ate breakfast.
It is raining outside.
The lights are on in our classroom.
All of the lights are on in our classroom.
I come to school on a bus.
I go home on a bus.
I have long hair.
I am short.

There are 10 girls in our class.
There are a lot of girls in our class.
There are five different colors in our carpet.
There are six tables in our classroom (the students need to define what a table is).
This is a set of seven (present a group of 7 (or not) items).
This is a set of twelve (using a ten frame and two more to make this easier to respond to quickly).
This is a set of nine (same procedure).
This is two sets. (First present an image of two sets of items far enough apart to see two separate sets. Then put the sets close enough together that students may consider it one, in order to generate discussion).
This is three sets.

3+3=6
3+2+1=6
6=0+6
5+1=0+6
5-1=4

4=1-5
4=5-1
4+?=7
12=?+7
5+2=4+2
4+3+2+1=11

Appendix E
Math Connects UPSC Organizer

Problem Solving Worksheet

Step 1: Understanding the problem:

- This is the information I know from the problem:

- This is what the problem is asking me to find:

Step 2: Making a plan:

1) These are the steps I will use to solve the problem:

Draw a picture --- Find a Pattern --- Write a Number Sentence --- Make a Table, a Graph or a List

Act It Out --- Work Backwards --- Guess and Check --- Solve a Simpler Problem

Step 3: Solving the problem:

2) This is how I solved the problem (show your work):

Step 4: Checking my answer:

1) The strategy I used makes sense?	Yes	No
2) I can explain why my answer makes sense?	Yes	No
3) I checked to make sure I added, subtracted, multiplied, or divided correctly?	Yes	No

Appendix F
Guess My Number Board Game Question Cards

Is your number the sum of a doubles fact? (even)	Is your number the sum of a doubles plus one fact? (odd)	Is your number 10 less than _____?
Is your number in the _____ column?	Is your number in the _____ row?	Is your number near _____?
Does your number have ____ tens?	Does your number have ____ ones?	Is your number less than < _____?
Is your number between ____ and _____?	Is your number 10 more than _____?	Is your number greater than > _____?

Resources

Student Resources

Crews, Donald. *Ten Black Dots*. New York: Greenwillow Books, 1986.
Curry, Don L. *More Bugs? Less Bugs?* Mankato, Minn.: Capstone Curriculum Pub., 2000.
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Greydanus, Rose, and Roland Rodegast. *Double Trouble*. Mahwah, N.J.: Troll Associates, 1981.
Merriam, Eve, and Bernie Karlin. *12 Ways to Get to 11*. New York: Simon & Schuster Books for Young Readers, 1993.
Murphy, Stuart J., and G. Brian Karas. *Elevator Magic*. New York, NY: HarperCollins,

1997.
Pluckrose, Henry. *Sorting*. Chicago: Childrens Press, 1995.
Samton, Sheila White. *Ten Tiny Monsters*. Crown Publ. U.S., 1998.
Tang, Greg, and Heather Cahoon. *Math Fables: Lessons That Count*. New York: Scholastic Press, 2004.
Wise, William, and Victoria Chess. *Ten Sly Piranhas: A Counting Story in Reverse, (a Tale of Wickedness-- and Worse!)*. New York: Dial Books for Young Readers, 1993.

Teacher Resources

Ball, Deborah L., and Hyman Bass. "Making Mathematics Reasonable in School." In *A Research Companion to Principles and Standards for School Mathematics*, 27-44. Reston, VA: National Council of Teachers of Mathematics, 2003.
I like the analogy of reasoning in mathematics to that of comprehending when reading. The authors equate rote memorization of facts and algorithms to knowing how to decode and read words. Without comprehension of the sentences made up of those words, what's the point of knowing how to read the words? Thus, without being able to reason through mathematics, what's the point of knowing the facts and algorithms? You won't know the purpose for them and you won't be able to use the information they provide.

Berenson, Sarah, Tyrette Carter, and Kerri Richardson. "Connected Tasks: The Building Blocks of Reasoning and Proof." Australian Primary Mathematics Classroom 15, no. 4 (2010): 17-23.
This article gave many examples of activities to use with young children which encourage critical thinking and open-ended questions to move students toward having a better number sense.

Burns, Marilyn. "The Role of Questioning." *Arithmetic Teacher 32*, no. 6, (1985): 14-16.
This article gave insight into the process of reasoning and how a teacher can help his/her students develop their mathematical understanding through that process

The Park City Mathematics Institute at The Math Forum. Nature and Role of Reasoning and Proof. Retreived from http://mathforum.org/pcmi/nature11.05.07Final.pdf. Not dated.

Carpenter, Thomas P., James M. Moser, and Thomas A. Romberg. *Addition and Subtraction: A Cognitive Perspective*. Hillsdale, N.J.: L. Erlbaum Associates, 1982. 4, 23, 28-29.
To a certain degree this book used quite a bit of technical language so it took a good deal of focus and rereading, however, the summary paragraphs at the end of each chapter helped simplify some very important information for teachers who deliver math instruction to early elementary students. After reading this book, I can see why the NCTM and Common Core developers have changed the way math is taught.

Additionally, the first few chapters are enlightening for teachers to realize just how much brain power is required of a six or seven year to learn each math concept.

Chazan, Daniel, and Loewenberg Ball, Deborah. *Beyond exhortations not to tell the teacher's role in discussion-intensive mathematics classes*. East Lansing, Mich: National Center for Research on Teacher Learning, Michigan State University. (1995)

Crosby, Cara. *What Goes in the Middle? A Curriculum Unit on Axioms, Conjectures and Logical Thinking*. http://www.tip.sas.upenn.edu/curriculum/units/2009/01/09.01.02.pdf (2009)
A Yale National Initiative curriculum unit.

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Lawler, Robert W. "The Progressive Construction of Mind." *Cognitive Science* 5, no. 1 (1981): 1-30.

Maher, Carolyn A., and Amy M. Martino. "The Development of the Idea of Mathematical Proof: A 5-Year Case Study." *Journal for Research in Mathematics Education* 27, no. 2 (1996): 194-214.
The researchers followed a student from first to fourth grade and observed how she came to grow a mathematical mind.

Warshauer, Hiroko K. "Strategies to Support Productive Struggle." *Mathematics Teaching in the Middle School* 20, no. 7 (2015): 390-93.
An article which provides a process to helping students be successful with struggling through problem solving.

Notes

[1] Carpenter, Moser, and Romberg. *Addition and Subtraction: A Cognitive Perspective*. (Hillsdale, N.J., 1982).
[2] Ball and Bass. "Making Mathematics Reasonable in School." In *A Research Companion to Principles and Standards for School Mathematics*. (Reston, VA, 2003).
[3] Lawler. The progressive construction of mind. (1980)
[4] Ball and Bass, 2003.
[5] Crosby. *What Goes in the Middle? A Curriculum Unit on Axioms, Conjectures and Logical Thinking*, (Yale, NH, 2009).
[6] http://www.corestandards.org/Math/
[7] Cai, Jinfa. Nature and Role of Reasoning and Proof
[8] Berenson, Carter, and Richardson . "Connected Tasks: The Building Blocks of Reasoning Proof." (2010)

[9] Burns, Marilyn. "The Role of Questioning." (1985): 14-16.
[10] Chazan and Ball. *Beyond exhortations not to tell the teacher's role in discussion-intensive mathematics classes* (1995)
[11] Maher and Martino. "The Development of the Idea of Mathematical Proof: A 5-Year Case Study." (1996)
[12] Cai, 2014
[13] Cai, 2014

Curriculum Unit Title	First Graders Have Something to Prove	Author	Janet B. Zegna

KEY LEARNING, ENDURING UNDERSTANDING, ETC.

Students must learn to justify why a particular mathematical statement is true (or not) or from where a mathematical rule comes. They need to be able to show their understanding of mathematical rules. They need to be able to argue their reasoning when problem solving.

ESSENTIAL QUESTION(S) for the UNIT

How can I use models and representations to show that I understand number sense and the properties of addition and subtraction?

CONCEPT A	CONCEPT B	CONCEPT C
Use a Graph as a Model of Subtraction	Develop an understanding of truth	Represent problems involving addition
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**	**ESSENTIAL QUESTIONS C**
How can I use a graph to represent a compare subtraction problem?	How do I know a statement to be true or false?	How do use a story to represent addition?
VOCABULARY A	**VOCABULARY A**	**VOCABULARY A**
compare, set "more than", greater "less than", lesser	Statement, prove mathematical statement, proof true false	addend, sum +, =

ADDITIONAL INFORMATION/MATERIAL/TEXT/FILM/RESOURCES

various manipulatives for counting

prepared graphs

CONCEPT D	CONCEPT E
Represent problems involving subtraction	Apply properties to draw conclusions
ESSENTIAL QUESTIONS A	**ESSENTIAL QUESTIONS B**
How do I use a story to represent subtraction?	How do I use what I know to figure out what I don’t know?
VOCABULARY A	**VOCABULARY A**
difference - compare	conclusion